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Theory Biosci. (1999) 118: 261–266

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Nonlinear time-series modelling by means of self-exciting threshold AR models

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Received: November 8, 1999; accepted: November 23, 1999

Key words: SETAR models, estimation, nonlinear dynamics, EEG, epilepsy

Summary: Self-exciting threshold autoregressive (SETAR) modelling has been used as an additional tool in nonlinear analysis of EEG data recorded during typical absence seizures in children suffering from childhood absence epilepsy. Firstly a SETAR model was fitted to the data by means of a novel adaptive estimation strategy. Secondly the real EEG data were compared with wave-forms generated from the resulting model. It was demonstrated that impulse responses (with infinite length) of the SETAR model reconstructed the TAS pattern with remarkable accuracy. Real and model EEG as well as their SW-shuffled surrogate signals were compared with respect to the autocorrelation function, correlation dimension, pointwise dimension and the largest Lyapunov exponent.

1. Introduction

The autoregressive (AR) model is very popular in the analysis of time series since its theoretical properties are thoroughly investigated and a number of efficient estimation procedures exist. Furthermore, a transition from time-domain to frequency-domain analysis can be easily performed by means of a z-transformation of the model coefficients. Nevertheless, the application of AR models to real data is limited for at least two reasons: First, such models are only suitable for analysing stationary processes. Second, their application is restricted to processes generated by linear systems.

The observation that linear models frequently fail to describe important properties of time series from many fields has led to the development of several nonlinear alternatives. At present, research interest is focused on the investigation of four particular nonlinear models: Bilinear models, threshold models, exponential autoregressive models and state-dependent models.

Despite the fact that the latter is the most general one there are several facts which motivate the use of SETAR models in practical applications:

- A wide class of non-linear time series which includes the exponential autoregressive and the invertible bilinear models can be approximated by threshold models (Petruccelli 1992).
- Compared with other non-linear models SETAR models are more easily interpretable and tractable.
- Patterns which are characteristic of non-linear systems such as oscillations with amplitude-dependent frequencies, (asymmetric) limit cycles, jump resonances and synchronization phenomena can be generated by means of SETAR models (Tong 1983).
- Many systems from the real world show saturation characteristics which can be modelled using thresholds.

Since SETAR models are especially useful to model nonlinear quasiperiodic oscillations it seems to be promising to apply this type of model in the analysis of spike-wave patterns in the EEG. A distinct type of such patterns occurs during typical absence seizures (TAS) in children suffering from childhood absence epilepsy (CAE). The EEG of such children is characterised by the sporadic emergence of clear rhythmic oscillations of high amplitude where a spike of high frequency is followed by a slow wave of lower frequency. Such periods of absence typically have a duration of about 10 seconds (ILEA 1981).

The aim of this study has been to get a new insight into the nonlinear structure of these EEG patterns using this novel modelling technique. This was mainly achieved by the comparison of the real EEG data with modelled data using the real data as the target signal.

2. Methods and Materials

The model. A SETAR model is given by a collection of AR models and a corresponding number of thresholds which define a partition of the real axes. At each instant one AR model is chosen to be active, i. e. it is assumed to generate the corresponding observation. The active regime is determined by detecting the interval which covers the observation at a defined time point in the past. This definition can be formalized as follows:

Let $\mathbb{R} = \bigcup_{j=1}^{l} R_j, R_j = (r_{j-1}, r_j), -\infty = r_0 < r_1 < \ldots < r_l = \infty$ be a disjunctive decomposition of the real axes and $d, p_1, p_2, \ldots, p_l$ positive integers. The process $(Y_n)_{n>0}$ with

$$Y_n + \sum_{j=1}^{l}\left[Y^{(j)}_{n-d}a_0^{(j)} + \sum_{i=1}^{p_j} Y^{(j)}_{n-d}a_i^{(j)}Y_{n-i}\right] = \sum_{j=1}^{l} Y^{(j)}_{n-d}\varepsilon_n^{(j)},$$

where $Y_n^{(j)} = \chi_{R_j}(Y_n)$ and $(\varepsilon_n^{(j)})_{n>0}$ are white noise sequences, is called a SETAR$(l, p_1, \ldots, p_l)$ process with delay parameter d.

Estimation. The estimation of the model parameters can be performed in two steps: First, estimation of structural parameters (model order, position of thresholds, delay parameter) and second, estimation of autoregressive parameters. Known estimation procedures use classical approaches such as minimization of least squared prediction errors or maximization of conditional density functions (Bayes estimation). Such techniques are described in Tsay (1989), Geweke and Terui (1993), Broemeling and Cook (1992) and Chan (1993).

In our application we used a procedure developed by Tsay (1989), which includes a test of threshold nonlinearity and the determination of thresholds by visual inspection of various scatterplots. Estimation of the AR order was performed by means of AIC.

The resulting structural parameters enable the identification of the regime sequence. This makes it possible to apply techniques from the linear community to estimate the autoregressive coefficients within each regime. One efficient approach is the adaptive estimation of coefficients via the so-called LMS algorithm (Günther 1983). We modified this technique in such a way that it can be used to adaptively estimate the AR coefficients of SETAR models if the structural parameters are known (Arnold and Günther, 1998). This method has the advantage that it can also be efficiently applied in time series with nonstationary characteristics.

Signal analysis. After fitting the model to the EEG data, impulse responses of the resulting nonlinear system were used as modelled EEG. Real and modelled EEG as well as their SW-shuffled surrogate signals were compared with respect to the autocorrelation (ACF), correlation dimension D_2, pointwise dimension D_p and the largest Lyapunov exponent LLE.

