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Abstract. *The present article offers a careful description of empirical identification of possible multiple changes in regime. We apply recently developed tools designed to select among regime-switching models among a broad class of linear and nonlinear regression models and provide a discussion of the impact on the formation of inflation expectations in the presence of multiple and recurrent changes in inflation regimes. Our empirical findings give a plausible explanation as to why the rational-expectations hypothesis based on direct measures of inflation expectations from survey series is typically rejected because of large systematic differences between actual and expected inflation rates. In particular, our results indicate that in the case of changing and not perfectly observed inflation regimes, inference about rationality and unbiasedness based on a comparison of ex ante forecasts from survey series and actual inflation rate based on ex post realizations will be ambiguous because of the presence of an ex post bias. The empirical findings are based on Danish inflation rates covering 1957–1998. We show that it is not possible to reject the hypothesis of multiple inflationary regimes and that the actual inflation rate can be represented by a two-state Markov regime-switching model. It turns out that the real-time forecasts produced from this model exhibit a large degree of similarity when compared to the direct measures of inflation expectations. The result illustrates the important impact of switching regimes on the formation of actual and expected inflation and hence of ex post bias as a main contributor to the difference between actual and expected inflation observed directly from survey series.*

Keywords. ex post bias, inflation, inflation expectations, Markov regime switching

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1 Introduction

It is by now well established that expectations about future inflation matter in many different macroeconomic contexts. Savings and investments decisions are well-known examples. The same is true of wage formation according to, for example, the expectations-augmented Phillips curve, and nominal interest rates should depend one-to-one on expected inflation according to the Fisher equation. Contrary to the monetary-business-cycles theory of Lucas focusing on expected actual rate of inflation, several new-Keynesian models predict a relationship between economic activity and the difference between actual and expected future inflation (see, e.g., Roberts 1995). In a large number of empirical studies, notably Phillips curve estimations, expected inflation is often proxied by current and lagged levels of inflation as suggested by simple backward-looking expectation formation. In many contexts, however, this approach turns out to be too crude, and more proper measures of expected inflation are warranted, for example, in accordance with rational, forward-looking expectations.

In many countries survey-based data on expected future inflation exist and hence constitute a natural approach to the problem. In this context perhaps the two most intensively studied survey series on inflation expectations are the Livingston price expectations data and the Survey Research Center (SRC) consumer survey data at the University of Michigan (see Rich 1989 and 1990 and the references therein). In general, the results on rationality/unbiasedness of expectations from these studies based on comparisons with actual inflation have been mixed. Engsted (1991) and Christensen (1996) examine survey-based studies of Danish households' inflation expectations. They find that inflation expectations have been markedly biased over extended periods. In particular, they find that the inflation expectations tend to overpredict actual inflation in periods with low inflation, especially in the 1990s, and underpredict in periods with high inflation, as in the 1970s, for example. These features also seem to be present in the Livingston and the SRC survey series, according to Evans and Wachtel (1993). This of course casts doubt on the rational-expectations hypothesis in the sense that agents persistently ignore relevant information. Evidence of bias and serial correlation in expectation errors does not, however, definitively refute the hypothesis of rationality, as pointed out by Jonung and Laidler (1988) and Evans and Wachtel (1993), who refer to "peso problems" and "possible changes in regime." They argue that if the inflation rate evolves according to a single regime for a sustained period, but households expect a regime shift in the inflation process with a nonzero probability, then expectations might actually be persistently biased and serially correlated.

The observed persistence in the difference between ex ante forecasts and ex post realizations will be denoted ex post bias in this article. In order to determine whether the observed wedge between the perceptions and expectations formed by the household sector could be due to ex post bias, we offer here a careful examination of the stochastic process underlying the actual inflation rate process. The basic premise of our study is that if it is not possible to reject the hypothesis that the actual inflation rate follows a regime-shifting model capable of producing a wedge between perceptions and expectations of the magnitude actually observed, then it is not possible to conclude—as is often done in the literature—that the household sector's inflation rate expectations are formed in contradiction to the rationality hypothesis. Another strand in the literature on the inflation process (cf., e.g., Baillie, Chung, and Tieslau 1996) emphasizes fractional integration as an explanation of long memory, that is, that shocks to the inflation rate have persistent, though not permanent, effects. As established by Diebold and Inoue (forthcoming), however, a regime-switching model in which only few shifts occur is easily confounded with a long-memory model, and furthermore, a long-memory process and a regime-switching process need not be considered alternatives, but rather two sides of the same coin.

Like Evans and Wachtel (1993) we use the Markov regime-switching approach as a workhorse in order to quantify the ex post bias. However, we elaborate extensively on their approach. First, we present numerous tests for identification of instability of the linear forecasting model. Second, we provide a battery of specification tests, ranging from tests based on flexible regression models, such as the neural network test of White (1992) and the test recently suggested by Hamilton (2001), to the more well-known Reset test, to identify nonlinear components in the inflation rate. Third, having identified the need for a nonlinear component in the regression model, possibly in the shape of a regime-shifting model, we test the Markov regime-switching model specification against the linear model using the test proposed by Hansen (1996). Fourth, using the approach recently suggested by Dahl and Hylleberg (1999) based on the flexible parametric regression model of Hamilton (2001) and the nonparametric projection pursuit regression model of Aldrin, Boelviken, and Schweder (1993), we are able to evaluate the adequacy of the Markov regime-switching model against a wide range of competing nonlinear representations of the inflation rate. Fifth, using the Bayesian approach suggested by Hamilton (2001), we provide evidence that the nonlinearity in the conditional mean function produced by the Markov regime-switching model is in accordance with the data. Finally, we test the Markov regime-switching model for parameter stability and for dynamic misspecification. Contrary to Evans and Wachtel (1993), who use *in-sample* forecasts, we suggest using *real-time* forecasts produced from the Markov regime-switching model when evaluating the size of the ex post bias. Using our approach we are able to produce a forecast based on the same information set as that of the household sector when it forms expectations. Our approach therefore constitutes a more natural means of comparison.

We find that the exact real-time forecasts produced from the Markov regime-switching model exhibit a high degree of similarity when compared to direct measures of inflation expectations. In particular, our results show that one-year-ahead expectation errors generated from the well-specified regime-switching model do exhibit severe bias and serial correlation, as do expectation errors based on survey data. We believe that the result illustrates the important impact of possible regime switches on the formation of actual and expected inflation and of ex post bias as a significant contributor to the difference between actual and expected inflation observed directly from survey series.

The article is organized as follows. In the next section we briefly discuss how the existence of unobserved shifts in regimes can lead to a persistent gap between actual and expected inflation. Against the background of existing studies on survey-based measures of Danish households' inflation expectations, we discuss common pitfalls and in particular why these studies tend to lead to a rejection of the rational-expectations hypothesis. In Section 3 we test the rational-expectations hypothesis based on the linear forecasting/regression model and actual inflation rates in Denmark, and in Section 4 we present and carefully test the Markov switching forecasting/regression model. Section 5 contains economic interpretations of the results, and Section 6 concludes.

2 Preliminaries

2.1 Inflation forecasting in the presence of changes in regime: The notion of ex post bias

The consequences for the inflation rate forecast of a switch in regime can most easily be illustrated by a simple example with two inflation regimes. Let us assume that the inflation rate π_t evolves over time according to the following rule

$$\pi_t = (1 - S_t)\pi_{0t} + S_t\pi_{1t} \tag{1}$$

where S_t is a dichotomous unobserved stochastic variable that can take only the values zero and one. Let us for simplicity assume that π_{0t} and π_{1t} are stochastic variables, independent of the current realizations of S_t. The interpretation of the above equation is that if $S_t = 1$, the inflation rate is said to be in regime 1, where it is governed by the process of π_{1t}. Otherwise, if $S_t = 0$, we say that the inflation rate is in regime 0 and is determined solely by the development in π_{0t}, which is supposed to be different from that of π_{1t}. If we assume that expectations are formed rationally and we let Y_{t-1} denote all the available information at the beginning of period t, the expected inflation rate conditional on $S_t = i$ for $i = \{0, 1\}$ is given by

$$E(\pi_t \mid S_t = i, Y_{t-1}) = E(\pi_{it} \mid Y_{t-1}) \tag{2}$$

Letting $P(S_t = 0 \mid Y_{t-1})$ and $P(S_t = 1 \mid Y_{t-1})$ denote the probability measures of being in regime 0 and regime 1, respectively, the unconditional[1] expected inflation rate can be found by summation over all the possible realizations of S_t. This yields the following expression for the expected inflation rate:

$$E(\pi_t \mid Y_{t-1}) = P(S_t = 0 \mid Y_{t-1})E(\pi_{0t} \mid Y_{t-1}) + P(S_t = 1 \mid Y_{t-1})E(\pi_{1t} \mid Y_{t-1}) \tag{3}$$

and the inflation forecast errors become

$$\pi_t - E(\pi_t \mid Y_{t-1}) = (1 - S_t)\pi_{0t} + S_t\pi_{1t} - P(S_t = 0 \mid Y_{t-1})E(\pi_{0t} \mid Y_{t-1}) - P(S_t = 1 \mid Y_{t-1})E(\pi_{1t} \mid Y_{t-1}) \tag{4}$$

From Equation (4) it is fairly easy to verify that ex ante the forecast error has a mean value equal to zero, that is, $E\{\pi_t - E(\pi_t \mid Y_{t-1}) \mid Y_{t-1}\} = 0$. The major implication of the model, however, is that rationally formed expectations can appear to be biased when viewed ex post. To illustrate this, let us assume that the actual inflation rate currently follows the regime-1 process. Recall that economic agents do not observe the current regime, so the best guess they can come up with is given by the unconditional expectation $E(\pi_t \mid Y_{t-1})$. Subtracting this term from the inflation rate currently following a regime-1 process equals

$$\pi_{t|S_{t=1}} - E(\pi_t \mid Y_{t-1}) = \{\pi_{1t} - E(\pi_{1t} \mid Y_{t-1})\} + P(S_t = 0 \mid Y_{t-1})\{E(\pi_{1t} \mid Y_{t-1}) - E(\pi_{0t} \mid Y_{t-1})\} \tag{5}$$

The first term on the right-hand side of Equation (5) equals zero on average because of the assumption of rationally formed expectations and because the inflation rate actually develops according to the regime-1 process in period t. If economic agents place some weight on the probability of the inflation rate following a regime-0 process, however, the second term on the right-hand side of Equation (5) will not have zero mean as long as the expected values of π_{1t} and π_{0t} differ. This implies that the forecast $E(\pi_t \mid Y_{t-1})$ will appear biased when viewed ex post, even though agents are using all available information efficiently in making their forecast. This is the phenomenon known as *ex post bias*. In general ex post bias occurs when inflation follows a process allowing for several regimes, and agents, when forming their expectations, incorporate the possibility of a regime switch. As long as the actual process continues in, say, regime 1 and the possibility of a switch to regime 0 persists, rational forecasts will appear biased. To estimate the size of the ex post bias, one must specify the processes generating π_{1t} and π_{0t}. In addition, one also has to specify a probability distribution of the stochastic variable S_t. In this setup the h-steps-ahead forecast will be given by

$$E(\pi_{t+h-1} \mid Y_{t-1}) = P(S_{t+h-1} = 0 \mid Y_{t-1})E(\pi_{0t+h-1} \mid Y_{t-1}) + P(S_{t+h-1} = 1 \mid Y_{t-1})E(\pi_{1t+h-1} \mid Y_{t-1}) \tag{6}$$

Evans and Wachtel (1993) suggest using a two-state Markov regime-switching approach with constant transition probabilities to determine the size of the ex post bias. In that case

$$\begin{bmatrix} P(S_{t+h-1} = 0 \mid Y_{t-1}) \\ P(S_{t+h-1} = 1 \mid Y_{t-1}) \end{bmatrix} = \begin{bmatrix} p & 1-q \\ 1-p & q \end{bmatrix}^h \begin{bmatrix} P(S_{t-1} = 0 \mid Y_{t-1}) \\ P(S_{t-1} = 1 \mid Y_{t-1}) \end{bmatrix} \tag{7}$$

where $p = P(S_t = 0 \mid S_{t-1} = 0)$ and $q = P(S_t = 1 \mid S_{t-1} = 1)$ denote the constant transition probabilities. Furthermore, if it is assumed that we can represent the inflation rate as autoregressive processes in both regimes such that

$$\pi_{st} = \Gamma'_{t-1}\beta_s + \epsilon_{st}, \text{ for } s = 0, 1 \tag{8}$$

[1]Unconditional with respect to the unobserved state variable S_t.

where $\Gamma_t = \{1, \pi_t, \pi_{t-1}, \ldots, \pi_{t-k+1}\}$. The rational forecasting rule is given according to the following recursive scheme for $j = 1$ to h

$$\begin{aligned} E(\pi_{t+j-1} \mid Y_{t-1}) &= \widehat{\Gamma}'_{t+j-2}[\beta_0 + P(S_{t+j-1} = 1 \mid Y_{t-1})(\beta_1 - \beta_0)] \\ \widehat{\Gamma}'_{t+j-2} &= \{1, E(\pi_{t+j-2} \mid Y_{t-1}), E(\pi_{t+j-3} \mid Y_{t-1}), \ldots, E(\pi_{t+j-k-1} \mid Y_{t-1})\} \end{aligned} \tag{9}$$

The analytical expression for the inflation expectations is very tedious to write out for high-dimensional models because of the assumptions about the dynamics of the inflation process given by Equation (8), which differ from the assumptions about those dynamics made by Evans and Wachtel. To illustrate the implications of these "new dynamics" for the agents' rational forecasting rule and to compare that with the forecasting rule used in Evans and Wachtel 1993, let us write out the case of $k = 1$. If we let the inflation rate in the two regimes be given by

$$\pi_t = \begin{cases} \alpha_0 + \alpha_1 \pi_{t-1} + \epsilon_{0t}, & \text{for } S_t = 0 \\ \vartheta_0 + \vartheta_1 \pi_{t-1} + \epsilon_{1t}, & \text{for } S_t = 1 \end{cases} \tag{10}$$

the expectations h periods ahead will be given as

$$\begin{aligned} E(\pi_{t+h-1} \mid Y_{t-1}) = \quad & [\alpha_0 + P(S_{t+h-1} = 1 \mid Y_{t-1})(\vartheta_0 - \alpha_0)] \\ & + \sum_{j=1}^{h} \{[\alpha_0 + P(S_{t+j-1} = 1 \mid Y_{t-1})(\vartheta_0 - \alpha_0)] \\ & \times \prod_{v=j+1}^{h} [\alpha_1 + P(S_{t+v-1} = 1 \mid Y_{t-1})(\vartheta_1 - \alpha_1)]\} \\ & + \prod_{j=1}^{h} [\alpha_1 + P(S_{t+j-1} = 1 \mid Y_{t-1})(\vartheta_1 - \alpha_1)]\pi_{t-1} \end{aligned} \tag{11}$$

This expression is by far more complicated than the analogous equation (6) in Evans and Wachtel 1993 (p. 491). This hinges on the fact that Evans and Wachtel assume the two underlying processes to be completely independent, that is, $\pi_{st} = f_s(\pi_{st-1}),\ s = 0, 1$, hence allowing for discrete jumps in the inflation rate process despite the underlying process being continuous, whereas we only allow for jumps in the parameters by assuming $\pi_{st} = f_s(\pi_{t-1}),\ s = 0, 1$. Our two underlying processes are hence interrelated, since both depend on the lagged values of the observed inflation rate, and this means that the possibility of the inflation rate shifting back and forth between regimes within the forecast interval must be taken into account. Under the simpler approach by Evans and Wachtel, this complicating—but more realistic—feature can be disregarded. A full description of the methodology for estimating the parameters of the Markov regime-switching model and on how to construct the filter probabilities—$P(S_t = 1 \mid Y_{t-1})$—is given in Section 4. Before turning to the more complex nonlinear modeling procedures, however, we will verify that a linear representation cannot provide an adequate description of the inflation rate process. Since parsimonious models often outperform less parsimonious models in terms of real-time forecast accuracy, linear models will typically be preferred to nonlinear models, implying that we have to make sure that a nonlinear representation is actually needed.

2.2 Quantification of qualitative data on inflation expectations

For most countries in the European Union, qualitative data exist on households' expectations about inflation one year ahead as well as the households' perceptions of last year's price development.[2] The data are

[2]The Danish data have been collected by Statistics Denmark since 1974, three times a year until 1983. Since then the data frequency has increased, and since 1987 figures are available on a monthly basis, though not in June before 1997. After 1988 gross figures on the number of respondents in the different categories are no longer reported.

produced by interviewing a representative sample of people between 16 and 74 years of age. The interviewees are asked two questions: "How is the price level today compared to the price level one year ago?" and "How will the price level be one year ahead compared to the price level today?" The five possible answers are "much higher" (weight: +1), "somewhat higher" (weight: $+\frac{1}{2}$), "slightly higher" (weight: 0), "unchanged" (weight: $-\frac{1}{2}$), and "slightly lower" (weight: −1). A single measure of the household sector's inflation expectations is then produced by adding up the weights of all the households. Assuming the same relationship for both past and future inflation, between qualitative and quantitative data on inflation, a quantitative measure of inflation expectations can be deduced by simple regression (cf. Pesaran 1987). Such a regression represents nothing but a simple conversion from qualitative into quantitative measures of inflation expectations and gives no causal explanation of inflation expectations. The categorical/ordinal responses are quantified using the regression method, assuming the relationship between actual price changes and survey responses to be given by

$$\pi_t = \alpha_1 MH_t^p + \alpha_2 SH_t^p + \alpha_3 LH_t^p + \alpha_4 U_t^p + \alpha_5 LL_t^p + \epsilon_t \tag{12}$$

where ε_t is a measurement error, assumed to follow a Gaussian distribution. MH_t^p denotes the fraction of households with a perception that prices today are much higher than one year ago, SH_t^p somewhat higher, LH_t^p a little higher, U_t^p unchanged, and LL_t^p a little lower. Let $MH_{t-1|t}^e$, $SH_{t-1|t}^e$, $LH_{t-1|t}^e$, $U_{t-1|t}^e$, and $LL_{t-1|t}^e$ represent the same fractions with respect to one-year-ahead expectations, respectively. Expected inflation can then be derived by the following conversion formula

$$\widehat{\pi}_{t-1|t}^e = \widehat{\alpha}_1 MH_{t-1|t}^e + \widehat{\alpha}_2 SH_{t-1|t}^e + \widehat{\alpha}_3 LH_{t-1|t}^e + \widehat{\alpha}_4 U_{t-1|t}^e + \widehat{\alpha}_5 LL_{t-1|t}^e \tag{13}$$

where $\widehat{\alpha}_i$, for $i = \{1, \ldots, 5\}$ are ordinary least squares estimates of α_i in Equation (12). One very crucial assumption upon which Pesaran's approach rests is that the perceptions are rational, so that the perception errors—given by ϵ_t in Equation (12)—are without any bias and serial correlation. Second, it requires that the regression coefficients in Equation (12) be stable and that the actual inflation be normally distributed. The consequences of the actual inflation rate following a nonlinear regime-shifting model can now easily be understood. It basically implies failure of all the critical assumptions of Pesaran's regression model approach to hold. In particular, if the actual inflation rate follows a two-state Markov regime-switching model, the perception errors can actually be temporally biased and serially correlated, because regimes are not perfectly observable ex post. Obviously, the inflation rate will not be normally distributed asymptotically in this situation but instead distributed as a mixture of two normals. Finally, this implies that it is very unlikely that the regression coefficients in (12) will be even approximately stable. Consequently, before undertaking Pesaran's regression model approach, one has to check carefully whether the basic assumptions stated above hold.

Conditional on Danish survey data, Engsted (1991) and Christensen (1996) represent two studies of the expectation hypothesis based on Pesaran's regression model approach. The data that they use on the household sector's inflation perceptions and expectations are depicted in Figure 1. Notice that when the actual inflation rate is high, as in the 1970s and the beginning of the 1980s, inflation expectations are below inflation perceptions, whereas in the 1990s, when actual inflation rates are very low, inflation expectations are well above inflation perceptions. For the subperiod 1986 to 1996, Christensen (1996) shows that Danish households' inflation expectations may have a serious upward bias in the latter part of the period investigated. According to his study, expected inflation has been relatively stable around 3 percent per year throughout the 1990s, whereas actual inflation has been persistently lower. Engsted (1991) studies gross figures for 1975 to 1990, and his analysis confirms that inflation expectations have a marked upward bias in periods with low inflation, and the opposite is also true. Unfortunately, in neither of the two studies are the crucial assumptions underlying Pesaran's (1987) approach checked carefully. Hence one of the main contributions of our article will be a careful discussion of how to perform a proper model evaluation and selection, which should enable us to evaluate the validity of the results of Engsted (1991) and Christensen (1996). Furthermore, the methods

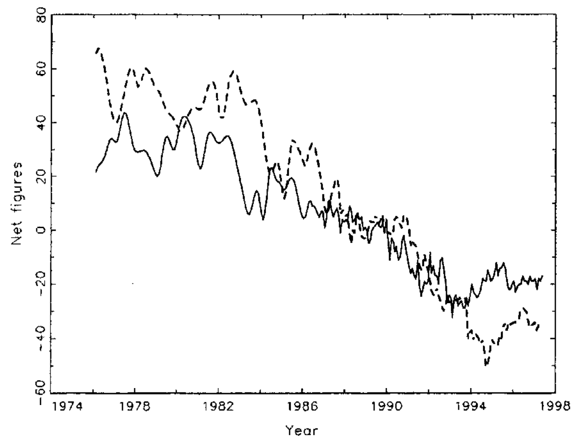

Figure 1
Perceived and expected inflation rates of Danish household sector, 1976m1–1997m10.
Note: Full line: expected changes in price level over previous year; dashed line: perceived change in price level one year ahead.

discussed will provide us with extensive guidance on how to specify the proper Markov regime-switching model as a superior alternative to the linear representation of the actual inflation rate.

Finally, it is worth mentioning that similar systematic overpredictions of the Danish inflation rate found by Engsted (1991) and Christensen (1996) have been made by official forecasters, such as the government and the Organisation for Economic Cooperation and Development (OECD). Despite the difficulties involved in tracking the actual inflation rate, it is striking that apparently systematic forecast errors of opposite signs are present, that is, underpredictions of the inflation rate in the period with high rates of inflation and overpredictions when inflation is low.