To obtain a surrogate time series the SW cycles were separated at the beginning of each spike, i. e. where the rising part crossed an appropriate threshold. Then the resulting blocks of spike-wave complexes were randomly arranged.

EEG recordings. EEG raw data (Schwarzer Picker ED 24 electroencephalograph, bandwidth set at 0.3–70 Hz, 19 scalp electrodes placed in standard 10/20 montage positions, referenced to average ears, impedance kept below 5 kΩ; data were digitised at 256 Hz) were collected from the data base of the Department for Seizure Disorders at the Vienna University Clinic of Neuropsychiatry for Children and Adolescents. Data were selected from recordings of 5 children. The channel with the most pronounced SW rhythm in conjunction with high-amplitude spikes was selected for detailed investigation of dynamcis and as a target signal for modelling. A detailed description of data and signal analysis can be found in Feucht et al. (1998).

3. Results

By means of the procedures described in section 2, a SETAR model with 5 thresholds ($l = 6$ regimes), a delay $d = 2$ and an order $p = 20$ was identified. These structural parameters determine the regime sequence. The model estimation was completed by an adaptive estimation of AR coefficients within the regimes. To obtain a better approximation the data sequence was processed twice where the estimates at the end of the interval were choosen as initial values of the second run.
The resulting nonlinear system was investigated by analysing impulse responses. It turned out that the impulse response of the system reconstructed the signal with high accuracy. Furthermore, it was interesting to observe that a short impulse led to a neverending excitation of the system. This way it was possible to generate SWD patterns with an infinite duration (Fig. 1).
Comparison of parameters which describe linear or nonlinear properties for the real and modelled EEG produced the following results:
The ACFs of the EEG signal and its surrogates are considerably different. Consequently this signal could be a realization of a nonstationary process. This is not the case for the modelled signal.
The pointwise dimension of the EEG was 3, that of the modelled signal 2. This can be interpreted in such a way that the EEG signal reflects three in-

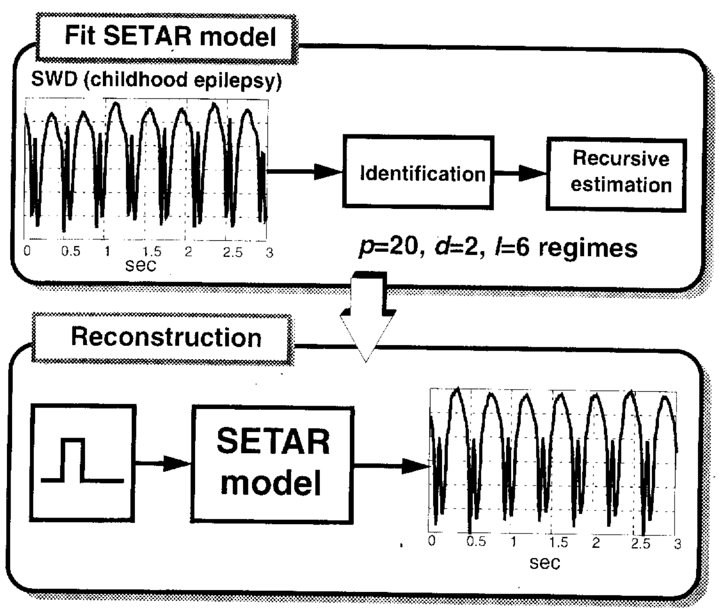

Fig. 1. Modelling of a TAS by means of SETAR models. The two waveforms are part of the input EEG (above) and part of the reconstruction by means of an impulse response, respectively.

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Investigation of LLE revealed no evidence of chaos in either signal. It was interesting to find that the LLE of the simulated signal was clearly smaller than the LLE of its surrogates. This indicates that the SETAR model possesses some deterministic nonlinear dynamical structure which is lost after randomization.

4. Discussion

SETAR models are useful to model systems with saturation nonlinearity or processes with excitatory and inhibitory states. They are able to generate oscillations with typical nonlinear properties which can be frequently observed in physiological recordings.
A crucial task in the estimation of SETAR models is the determination of the structural parameters, i. e. delay and thresholds. Methods exist which can be used to solve this problem but they are not practical in many situations, especially in the analysis of huge data sets. Once the structural parameters are identified techniques from linear modelling can be used to estimate the AR coefficients within each regime. An adaptive estimation is possible by means of a stochastic gradient procedure. This approach works well also in situations where the data show a mild nonstationary behaviour.
It was demonstrated that SETAR processes can be used to model the dynamics of SW patterns in the EEG during TAS of children suffering from CAE. The resulting system is able to reproduce the dynamics of this special pattern. By means of impulse responses it was possible to generate patterns with inifite duration which approximate the original waveform with high accuracy.
Comparison of descriptive parameters of real and modelled EEG was helpful to interprete the linear and nonlinear characteristics of the data. The surrogate data set made it obvious that the model SW sequence involves nonlinear structure (i. e. relationship over more than one SW cycle, not represented by the ACF). This time series seems to reflect a stationary process in the statistical and the dynamic sense. This is indicated by the invariance of the ACF with respect to SW shuffling and the invariance of the spatial extent of the reconstruction set with respect to the signal length. The estimated dimension of 2 for the modelled EEG suggest that two oscillatory modes are necessary in order to reproduce the SW morphology of the EEG adequately. Thus, a dimension of 3 is the expected result for the EEG because it additionally exhibits an organized alteration of the patterns possibly associated to a third variable.

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Theory Biosci. (1999) 118: 267–2
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Received: October 28,

Key words: Neurodyr

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1. Introduction

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