2.3 Flexible regression models

To determine the existence of nonlinear components in the inflation rate process, we use two different flexible regression models. We use a parametric approach recently suggested by Hamilton (2001), and we use the more familiar nonparametric projection pursuit approach in a form suggested by Aldrin, Boelviken, and Schweder (1993). The basic idea underlying both approaches is to estimate the conditional mean function of the time series y_t without imposing any restrictions on the functional form of the function. The flexible regression model can be written as

$$y_t = \mu(\boldsymbol{x}_t, \delta) + \epsilon_t \tag{14}$$

where ϵ_t is a sequence of $N(0, \sigma^2)$-distributed error terms and $\mu(\boldsymbol{x}_t, \delta)$ is the conditional mean function. $\boldsymbol{x}_t$ is a $k \times 1$ vector that may include lagged dependent variables. In Hamilton's approach, the conditional mean function, that is, $\mu_{fnl}(\boldsymbol{x}_t, \boldsymbol{\delta})$, is represented as having a linear part and a stochastic nonlinear part according to[3]

$$\mu_{fnl}(\boldsymbol{x}_t, \boldsymbol{\delta}) = \boldsymbol{x}_t'\boldsymbol{\beta} + \lambda m(\mathbf{g} \odot \boldsymbol{x}_t) \tag{15}$$

where only the linear part is perfectly observable up to an unknown parameter vector $\boldsymbol{\beta}$, and the nonlinear random function $m(\cdot)$ depends on the parameter vector $\boldsymbol{g}$ determining the curvature of the function and on the regressors $\boldsymbol{x}_t$. λ is a parameter that determines the weight to assign to the nonlinear component in the conditional mean function. Note that if the hypothesis $\lambda = 0$ cannot be rejected, the model is purely linear.

[3]Here $\boldsymbol{g}$ is a $k \times 1$ vector of parameters and $\odot$ denotes element-by-element multiplication, that is, $\boldsymbol{g} \odot \boldsymbol{x}_t$ is the Hadamard product. $\boldsymbol{\beta}$ is a $k \times 1$ vector of coefficients.

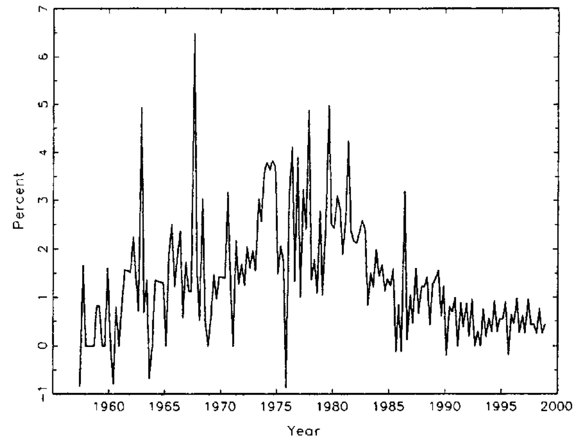

Figure 2
Quarterly growth rates in Danish consumer prices (seasonally adjusted), 1957q2–1998q4.

Note also that inference about whether the regressors x_{it}, $i = 1, \ldots, k$ should enter nonlinear in the conditional mean function can be based on the null that $g_i = 0$. Estimation of the parameters of the model given by $\boldsymbol{\delta} = \{\boldsymbol{\beta}, \lambda, \boldsymbol{g}, \sigma\}$ as well as the estimation of the nonlinear random function $m(\cdot)$ is carried out in accordance with the principle of maximum likelihood (see Hamilton 2001 for details).

According to the projection pursuit approach, the conditional mean function is represented as

$$\mu_{ppr}(\boldsymbol{x}_t, \varrho) = \boldsymbol{x}'_t\boldsymbol{\beta} + \sum_{j=1}^{v} \omega_j \varphi_j(\boldsymbol{x}'_t \Phi_j) \tag{16}$$
$$\varrho = \{\boldsymbol{\beta}, \omega_1, \ldots, \omega_v, \Phi_1, \ldots, \Phi_v\}$$

The parameters Φ_j define the projection of the input vector $\boldsymbol{x}_t$ onto a set of planes labeled by $j = 1, \ldots, v$. These projections are transformed by the nonlinear activation functions denoted $\varphi_j(\cdot)$, which in our case are taken to be a cubic spline function, and these in turn are linearly combined with weight ω_j to form the output variable y_t. The algorithm for estimating the parameters is described in detail by Aldrin, Boelviken, and Schweder (1993) and Dahl and Hylleberg (1999).

3 Linear Representation of the Inflation Rate

As a first attempt to model the actual inflation rate, we begin by considering linear representations. But before we turn to the actual modeling, let us have a short look at the development in the Danish quarterly inflation rate in the period 1958q3–1998q4, depicted in Figure 2. The most striking feature in the figure is the shift in level that seemed to occur very rapidly in the middle of the 1980s: a transition from very high and volatile inflation rates in the 1970s to low and much more stable inflation rates in the 1990s. Over the whole period the mean inflation rate was about 1.4 percent per quarter, with a standard deviation around 1.2. Inspection of the results reported in Table 1 reveals that it is very unlikely that the inflation rate follows a normal distribution based on a Jacque-Bera test. This evidence is confirmed in Figure 3, which is a plot of the estimated density of the inflation rate over the same period. The right tail of the density seems to be too fat for the density to be approximated by a normal density. Notice further that a mean and variance shift of the inflation rate could produce a mixture density of a shape consistent with the estimated density. We will return in detail to this subject later in the article.

The augmented Dickey-Fuller tests for stationarity on the inflation rate are reported in Table 2. In general it seems that the hypothesis of nonstationarity is strongly rejected when the number of lags included in the auxiliary regression is less than two. Furthermore, a closer look at the auxiliary regression equations in the

Table 1

Quarterly growth rates in Danish consumer prices, 1957q1–1998q4: Simple descriptive statistics and the Jacque-Bera test for normality

Mean	1.368
Standard deviation	1.222
Skewness	1.109
Excess kurtosis	1.785
Minimum	−0.863
Maximum	6.489
Normality test	
Statistics	34.804
p-value	0.000

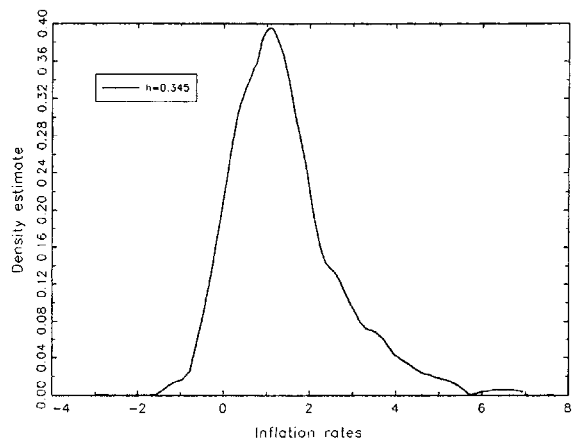

Figure 3

Density estimate of quarterly growth rates in Danish consumer prices, 1957q1–1998q4.

Note: Density estimate is based on the Epanechnikov kernel with data-determined bandwidth $h = 0.345$. For details about the data-dependent bandwidth selection procedure, see Silverman 1986, Equation (3.31).

Table 2

Augmented Dickey-Fuller tests for nonstationarity of Danish inflation rate, 1958q2–1998q4

	ADF coefficient on lagged level (t-ratio) For model including			
Lags	No constant No trend	Constant No trend	Constant Trend	No. of Obs.
0	−0.286 (−5.26)***	−0.654 (−9.01)***	−0.669 (−9.18)***	166
1	−0.152 (−2.92)***	−0.437 (−5.16)***	−0.453 (−5.29)***	165
2	−0.102 (−2.00)**	−0.329 (−3.70)***	−0.349 (−3.86)**	164
3	−0.083 (−1.64)*	−0.292 (−3.15)**	−0.316 (−3.35)*	163
4	−0.078 (−1.53)	−0.296 (−3.07)**	−0.324 (−3.31)*	162

*rejection of the null of nonstationarity at the 10 percent level.
**rejection of the null of nonstationarity at the 5 percent level.
***rejection of the null of nonstationarity at the 1 percent level.

Table 3
Estimated AR(4) representation of quarterly Danish inflation rate, 1958q3–1998q4

RHS variable	Linear model (LR)		
	Estimate	Std. error	p-value
π_{t-1}	0.127	0.079	0.111
π_{t-2}	0.221	0.077	0.005
π_{t-3}	0.232	0.077	0.003
π_{t-4}	0.121	0.079	0.126
Constant	0.423	0.155	0.007
Log-likelihood		−235.910	
R^2		0.269	
Residual sum of squares		174.552	

Note: π_t denotes the inflation rate.

case of up to three and up to four lags (included in Appendix Tables A1 and A2, respectively) reveals that the third and fourth lags do not enter significantly into the auxiliary regression, which results in a potential loss in power of the augmented Dickey-Fuller statistics.

We are now ready to turn to the linear modeling of a univariate dynamic representation of the inflation rate. The aim is to analyze whether it is consistent to model the inflation rate with a dynamic one-regime econometric model. The model is represented as

$$\begin{aligned} \pi_t &= \alpha + \sum_{j=1}^{k} \beta_j \pi_{t-j} + \epsilon_t \\ \epsilon_t &\sim N(0, \sigma^2) \end{aligned} \tag{17}$$

The AR(4) model is estimated using recursive least squares, and the estimation results are presented in Table 3.

A first question of interest is whether the one-year-ahead forecasts produced by the estimated AR(4) model share some of the same characteristics as the survey-based inflation expectations. It turns out that the inflation rate forecasts produced by the AR(4) reveal a similarity with inflation expectations based on gross survey data, in the sense that forecasts based on an AR(4) model produce a negative wedge between actual inflation and expectation in the 1970s and a clear-cut positive wedge in the 1990s (see Figure 4 for the one-quarter-ahead forecasts and Figure 5 for the one-year-ahead forecasts).

The distribution of the one-period-ahead predictions of the linear model depicted in Figure 6 confirms that the estimated median of the inflation rate expectation is higher numerically than the median of the actual inflation rate. The density plot also reveals that the linear model cannot explain the observed fatness in the right-hand tail of the inflation rate distribution.

The next question of interest is whether the observed expectation bias could arise from a misspecified model. If this is the case, and the similarity of the forecasts is in fact because the household sector uses an AR(4) representation when forecasting inflation, the observed bias is indeed a result of nonrational behavior and a clear-cut violation of the rational-expectations hypothesis. Looking at the dynamic specification tests reported in Table 4, we see that it is not possible to reject the hypothesis that the linear model is dynamically well specified because of the absence, at a 5 percent significance level, of neglected autocorrelation and of ARCH effects up to an order of four.

However, the stability of the model seems more critical. Based on the well-known one-step-up Chow test for structural breaks within the sample period, depicted in Figure 7, and the forecast (n steps up) Chow tests, depicted in Figure 8, the null of stability of the model is rejected at a 5 percent level, and the tests give some

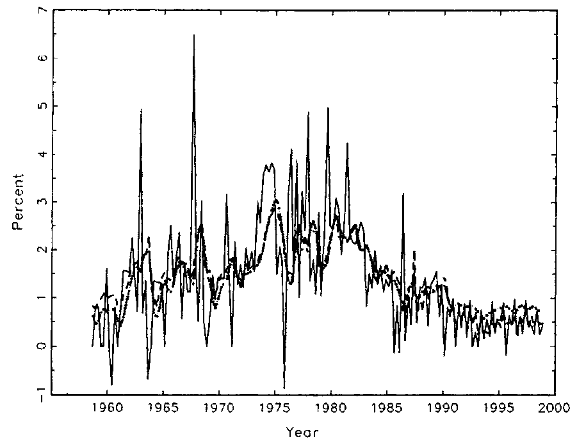

Figure 4
Actual and one-period-ahead predictions of quarterly growth rates in Danish consumer prices, 1958q3–1998q4.
Note: Full line: actual inflation; dashed line: in-sample predictions from Markov regime-switching model; dotted line: in-sample predictions from AR(4) model.

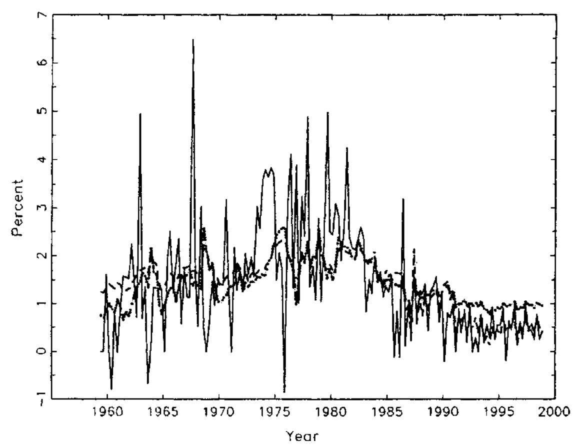

Figure 5
Actual and four-periods-ahead predictions of quarterly growth rates in Danish consumer prices, 1958q3–1998q4.
Note: Full line: actual inflation; dashed line: in-sample predictions from Markov regime-switching model; dotted line: in-sample predictions from AR(4) model.

Table 4
Testing adequacy of AR(4) model: Lagrange multiplier tests for detecting dynamic misspecification in terms of neglected autocorrelation and ARCH effects

Test	Statistics	*p*-value
Lagrange multiplier tests for no autocorrelation		
AR(1;1)	0.009	0.920
AR(1;2)	5.743	0.056
AR(1;3)	5.688	0.128
AR(1;4)	6.828	0.145
Lagrange multiplier tests for no ARCH effects		
ARCH(1;1)	3.814	0.051
ARCH(1;2)	5.103	0.078
ARCH(1;3)	4.895	0.179
ARCH(1;4)	4.917	0.296

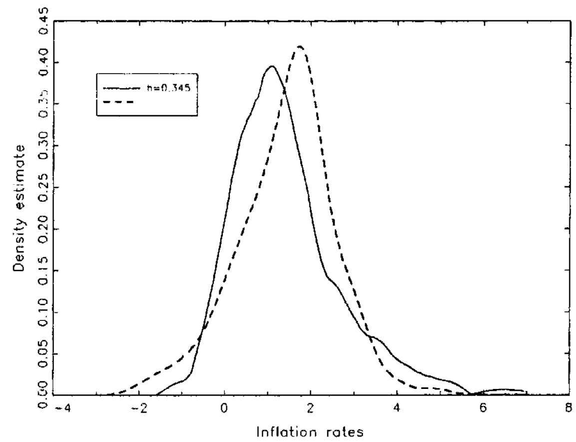

Figure 6
Density estimates, 1958q3–1998q4.
Note: Full line: actual quarterly growth rates in Danish consumer prices; dashed line: one-step-ahead in-sample predictions produced from AR(4) model. Density estimates are based on Epanechnikov kernel with data-determined bandwidth $h = 0.345$.

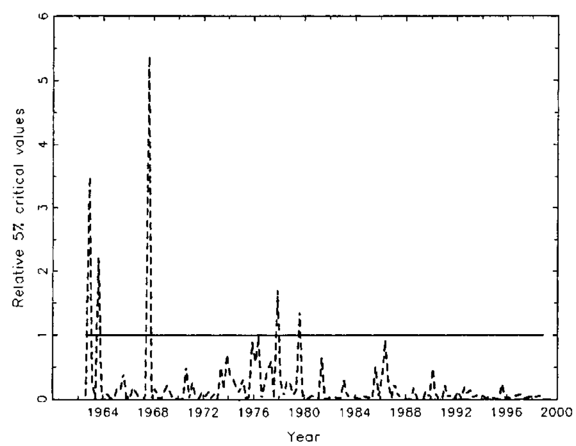

Figure 7
One-step-up Chow test for stability of AR(4) model of quarterly growth rates in Danish consumer prices, 1958q3–1998q4.
Note: Dotted line: Chow statistics relative to 5 percent critical value under null of stability; horizontal full line: Values above indicates rejection of null of stability at 5 percent level.

indications of one or more structural breaks over the period. In contradiction to this evidence stands the break point (n steps down) Chow test, where stability cannot be rejected at a 5 percent level (see Figure 9).

To add more information, we now turn to some more powerful tests for detecting multiple structural break suggested by Andrews, Lee, and Ploberger (1996). Based on an extensive Monte Carlo study, these authors cast some doubt on the power of the traditional Chow tests for various kinds of structural breaks and regime shifts. Instead, they suggest three optimal change point tests for detecting the presence of nonstable coefficients in a normal linear-regression model with unknown breakpoints.[4] The three test statistics denoted *SupF*, *ExpF*, and *AvgF* are presented in Table 5. Based on all three statistics, it is possible to test the joint hypothesis of stability of all the parameters in the model as well as the hypothesis that a single parameter is

[4]Based on a wide range of Monte Carlo experiments, Andrews, Lee, and Ploberger (1996) show that their test statistics appear to be superior in power. The probability distributions of the test statistics are nonstandard under the null, which makes them a bit more tedious to use. Hansen (1997) recently constructed a response surface for calculating the p-values for the three optimal change point tests suggested by Andrews, Lee, and Ploberger (1996), making inference straightforward.

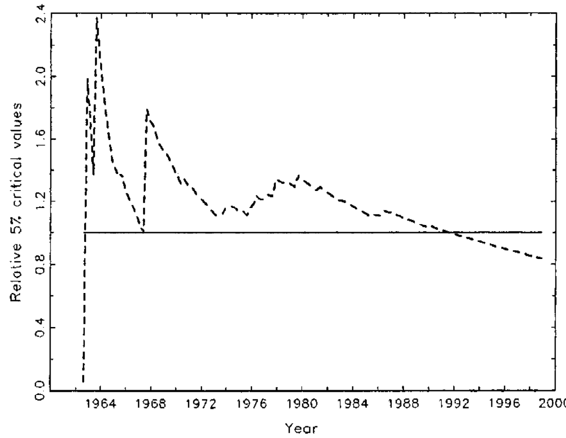

Figure 8
Forecast (n steps up) Chow test for stability of AR(4) model of quarterly growth rates in Danish consumer prices, 1958q3–1998q4.
Note: Dotted line: Chow statistics relative to 5 percent critical value under null of stability; horizontal full line: Values above indicate rejection of null of stability at 5 percent level.

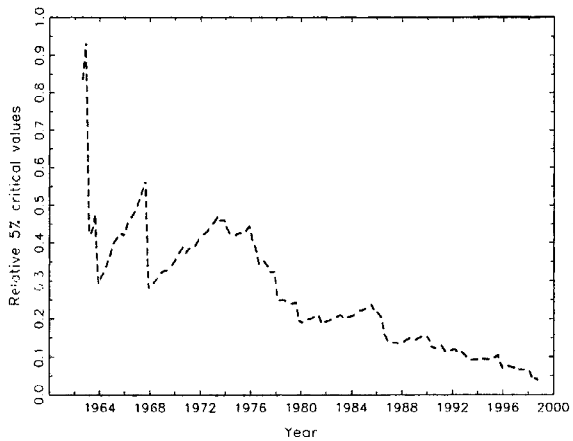

Figure 9
Breakpoint (n steps down) Chow test for stability of AR(4) model of quarterly growth rates in Danish consumer prices, 1958q3–1998q4.
Note: Dotted line: Chow statistics relative to 5 percent critical value under null of stability; horizontal full line: Values above indicate rejection of null of stability at 5 percent level.

stable. By inspection of Table 5 we can clearly reject the hypothesis that all the parameters are stable at a 5 percent level under all three tests. In fact the three tests all indicate that only the second and fourth lag of the inflation rate enter stably in the linear-regression model, and the stability of the constant term and the coefficient related to the first and third lag of the inflation rate clearly is rejected. Finally, we consider Nyblom's (1989) test for parameter stability against the hypothesis that the regression coefficients follow a martingale process. This test for stability of the parameters of the linear model is also rejected at a 5 percent level.

We conclude this section by testing the linear AR(4) model for neglected nonlinearity. If instability of the linear model of the inflation rate is due to the fact that the true inflation rate is generated by a regime-shifting process, we would expect these specification tests to reject the null hypothesis of linearity. The results from the specification tests are reported in Table 6. The battery of specifications tests include Hamilton's (2001) test, the neural network test due to White (1992), Tsay's (1986) polynomial test, White's (1987) information matrix test, Ramsey's (1969) Reset test, and finally the nonparametric BDS test due to Brock, Deckert, and

Table 5
Testing the adequacy of the AR(4) model: Nyblom's score test and the Andrews-Lee-Ploberger *SupF*, *ExpF*, and *AvgF* tests for detecting parameter instability

Test	Statistics	p-value
Tests for structural stability		
Nyblom		
$\{\beta_1, \ldots, \beta_5\}$	1.659	.
SupF		
$\{\beta_1, \ldots, \beta_5\}$	19.336	0.033
β_1	10.980	0.017
β_2	11.993	0.010
β_3	6.479	0.130
β_4	13.121	0.006
β_5	3.183	0.517
ExpF		
$\{\beta_1, \ldots, \beta_5\}$	7.618	0.013
β_1	3.517	0.006
β_2	3.923	0.003
β_3	1.355	0.126
β_4	4.534	0.001
β_5	0.704	0.342
AvgF		
$\{\beta_1, \ldots, \beta_5\}$	10.930	0.008
β_1	3.963	0.012
β_2	4.056	0.011
β_3	1.969	0.104
β_4	4.515	0.006
β_5	1.206	0.267

Note: The 1, 5, and 10 percent critical values of Nyblom's statistics equal 1.870, 1.462 and 1.275, respectively. The p-values associated with the Andrews-Lee-Ploberger test are calculated according to the method described by Hansen (1997).

Table 6
Testing adequacy of linear AR(4) representation of the Danish inflation rate

	AIC (lags = 4)		BIC (lags = 3)		CV (lags = 3)	
Hamilton	1.57	(0.210)	1.65	(0.200)	1.65	(0.200)
Neural network	7.90	(0.000)	4.46	(0.030)	4.46	(0.030)
Tsay	1.29	(0.240)	1.22	(0.300)	1.22	(0.300)
White	18.90	(0.460)	13.73	(0.390)	13.73	(0.390)
Reset	0.56	(0.450)	0.64	(0.420)	0.64	(0.420)
BDS	25.34	(0.000)	15.34	(0.000)	15.34	(0.000)

Note: The linear models under the null are selected by Akaike's infomation criteria (AIC), Schwarz's infomation criterion (BIC), and the cross-validation selection criterion (CV). Critical values are reported with the associated p-values in parentheses.

Scheinkman (1987). The evidence for the presence of a neglected nonlinear component seems a little mixed. Whereas Hamilton's test, Tsay's test, White's information matrix test, and the Reset test support the null of linearity, the neural network test and the BDS test clearly rejects the null of linearity.

To summarize, there seems to be significant evidence of the presence of one or more structural breaks in the Danish inflation rate process during 1958–1998. On this account it is fairly safe to conclude that the persistent expectation bias observed is due to shifts in the parameters of the AR(4) model. This implies that if the households use this model to forecast the inflation rate, their behavior is indeed nonrational. This raises the issue of whether there exists an alternative empirical—possibly nonlinear—representation that takes into account the regime shifts that apparently occur in the inflation process and at the same time produces the same degree of similarity when compared to the survey-based measures of expected inflation, as the forecasts from the AR(4) model apparently do. In the next section such a new candidate, the Markov regime-switching model, will be presented and discussed.

In recent years there has been an important literature highlighting fractional integration as an alternative description of the inflation process. Baillie, Chung, and Tieslau (1996) and Hassler and Wolters (1995) find that inflation in the major Western economies can be described by fractional integration allowing long memory, implying that shocks to the inflation rate have a long-lasting, though not permanent, effect. Baum, Barkoulas, and Caglayan (1999) extend the same results to a broader range of countries, including Denmark. Potentially, long memory reflecting the long swings in the Danish inflation rate could be an alternative explanation of the ex post bias. In practice, however, it turns out to be difficult to distinguish between the two phenomena. Diebold and Inoue (forthcoming) demonstrate that according to asymptotic theory, long memory and a regime-switching model with only a few break points are easily confounded, and show through Monte Carlo simulations that the same result is valid in finite samples as well. In fact, there is no contradiction between a stationary Markov switching model where regime shifts occur infrequently and a long-memory model.

4 A Markov Regime-Switching Representation of the Inflation Rate

In this section we investigate whether the Danish inflation rate can be represented by a two-state Markov regime-switching model. The section begins with a short description of the statistical representation of the model and a discussion of how the unknown parameters of the model are estimated. We consider a *noncentered* version of Hamilton's (1989) Markov regime-switching model.[5] In addition to Hamilton's *centered* model we allow both the constant term and the autoregressive coefficients of the AR(4) representation of the inflation process to switch among possible regimes, thereby taking into account the evidence presented in the last section on the instability of these parameters in the one-regime AR(4) model. We also allow the variance of the inflation rate to shift across the different regimes. The inflation regime at date t is indexed by an unobserved random variable S_t. In our setup S_t has two possible outcomes: $S_t = 0$ if the inflation process is in regime 0, and $S_t = 1$ if the inflation process is in regime 1. We assume that the transitions between the regimes are governed by a two-state Markov chain. The noncentered Markov regime-switching model can be given a simple state space representation, with one measurement equation representing the actual inflation rate and one transition equation representing the unobserved state variable S_t. This is written as

$$\begin{aligned} \pi_t &= (1 - S_t)\left\{\alpha_0 + \sum_{j=1}^{k_0} \beta_{0j}\pi_{t-j} + \epsilon_{0t}\right\} + S_t\left\{\alpha_1 + \sum_{j=1}^{k_1} \beta_{1j}\pi_{t-j} + \epsilon_{1t}\right\} \\ \begin{bmatrix}\epsilon_{0t}\\ \epsilon_{1t}\end{bmatrix} &\sim N\left(0, \begin{bmatrix}\sigma_0^2 & 0\\ 0 & \sigma_1^2\end{bmatrix}\right) \end{aligned} \tag{18}$$

with the constant transition probabilities of the Markov chain defined as

$$\begin{aligned} p(S_t = 0 \mid S_{t-1} = 0) &= p \\ p(S_t = 1 \mid S_{t-1} = 0) &= 1 - p \\ p(S_t = 0 \mid S_{t-1} = 1) &= 1 - q \\ p(S_t = 1 \mid S_{t-1} = 1) &= q \end{aligned} \tag{19}$$

[5] Hamilton typically represents the dependent variable in deviations from its mean (the variable is said to be centered around its mean), where the mean of the lagged dependent variable is a function of the lagged value of the state variable. In our setup we do not represent the inflation rate in deviations from its mean, and only the current value of the state variable matters.

There are $6 + k_0 + k_1$ unknown parameters to be estimated in the model. These are collected in the vector $\boldsymbol{\lambda} = \{\alpha_0, \alpha_1, \beta_{01}, \ldots, \beta_{0k_0}, \beta_{11}, \ldots, \beta_{1k_1}, \sigma_0^2, \sigma_1^2, p, q\}$. As shown by Hamilton (1989) the conditional density function for π_t in this setup is a mixture of two normal densities weighted by the filter probabilities of being either in state $S_t = 0$ or $S_t = 1$. If we let Y_t denote the information set containing the observations $\pi_1 \ldots \pi_t$ and use successive conditioning, the conditional likelihood function, given an initial observation for π_0, can be written as

$$l(\boldsymbol{\lambda}) = \prod_{t=1}^{T} \sum_{i=1}^{2} \frac{1}{(2\pi\sigma_i^2)} \exp\left(\frac{-(\pi_t - \alpha_i - \sum_{j=1}^{k_i} \boldsymbol{\beta}_{ij}\pi_{t-j})^2}{2\sigma_i^2}\right) P(S_t = i \mid Y_{t-1}\colon\ \boldsymbol{\lambda}) \tag{20}$$

Provided that the conditional probabilities $P(S_t = i \mid Y_{t-1}\colon\ \boldsymbol{\lambda})$ exist and can be evaluated at $t = 1, \ldots, T$, the maximum-likelihood estimates $\widehat{\boldsymbol{\lambda}}$ can be found solving arg max $l(\boldsymbol{\lambda})$ using suitable numerically constrained optimization algorithms.[6] Before turning to the actual estimation of parameters of the Markov regime-switching model, we first consider a specification test, proposed by Hansen (1992, 1996), to draw some inference whether the null of the AR(4) representation can be rejected in favor of the Markov switching model. Second, using an approach based on forecast accuracy suggested by Dahl and Hylleberg (1999), we test the null of the Markov regime-switching model against a wide class of linear and nonlinear models. In this approach the model under the alternative is assumed to be represented by one of two different flexible-regression-model approaches consisting of Hamilton's parametric flexible regression model (denoted FNL) and the nonparametric projection-pursuit regression model of Aldrin, Boelviken, and Schweder (1993) (denoted PPR-L).

As pointed out by Hansen (1992) classical test statistics such as the *LR*, *Wald*, and *LM* statistics are not asymptotically chi-squared distributed in a situation where the null hypothesis of linearity is tested against the Markov regime-switching model. The statistics are all based on regularity conditions ensuring that the likelihood surface is locally quadratic and that the score vector has a nonzero variance. These conditions are all violated here, first of all because there are two nuisance parameters (the transition probabilities) not identified under the null, making the likelihood surface flat at the optimum; and second, because the null hypothesis yields a local optimum of the likelihood surface, implying that the score vector is identically equal to zero, hence contradicting the nonzero variance condition. By working directly with the likelihood surface, viewing the likelihood function as an empirical process for the unknown parameters, Hansen (1992, 1996) derives a test statistic that does not require the likelihood function to be locally quadratic or require the scores (or for that matter any other higher-order derivative) to have positive variance. The cost of working with empirical process theory is that it is only possible to derive a boundary and not an asymptotic distribution for the standardized likelihood ratio test suggested by Hansen (1992, 1996). Based on a Monte Carlo experiment, however, Hansen (1992) shows that the size and power properties of his standardized likelihood ratio statistics are reasonably good. Hansen's standardized likelihood ratio test is known to be rather cumbersome to compute even in small parametric models. In our fairly parsimonious representation this was not a serious problem. The results from conducting Hansen's (1996) tests are reported in Table 7, where LR_T^* denotes the standardized likelihood ratio test statistics and M denotes the chosen bandwidth (see Hansen 1996 for details). Independent of the choice of bandwidth, the associated p-values are all very small, suggesting that we can reject the null of a linear AR(4) model as an adequate description of the Danish inflation rate in favor of the two-state Markov regime-switching model.

A crucial question is whether the Markov regime-switching model is the preferred nonlinear specification among the large class of nonlinear models that can mimic multiple regime shifts or smooth transitions between regimes. Recently, Dahl and Hylleberg (1999) have suggested a general-to-specific approach as a

[6]In a series of papers Hamilton carefully describes an algorithm for obtaining the sequences of $\{P(S_t = i \mid Y_{t-1}\colon\ \boldsymbol{\lambda})\}_{t=1}^{T}$. The reader is referred to Hamilton 1994 for further details.

Table 7

Hansen's (1996) standardized likelihood ratio test of null of linear AR(4) representation of Danish inflation rate against Markov regime-switching AR(4) model under the alternative, 1958q3–1998q4

Statistics		*p*-value				
		$M = 0$	$M = 1$	$M = 2$	$M = 3$	$M = 4$
LR_T^*	5.009	0.000	0.000	0.000	0.001	0.001

Note: Grid used: p, q from 0.1 to 0.925 in steps of 0.075. Regression parameters from 0.01 to 0.61 in steps of 0.3. Total number of grid points: 104,976. M denotes bandwidth. Internal Monte Carlo replications: 1,000. $p_0 = p_1 = 4$.

device for selecting among nonlinear models. In particular they suggest using flexible nonlinear regression models as baseline models under the null and testing (in this case) the Markov regime-switching model against this broad class of alternative models in terms of recursive real-time forecast accuracy. Because the statistics suggested by Dahl and Hylleberg do not depend on any unidentified parameters, the approach turns out to be particularly useful for discriminating among models containing nonlinear components and parameters not identified under the alternative. The first step in the Dahl-Hylleberg procedure consists of generating a sequence of h-steps-ahead forecasts in real time. In our case we set $h = 1$ and select and estimate the models based on the initialization period, given by 1958q3–1979q4. Conditional on this information set a forecast of 1980q1 is produced. In the next step, the various models are selected and estimated on the period 1958q3–1980q1 and a real-time forecast for 1980q2 is made. This procedure continues until we have generated a sequence of true out-of-sample forecasts on the period 1980q1–1998q4 from the linear model, the Markov regime-switching regression model specification, Hamilton's flexible regression model, and the projection-pursuit regression model. Note that model selection, in terms of choosing the lags included in the regression model, occurs every time the data window is expanded. This is just a simple way to allow for time-varying influence of the nonlinear components in the regression model. LeBaron (1992) underlines the importance of this point by showing that nonlinearities often kick in for some periods and completely vanish in other periods. Since it is a relatively difficult task to apply the cross-validation model selection principle to the Markov regime-switching model, we base the model selection entirely on the Schwarz's information criterion (BIC), as recommended in Dahl and Hylleberg (1999).[7] Measures of absolute forecast performance are reported in the top part of Table 8. It is apparent that all of the nonlinear models have lower mean squared forecast error loss (MSE) than the linear model as well as lower mean absolute forecast error loss (MAD). Note also that the lowest point estimate of MSE is obtained using the Markov regime-switching model. Theil's U statistic gives the ratio of the mean squared error from the model under consideration relative to the mean squared error from the pure random walk model. Since the statistics are far below one in all cases, we can conclude that all the models forecast the inflation rate better than the simple random walk model. In Table 8 the outcome from the Granger-Newbold (1977) version of the Mincer-Zarnowitz regression of the actual value on a constant and the real-time forecasts is also reported. The coefficient of determination (R^2) from this regression can be applied directly as a measure of goodness of fit if the intercept in the Mincer-Zarnowitz regression is zero and the slope coefficient equals one. The reported values for the t-stat. ($intc = 0$) and t-stat. ($slope = 0$) are the p-values associated with the t-statistics of the null that the intercept equals zero and the t-statistics of the null that the slope equals one, respectively. The reported value in the F-stat. entry is the p-value associated with the joint hypothesis that the intercept and slope equal zero and one respectively. Again, the Markov regime-switching model performs rather well in the sense that it has the highest R^2 measure associated with it and that it is not possible, based on the simple t-statistics, to reject the null that the

[7] For the flexible regressions models cross-validation and Akaike's information criterion were also used, as in Dahl and Hylleberg (1999), however, the BIC-selected models in general turned out most successful in terms of "best" perfomance under the accuracy measures applied.

Table 8

One-period-ahead forecast performance in real time of quarterly growth rates in Danish consumer prices, 1980q1–1998q4

	MS-AR	LR	FNL	PPR-L
Mean BIC	0.792	0.443	0.585	0.503
Absolute forecast performance				
MSE	0.524	0.576	0.525	0.564
MAD	0.534	0.562	0.524	0.542
Theil's U	0.824	0.863	0.824	0.854
t-stat.($intc = 0$)[p-val.]	0.292	0.033	0.030	0.132
t-stat.($slope = 1$)[p-val.]	0.584	0.185	0.235	0.503
F-stat.[p-val.]	0.005	0.015	0.010	0.047
R^2	44.3	36.4	42.8	35.3
Directional forecast performance				
HM [p-val.]	0.031	0.031	0.000	0.000
χ^2 [p-val.]	0.030	0.030	0.000	0.000
CR	39.5	39.5	34.2	36.8
ϕ	24.9	24.9	35.5	30.8

Note: All models were selected by BIC. MS-AR: Markov switching AR(4); LR: linear regression; FNL: Hamilton's flexible regression; PPR-L: projection-pursuit regression.

intercept equals zero and the slope equals one. This feature is only shared by the PPR-L. A note of caution is needed here, however, since the F-statistics reject the joint hypothesis at a 5 percent level, implying that the R^2 is not a meaningful measure. We have also reported measures of the directional forecast performance of the various models in the lower part of Table 8. These consist of the Henriksson-Merton (1981) test (HM), the chi-squared test for independence, the confusion rate (CR), indicating how frequent the forecast is in a wrong direction, and finally the measure of the degree of diagonal concentration (ϕ), which can be interpreted almost like R^2 and hence is a measure of goodness of directional fit. Based on the HM statistics and the χ^2 statistics, independence between the directional forecast and the actual change in the inflation rate is rejected at a 5 percent level for all the models presented in Table 8. It is apparent, however, that the Markov regime-switching model does not perform as well as the two flexible regression models when it comes to predicting the change in the inflation rate one period ahead. Upon inspection of the point estimates on absolute forecast performance, the Markov regime-switching specification seems very promising, but to determine whether this performance actually is significantly more accurate, we turn to the Diebold-Mariano tests and to a range of forecast-encompassing tests. The results of the Diebold-Mariano test and the modified Diebold-Mariano test for evaluating relative predictive accuracy based on MSE and MAD loss function are reported in Tables 9 and 10, respectively. (For details on the Diebold-Mariano test and on the modified version, see Diebold and Mariano 1995 and Harvey, Leybourne, and Newbold 1997.) Even though the MSE and MAD produced from the Markov regime-switching model seem much lower than the MSE and MAD from the linear model, we are not able to reject the hypothesis that the two models have equal forecast accuracy. The predictive accuracy of the FNL model, however, turns out to be significantly better than the forecast accuracy of the linear model based on the MSE loss function, whereas both of the two flexible regression models significantly outperform the linear model in terms of accuracy when the MAD loss function is used.

It is important to note, however, that it is not possible to reject, at any reasonable significance level, the hypothesis that the Markov regime-switching model possesses the same predictive accuracy as the two flexible regression models, based on the Diebold-Mariano test. This result is independent of the choice of loss function.

As pointed out by Dahl and Hylleberg (1999), a statistic that is probably more powerful for discriminating among the forecast accuracy of nonlinear models is the forecast encompassing principle, particularly when

Table 9
Diebold-Mariano tests for relative predictive ability in real time: Quarterly growth rates in Danish consumer prices, 1980q1–1998q4

H_0	DM	MDM
MS-AR $\approx$ LR	0.475	0.480
FNL $\approx$ LR	0.001	0.012
PPR-L $\approx$ LR	0.482	0.487
MS-AR $\approx$ FNL	0.999	0.990
MS-AR $\approx$ PPR-L	0.589	0.801

Note: The squared error loss function is used in the DM and modified DM statistics. $\approx$ denotes equal relative predictive ability. Associated *p*-values reported. DM: Diebold-Mariano; MDM: modified Diebold-Mariano; MS-AR: Markov switching AR(4); LR: linear regression; FNL: Hamilton's flexible regression; PPR-L: projection-pursuit regression.

Table 10
Diebold-Mariano tests for relative predictive ability in real time: Quarterly growth rates in Danish consumer prices, 1980q1–1998q4

H_0	DM	MDM
MS-AR $\approx$ LR	0.387	0.393
FNL $\approx$ LR	0.000	0.000
PPR-L $\approx$ LR	0.027	0.031
MS-AR $\approx$ FNL	0.725	0.728
MS-AR $\approx$ PPR-L	0.801	0.803

Note: The absolute error loss function is used in the DM and modified DM statistics. $\approx$ denotes equal relative predictive ability. Associated *p*-values reported. DM: Diebold-Mariano; MDM: modified Diebold-Mariano; MS-AR: Markov switching AR(4); LR: linear regression; FNL: Hamilton's flexible regression; PPR-L: projection-pursuit regression.

based on the battery of robust statistics suggested by Harvey, Leybourne, and Newbold (1998). The results of forecast encompassing are reported in Table 11. Based on this test principle, it is not possible to reject the hypothesis that the Markov regime-switching model forecast encompasses the linear model, whereas it can clearly be rejected on the basis of all the statistics considered that the linear model forecast encompasses the Markov regime-switching model. Also, the FNL model forecast encompasses the linear model, whereas the opposite hypothesis can be rejected, again indicating and supporting the existence of a nonlinear component in the inflation rate process. In addition, the Markov regime-switching model forecast encompasses the FNL and PPR-L models, whereas the opposite hypothesis can be rejected on the basis of some of the statistics reported. Hence, from the empirical evidence reported in Tables 9–11, based on the general-to-specific

Table 11
Forecast encompassing tests for relative predictive ability in real time: Quarterly growth rates in Danish consumer prices, 1980q1–1998q4

H_0			R	R_s	R_1	R_{dm}	R_{mdm}
MS-AR	$\sqsubset$	LR	0.221	0.476	0.348	0.391	0.394
LR	$\sqsubset$	MS-AR	0.000	0.000	0.022	0.013	0.013
FNL	$\sqsubset$	LR	0.060	0.121	0.214	0.091	0.093
LR	$\sqsubset$	FNL	0.000	0.000	0.027	0.000	0.000
PPR-L	$\sqsubset$	LR	0.852	0.156	0.853	0.852	0.853
LR	$\sqsubset$	PPR-L	0.216	0.020	0.219	0.223	0.223
MS-AR	$\sqsubset$	FNL	0.061	0.324	0.170	0.230	0.233
FNL	$\sqsubset$	MS-AR	0.060	0.026	0.169	0.138	0.141
MS-AR	$\sqsubset$	PPR-L	0.199	0.141	0.347	0.379	0.382
PPR-L	$\sqsubset$	MS-AR	0.000	0.415	0.046	0.040	0.041

Note: $\sqsubset$ denotes encompassing. Associated p-values reported. DM: Diebold-Mariano; MDM: modified Diebold-Mariano; MS-AR: Markov switching AR(4); LR: linear regression; FNL: Hamilton's flexible regression; PPR-L: projection-pursuit regression.

approach for selecting among nonlinear models suggested by Dahl and Hylleberg (1999), it is not possible to reject the hypothesis that the Markov regime-switching model is an adequate nonlinear specification of the Danish inflation rate.

To specify the number of lags to include in the Markov switching model, we use Hamilton's flexible regression model approach. Without putting any assumption on the functional form of the regression model, it is possible to determine which lags contribute mostly to the nonlinear component of the inflation rate process, as described in section 2.3. The estimation results are reported in Table 12. The value of λ (the weight placed on the nonlinear component) is close to one and based on the p-value from a t-statistic, the presence of the nonlinear component in the inflation rate process seems highly significant. From the estimated FNL model we infer that the third and fourth lag enter significantly in the nonlinear component at a 5 percent level. Also the second lag seems to make a significant contribution to the nonlinear component, whereas the influence of the first lag is more limited. Hence, the basic message from the FNL model is that we should include up to four lags when modeling the nonlinear component. As a consequence, we chose to include four lags in the conditional mean function of the Markov regime-switching model.

The estimated parameters of the Markov switching model and the corresponding standard errors are reported in Table 13. As expected from the inference based on the FNL model, the dynamics of the inflation rate seem to differ substantially between the two regimes, and we see that the largest observed numerical difference between the estimated autoregressive coefficient is associated with the third and fourth lag, which corresponds almost perfectly to the results obtained from the FNL model.

Not surprisingly, there is a considerable difference between the mean and variance of the inflation rate across the two different regimes. In the high inflation rate regime ($S_t = 1$) the unconditional mean of the quarterly inflation rate ($\widehat{\mu}_1$) equals 1.7, whereas in the low inflation rate regime ($S_t = 0$) the quarterly inflation rate ($\widehat{\mu}_0$) equals 0.8. The estimators also confirm the conjecture of a positive relationship between the level of inflation and its volatility, which is often cited as a reason to pursue an economic policy aiming at low inflation (see Barro 1997). In the high inflation rate regime the degree of inflation rate uncertainty is clearly higher compared to the low inflation rate regime, as demonstrated by the fact that $\widehat{\sigma}_0^2 = 0.3$ whereas $\widehat{\sigma}_1^2 = 1.2$.

A comparison of the distribution of the predicted inflation rate based on the Markov switching model and the actual inflation rate reveals that the model provides a good estimate of the median of the distribution and that it seems to be able to produce the same fatness in the right-hand tail (see Figure 10). Contrary to that

Table 12
Maximum log-likelihood estimates of Hamilton's flexible regression model representation of Danish inflation rate, 1958q3–1998q4

	Linear part		
	Estimator	Std. error	p-value
π_{t-1}	0.117	0.084	0.083
π_{t-2}	0.197	0.085	0.011
π_{t-3}	0.221	0.086	0.005
π_{t-4}	0.087	0.089	0.165
Constant	0.608	0.241	0.006
σ	0.194	0.427	0.325
	Nonlinear part		
	Estimator	Std. error	p-value
$g_{\pi_{t-1}}$	0.368	0.234	0.059
$g_{\pi_{t-2}}$	1.413	0.784	0.037
$g_{\pi_{t-3}}$	3.230	0.924	0.000
$g_{\pi_{t-4}}$	2.132	0.988	0.016
λ	1.080	0.111	0.000
Log-likelihood			−229.18
Likelihood ratio (LRA)			6.73
2*LRA			13.46

Note: Calculation of the likelihood ratio test is done under the null that the inflation rate follows an AR(4) model. Again it should be emphasized that the test statistics do not follow any standard distribution asymptotically and hence should only be considered suggestive.

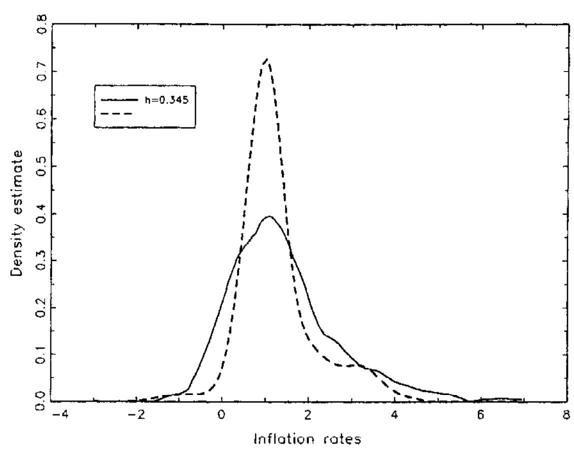

Figure 10
Density estimates, 1958q3–1998q4.
Note: Full line: Actual quarterly growth rates in Danish consumer prices; dashed line: one-step-ahead in-sample predictions produced from the Markov regime-switching model. Both density estimates are based on the Epanechnikov kernel with data-determined bandwidth $h = 0.345$.

from the AR(4) model the expectation bias from the Markov switching model is generated by a well-specified model.

Dynamic misspecification tests based on the prediction errors of the estimated Markov regime-switching model (see Table 14) do not reveal any violations of the underlying assumptions of the model. In neither inflation regime does there seem to be autocorrelation in the forecasts, nor are there signs of any serious

Table 13
Maximum log-likelihood estimates of Markov switching AR(4) representation of Danish inflation rate, 1958q3–1998q4

RHS variable	Regime $S_t = 0$		
	Estimate	Std. error	p-value
π_{t-1}	0.123	0.082	0.068
π_{t-2}	0.244	0.076	0.000
π_{t-3}	0.109	0.078	0.082
π_{t-4}	0.537	0.069	0.000
Constant	−0.013	0.085	0.559
σ_0	0.245	0.024	0.000
	Regime $S_t = 1$		
	Estimate	Std. error	p-value
π_{t-1}	0.128	0.101	0.103
π_{t-2}	0.162	0.099	0.052
π_{t-3}	0.241	0.099	0.001
π_{t-4}	0.018	0.105	0.431
Constant	0.790	0.262	0.000
σ_1	1.239	0.090	0.000
$p = P(S_t = 0 \mid S_{t-1} = 0)$	0.963	0.023	0.000
$q = P(S_t = 1 \mid S_{t-1} = 1)$	0.937	0.037	0.000
Log-likelihood			−191.941
Likelihood ratio (LRA)			43.972
2*LRA			87.944

Note: Calculation of the likelihood ratio test is done under the null that the inflation rate follows an AR(4) model. Again it should be emphasized that the test statistics do not follow any standard distribution asymptotically and hence should only be considered suggestive.

ARCH effects.[8] The same holds across regimes. Furthermore, it is not possible to reject the assumption that regime switching actually follows a stochastic, homogeneous Markov process at the traditional 5 percent significance level.

We also conduct a series of tests for the stability of the parameters of the linear model (see Table 15). Hamilton (1996) suggests the use of the statistics of Andrews (1993) in order to test whether there is evidence in favor of one or more additional shifts in the mean of the inflation rate not already accounted for. Based on Andrews' test statistics, the null hypothesis of a stable mean cannot be rejected.

Hansen (1992) suggests using the parameter stability test of Nyblom (1989) on the Markov regime-switching model. According to Nyblom's test as well, stability of the Markov regime-switching model cannot be rejected. Note, however, that the stability of the transition parameters is rejected at a 5 percent level but not at a 1 percent level. Hence, there appears to be no evidence of serious misspecification in the Markov regime-switching representation of the Danish inflation rate.

As a part of checking the selected nonlinear specification, Hamilton (2001) suggests comparing the shape of the conditional mean function of the flexible regression model with the shape of conditional mean function of the Markov regime-switching model by varying every variable included in the conditional mean in turn,

[8]Two types of tests were actually performed, namely, on one hand, well-known asymptotic tests for dynamic misspecification (cf. White 1987), with a chi-squared distribution, and on the other hand, Lagrange multiplier test (LM test), adopted recently by Hamilton (1996) for Markov switching time series models.

Table 14
Testing adequacy of Markov switching AR(4) representation of Danish inflation rate, 1958q3–1998q4

Test	Statistics	p-value
White's dynamic misspecification tests		
No autocorrelation	2.912	0.573
No ARCH effects	4.588	0.332
Validity of Markov assumption	5.031	0.284
Hamilton's LM tests for no autocorrelation		
No autocorrelation in regime $S_t = 0$	0.598	0.439
No autocorrelation in regime $S_t = 1$	0.736	0.391
No autocorrelation across regimes	0.034	0.853
Hamilton's LM tests for no ARCH		
No ARCH effects in regime $S_t = 0$	2.649	0.104
No ARCH effects in regime $S_t = 1$	2.078	0.149
No autocorrelation across regimes	4.581	0.032

Note: White's dynamic misspecification tests are all $\chi^2(4)$ distributed, whereas Hamilton's LM tests are $\chi^2(1)$ distributed asymptotically.

Table 15
Testing stability of Markov switching AR(4) representation of Danish inflation rate, 1958q3–1998q4

Test	Statistics	Asymptotical critical values		
		10%	5%	1%
Nyblom's stability tests				
All parameters stable	2.462	*2.295	*2.533	*3.035
Autoregressive parameters stable	0.926	2.295	2.533	3.035
Autoregressive parameters in $S_t = 0$ stable	0.482	**1.275	**1.462	**1.870
Autoregressive parameters in $S_t = 1$ stable	0.448	**1.275	**1.462	**1.870
Standard error parameters stable	0.123	0.607	0.748	1.107
Transition parameters stable	1.012	0.607	0.748	1.107
Andrew's stability tests				
No additional shifts in mean	4.689	7.170	8.850	12.350

Note: The critical values of Nyblom's statistics are only very sparsely tabulated and only up to 10 parameters/degrees of freedom. *critical value in the case of 10 degrees of freedom when 14 are actually needed. **approximate/interpolated critical value in the case of 5 degrees of freedom.

keeping the other variables fixed at some predetermined level. By using a Bayesian approach Hamilton provides a method to calculate 95 percent confidence bands around the partial conditional mean function of the FNL model. The partial conditional mean functions of the estimated FNL model, the estimated Markov regime-switching model, and the linear model with respect to all four lags, in addition to the 95 percent confidence bands produced from the FNL model, are presented in Figures 11–14.

Since the partial conditional mean functions produced by the Markov switching model all lie within the 95 percent confidence bands, it is not possible to reject the hypothesis that the inflation rate actually follows a Markov regime-switching model. Note, however, that the confidence bands actually are so wide that based on Hamilton's approach it is not possible to reject linearity either.

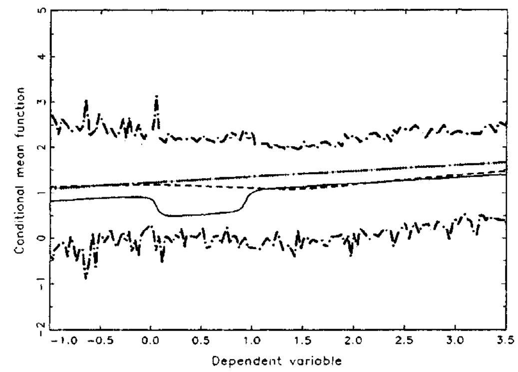

Figure 11
Posterior means $\widehat{E}(\mu(\pi_{t-1}, \overline{\pi}_{t-2}, \overline{\pi}_{t-3}, \overline{\pi}_{t-4}) \mid Y_T)$ as a function of π_{t-1} for $\overline{\pi}_{t-2}, \overline{\pi}_{t-3}, \overline{\pi}_{t-4}$ fixed at 1 and Y_T given sample observations on π_t, $\pi_{t-1}, \pi_{t-2}, \pi_{t-3}$, and π_{t-4}.
Note: Dashed line: posterior mean from the flexible regression model with 10,000 Monte Carlo draws with a fixed sample size of 162 observations covering the period 1958q3–1998q4; dotted-dashed line: 95% confidence intervals; full line: posterior mean from Markov regime-switching model; dotted line: posterior mean from linear model.

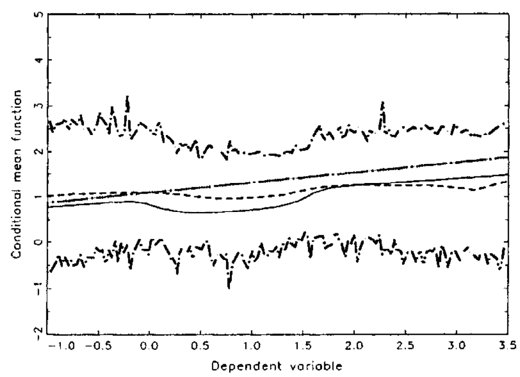

Figure 12
Posterior means $\widehat{E}(\mu(\overline{\pi}_{t-1}, \pi_{t-2}, \overline{\pi}_{t-3}, \overline{\pi}_{t-4}) \mid Y_T)$ as a function of π_{t-2} for $\overline{\pi}_{t-1}, \overline{\pi}_{t-3}, \overline{\pi}_{t-4}$ fixed at 1 and Y_T given sample observations on π_t, $\pi_{t-1}, \pi_{t-2}, \pi_{t-3}$, and π_{t-4}.
Note: Dashed line: posterior mean from the flexible regression model with 10,000 Monte Carlo draws with a fixed sample size of 162 observations covering the period 1958q3–1998q4; dotted-dashed line: 95% confidence intervals; full line: posterior mean from Markov regime-switching model; dotted line: posterior mean from linear model.

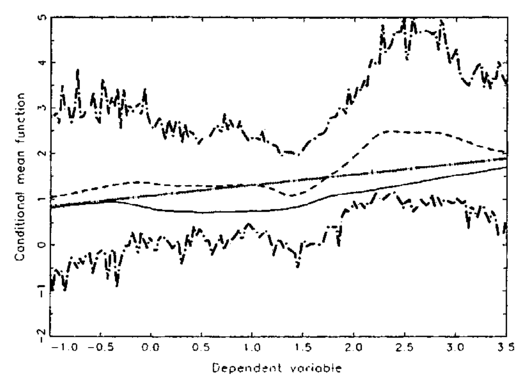

Figure 13
Posterior means $\widehat{E}(\mu(\overline{\pi}_{t-1}, \overline{\pi}_{t-2}, \pi_{t-3}, \overline{\pi}_{t-4}) \mid Y_T)$ as a function of π_{t-3} for $\overline{\pi}_{t-1}, \overline{\pi}_{t-2}, \overline{\pi}_{t-4}$ fixed at 1 and Y_T given sample observations on π_t, $\pi_{t-1}, \pi_{t-2}, \pi_{t-3}$, and π_{t-4}.
Note: Dashed line: Posterior mean from the flexible regression model with 10,000 Monte Carlo draws with a fixed sample size of 162 observations covering the period 1958q3–1998q4; dotted-dashed line: 95% confidence intervals; full line: posterior mean from Markov regime-switching model; dotted line: posterior mean from linear model.

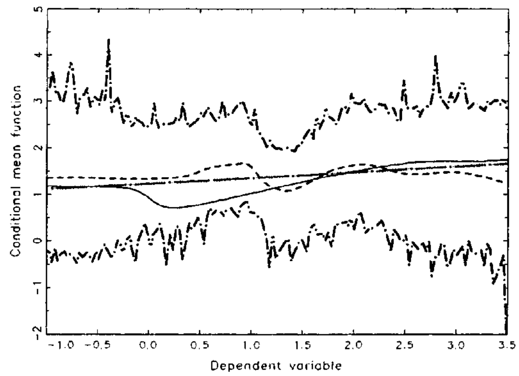

Figure 14
Posterior means $\widehat{E}(\mu(\overline{\pi}_{t-1}, \overline{\pi}_{t-2}, \overline{\pi}_{t-3}, \pi_{t-4}) \mid Y_T)$ as a function of π_{t-4} for $\overline{\pi}_{t-1}, \overline{\pi}_{t-2}, \overline{\pi}_{t-3}$ fixed at 1 and Y_T given sample observations on π_t, $\pi_{t-1}, \pi_{t-2}, \pi_{t-3}$, and π_{t-4}.
Note: Dashed line: Posterior mean from the flexible regression model with 10,000 Monte Carlo draws with a fixed sample size of 162 observations covering the period 1958q3–1998q4; dotted-dashed line: 95% confidence intervals; Full line: posterior mean from Markov regime-switching model; dotted line: posterior mean from linear model.

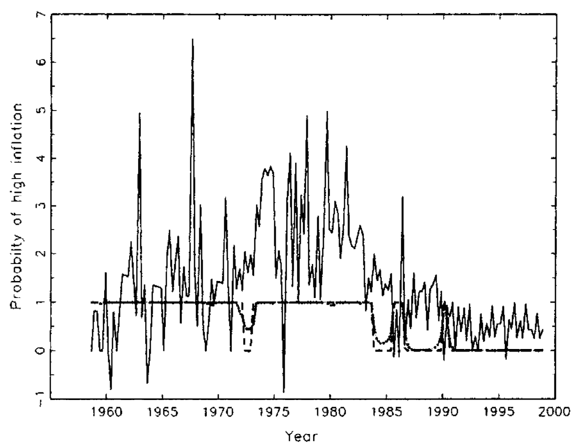

Figure 15
Classification of the inflation regimes in Denmark, 1958q3–1998q4.
Note: Full line: actual inflation; dashed line: smoothed probabilities of being in a regime with high inflation rates and high inflation uncertainty $\{P(S_t = 1 \mid Y_T)\}$; dotted line: classification of inflation regimes. High-inflation regime when $P(S_t = 1 \mid Y_T) > 0.5$.

5 Economic Interpretations

Having discussed all the statistical properties of the Markov regime-switching model, let us turn to the economic interpretations. From inspection of the smoothed probabilities and the classification of inflation regimes depicted in Figure 15, the inflation rate appears to be in the high-mean/high-variance regime in the periods from 1958 to the end of 1983 and 1986–1987, and again in 1990.

In the beginning of the 1980s the outlook for the Danish economy was very poor, in terms of both high unemployment and high government and of current account deficits as well. The credibility of economic policy was very low, and long-term interest rates were above 20 percent. In 1982 a new government took office and immediately announced radical measures to tackle the crisis situation. The program consisted of three main components: (1) an abolition of the wage indexation schedule, (2) a significant tightening of fiscal policy, and (3) a strong commitment to a fixed exchange rate policy. Important credibility was gained shortly after the announcement by keeping the exchange rate unchanged when Sweden, Denmark's second largest trading partner, devalued its currency by 16 percent. Evidence from financial data (see Christensen 1988) supports the hypothesis that this policy gained credibility for the country relatively quickly, leading to an almost instant reduction in the variability in exchange rates and a lowering of the inflation rate expectations. The fast reduction in both nominal interest rates and inflation after the policy shift confirms this. Our analysis confirms this claim to some extent, in the sense that the agents' perceptions of the underlying inflation rate regime shift from a high-mean/high-variance regime to a more stable low-mean/low-variance regime relatively quickly after the introduction of the new economic policy in 1982. Furthermore, our analysis indicates that the parameters of the inflation rate model are policy variant. In the one-regime AR(4) model this is exactly what leads to a contradiction of the basic assumptions of the model, implying nonrational forecasts. The Markov regime-switching model allows for the shifting parameters, but this may lead to ex post biased forecasts. In particular, the fixed exchange rate policy, despite the government's marked demonstration of its strong commitment to the policy shortly after the announcement in 1982, does not seem to have been considered fully credible, in terms of low inflation expectations, before the very late 1980s. This is confirmed by the gradual reduction in the country's long-term interest rate differential to Germany, from above 10 percentage points in 1982 to less than 1 percentage point in 1991.

In contrast to previous results, the credibility of the policy seems to disappear in 1986–1987, as implied by the smoothed probabilities of the model. Danish domestic demand was extremely buoyant in 1986–1987, driven by the fall of interest rates since 1982 and a sharp increase in house prices. The current account deficit peaked in 1986 at a level of 6 percent of GDP, and annual wage increases doubled in 1987 to around 10 percent. The return to high inflation expectations is confirmed by the fact that the long-term interest rate differential to Germany widened substantially in 1986 and 1987. The unification of Germany in 1990 and the implied massive growth in Danish exports to Germany also appear to have affected the inflation rate process to such an extent that there are indications of a temporary return to the high-mean/high-variance regime. In fact, the smoothed probabilities of the model reveal that every time there has been an upward pressure on the Danish inflation rate, agents have anticipated a return to the high-mean/high-variance regime.

Comparing the in-sample predictions from the Markov switching model with the actual inflation rate over the entire estimation period shows that the model is not able to generate ex post bias as of the size observed in the household-sector data, particularly not in the 1990s—not even when the forecast horizon is expanded to four quarters (see Figures 4 and 5). This conclusion changes when the inflation rate forecasts one year ahead are based on true out-of-sample predictions. In Figure 16 actual inflation rates and expected inflation from the linear model and the Markov regime-switching model one year ahead are compared. Our estimations seem to confirm the survey-based measures of inflation expectations (see above) in the sense of a protracted overprediction of the inflation rate in the low-inflation period of the 1990s, whereas the higher inflation rates, which occurred in much of the 1970s and early 1980s, were clearly underpredicted. Furthermore, note that the size of the ex post bias seems to diminish gradually, indicating that the household sector is attaching a decreasing probability to a return by the Danish economy to a high inflation rate regime. This seems plausible after a long period (seven to eight years) of historically low inflation rates in Denmark.

This suggests that the systematic forecast errors are due to a low, but non-zero probability of a regime shift one year ahead from today.

6 Conclusion

We have presented empirical evidence showing that the Danish inflation rate cannot be represented adequately by means of a simple linear regression model because of the presence of multiple and recurrent shifts in the mean and variance of the underlying stochastic process driving the inflation rate. By using Hansen's (1992) test and the general-to-specific approach of Dahl and Hylleberg (1999), we are not able to reject the Markov regime-switching representation as the preferred nonlinear specification of the conditional

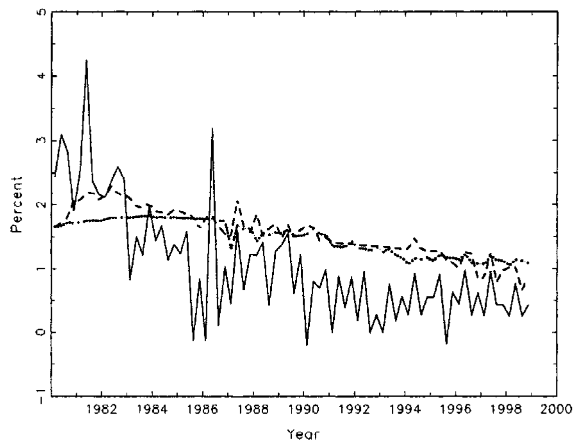

Figure 16
Actual and four-periods-ahead predictions in real time of the quarterly growth rates in Danish consumer prices.
Note: Full line: actual inflation; dashed line: real-time predictions from the Markov regime-switching model; dotted line: real-time predictions from the AR(4) model. Growing data window size, with initial period covering 1958q3–1979q1 and the final period covering 1958q3–1997q4.

mean function against a wide range of nonlinear and linear alternatives. Furthermore, we are not able to reject the hypothesis that the Markov regime-switching representation is dynamically well specified and that the assumption of constancy of the estimated parameters of this representation is satisfied.

Studies of survey data on households' inflation perceptions tend to support evidence of a persistent difference between actual and regression-based inflation rate expectations. Similar differences between actual and expected inflation are found in our analysis. Based on evidence of possible regime shifts in the Danish inflation rate process, we claim that part of this difference is due to an ex post bias. This implies that persistent deviations between actual and expected inflation cannot be taken as an argument against rational expectations but can rather be explained by perceived nonzero probabilities of a change in the inflation regime. If forecasters forming rational expectations about future inflation are confronted with an additional problem of identifying the inflation regime, systematic forecast errors over sustained periods may actually occur. Our result shows that in large parts of the 1970s and early 1980s the high rates of inflation in Denmark were underpredicted. The steep decline in inflation after 1982, followed by a more gradual disinflation throughout the second half of the 1980s and into the 1990s, has generally not been fully anticipated, hence implying an overprediction of the actual inflation rate. Moreover, contrary to what is sometimes argued in the literature (see, e.g., Christensen 1988), the estimated regime probabilities suggest that the hard currency policy introduced in Denmark in 1982 was not fully credible before well into the second part of the 1980s. This is also to some extent backed by the protracted decline in the interest rates. A short-lived shift to a perceived high-inflation regime in 1990 follows immediately after the German unification and is the forerunner of a persistent overprediction of the inflation rate in the 1990s, owing to a permanent nonzero probability of a change in regime. It must be emphasized that similar forecast errors were made by government and other official forecasters (see Christensen 1996) and hence do not indicate that useful information is systematically ignored.

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Appendix

Table A1
Augmented Dickey-Fuller test for nonstationarity of Danish inflation rate, 1958q1–1998q4: Estimation of auxiliary regression under inclusion of three lags of left-hand-side variable

RHS variable	ADF regression coefficients (t-ratio) For model including		
	No constant No trend	Constant No trend	Constant Trend
π_{t-1}	−0.083	−0.292	−0.316
	(−1.64)	(−3.15)	(−3.35)
$\Delta\pi_{t-1}$	−0.737	−0.578	−0.564
	(−8.55)	(−5.59)	(−5.44)
$\Delta\pi_{t-2}$	−0.459	−0.354	−0.347
	(−4.85)	(−3.52)	(−3.45)
$\Delta\pi_{t-3}$	−0.176	−0.127	−0.126
	(−2.27)	(−1.63)	(−1.62)
Constant	.	0.410	0.442
	.	(2.67)	(2.85)
Trend	.	.	−0.002
	.	.	(−1.30)
Residual sum of squares	183	175	173

Table A2
Augmented Dickey-Fuller test for nonstationarity of Danish inflation rate, 1958q1–1998q4: Estimation of the auxiliary regression under inclusion of four lags of left-hand-side variable

RHS variable	ADF regression coefficients (t-ratio) For model including		
	No constant No trend	Constant No trend	Constant Trend
π_{t-1}	−0.079 (−1.53)	−0.296 (−3.07)	−0.324 (−3.31)
$\Delta\pi_{t-1}$	−0.750 (−8.42)	−0.577 (−5.29)	−0.561 (−5.12)
$\Delta\pi_{t-2}$	−0.487 (−4.66)	−0.358 (−3.15)	−0.350 (−3.08)
$\Delta\pi_{t-3}$	−0.215 (−2.11)	−0.127 (−1.21)	−0.125 (−1.19)
$\Delta\pi_{t-4}$	−0.050 (−0.63)	−0.007 (−0.09)	−0.008 (−0.10)
Constant	. .	0.420 (2.65)	0.458 (2.86)
Trend	. .	. .	−0.003 (−1.42)
Residual sum of squares	182	175	172