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Bootstrap confidence intervals in a switching regressions model

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Abstract

Traditional switching regression methods produce slope and intercept estimates conditional on the change point estimate, with confidence intervals that overstate their precision. This paper describes the problem and a bootstrap alternative. Extensive sampling experiments confirm that the traditional methods overstate precision, and that bootstrap confidence intervals are far more accurate.

Keywords: Resampling; Change-point; Percentile; Parameter shift

JEL classification: C12; C14

1. Introduction

Applications of switching regression models are widespread in the recent empirical literature, inspired in part by recent advances in econometric theory and method. Much of the theoretical work, in articles such as Andrews (1993), has focused on obtaining optimal test statistics for parameter stability. This theoretical work is technically impressive and very useful, but it leaves some questions unanswered. For example, the applicability of asymptotic results to a finite sample, and the implications and validity of theoretical assumptions used in the proofs, is often unclear. Also, it is unclear how to extend the analytical results beyond the issue of testing for parameter stability.

In particular, the theoretical literature has largely ignored interval estimation for slope and intercept parameter estimates. Currently available techniques first estimate the location of the switch point, then calculate the regression slope and intercept parameters conditional on that location, *As if the switch point were known with certainty*. The results therefore overstate the precision of the slope and intercept estimates, and their naive use leads to faulty inference.

Theoretical work in this area has long been hampered by the intractability of the mathematics of the switch point estimator's distribution. Computation-intensive techniques such as the bootstrap complement theory because they use different assumptions, they are often more robust to small changes in the model or technique, and they may have better

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small-sample properties than methods based on asymptotic theory. Econometricians are making increased use of the bootstrap; see Jeong and Maddala (1993) and Vinod (1993) for analytic summaries of the literature. This paper provides evidence that the bootstrap can provide information about the timing of the regime switch, and that it can improve inference by accurately estimating the precision of regression parameter estimates.

This is not the first paper to use the bootstrap on the switching regression model. However, it is the first to explore the use of bootstrap confidence intervals of switching regression slope and intercept parameters. Douglas and Guilkey (1995) look at bootstrap standard errors in switching regressions, but they do not examine the much more interesting question of confidence intervals. Christiano (1992) presents a bootstrap test for a break in the trend in GNP. He does not consider interval or standard error estimators for parameter estimates, however, and his technique uses residuals generated under the null hypothesis of parameter stability. Hinkley and Schechtman (1987) use the 'conditional' boostrap to estimate the distribution of the switch point estimator. Their technique is far more expensive computationally than the technique under consideration in this paper, and its applicability to the unconditional distributions of $\hat{\tau}$ and $\hat{\beta}$ is unclear.

The paper is organized as follows. Section 2 describes the characteristics and problems of switching regressions estimation techniques in more detail, focusing on the least squares switching estimator. Section 3 describes the bootstrap approach to solving these problems. Section 4 presents the results of some Monte Carlo experiments, and Section 5 concludes.

2. Switching regressions methods

Quandt (1958) introduced the simple switching regression model with an unknown switch point to the econometrics literature. The model assumes a structural shift at some unknown point in the data set. If y_i and ε_i are scalars, x_i is a $1 \times k$ vector, and β_1 and β_2 are $k \times 1$ vectors, then the model with one switch point is

$$\begin{aligned} y_i &= x_i\beta_1 + \varepsilon_i\,, \quad i = 1, \ldots, \tau\,, \\ y_i &= x_i\beta_2 + \varepsilon_i \quad\;\; i = \tau + 1, \ldots, T\,. \end{aligned} \tag{1}$$

The switch point, τ, is the number of the last observation in the first regime, and is unknown. Its estimate, $\hat{\tau}$, is obtained by optimizing an objective function (e.g. minimizing the sum of squared residuals, or maximizing likelihood, or satisfying a moment restriction) separately for each reasonable value for τ. See Andrews (1993) for a 'partial-sample generalized method of moments' estimator that includes least squares (LS), maximum likelihood, and many others as special cases. Any such grid search for $\hat{\tau}$ will provide no interval estimates of $\hat{\tau}$ and will provide interval estimates for $\hat{\beta}_1$ and $\hat{\beta}_2$ that are conditional on the value of $\hat{\tau}$.

A few researchers have analyzed the distribution of the switch point estimator $\hat{\tau}$ in a regression model. Kim and Siegmund (1989) use the distribution of the likelihood ratio test statistic to derive confidence intervals for the switch point in a single equation linear regression model. Bai et al. (1991), following Picard (1985), derived asymptotic confidence intervals for the switch point estimator in a system of equations with stationary endogenous variables and regressors and a possibly changing intercept. (Asymptotic results are based on τ/T constant as $T \rightarrow \infty$.) These theoretical works, including Andrews (1993), agree that the moments of $\hat{\tau} - \tau$ are all asymptotically $O(1)$, and therefore $\hat{\tau}/T$ is consistent although $\hat{\tau}$ is not. Also, the precision of $\hat{\tau}$ increases with information about $\hat{\tau}$; that is, precision increases as the variance of the regression error ε decreases, and as the magnitude of the parameter change increases.

For finite samples, $\hat{\tau}$ and the slope and intercept estimator $\hat{\beta} \equiv [\hat{\beta}_1' \;\; \hat{\beta}_2']'$ are not independent, since a change in $\hat{\tau}$ changes the set of observations used to calculate $\hat{\beta}$. However, because the moments of $\hat{\tau} - \tau$ are all asymptotically $O(1)$, asymptotically the effect of changes in $\hat{\tau}$ on $\hat{\beta}$ becomes negligible. Therefore, the LS switching estimator, $\hat{\beta}$, is consistent, and the asymptotic covariance of $\sqrt{T}(\hat{\beta} - \beta)$ is asymptotically the same whether or not τ is known. But for finite samples, conditional ML or LS variance estimates for $\hat{\beta}$ are biased downward, resulting in confidence intervals that are 'too tight'. To see this, look at the variance decomposition of $\hat{\beta}_1$;

$$\mathrm{Var}(\hat{\beta}_1) = \mathrm{Var}_{\hat{\tau}}[\mathrm{E}(\hat{\beta}_1 \,|\, \hat{\tau})] + \mathrm{E}_{\hat{\tau}}[\mathrm{Var}(\hat{\beta}_1 \,|\, \hat{\tau})]\,, \tag{2}$$

so

$$\mathrm{E}_{\hat{\tau}}[\mathrm{Var}(\hat{\beta}_1 \,|\, \hat{\tau})] = \mathrm{Var}(\hat{\beta}_1) - \mathrm{Var}_{\hat{\tau}}[\mathrm{E}(\hat{\beta}_1 \,|\, \hat{\tau})]\,, \tag{3}$$

where $\mathrm{E}_{\hat{\tau}}$ and $\mathrm{Var}_{\hat{\tau}}$ are the expected value and variance with respect to the marginal distribution of $\hat{\tau}$. The traditional covariance matrix of $\hat{\beta}$ is calculated conditional on $\hat{\tau}$, and therefore estimates the left-hand side of (3), but the left-hand side of (2) is needed for inference on $\hat{\beta}$. The more sensitive $\mathrm{E}(\hat{\beta}_1 \,|\, \hat{\tau})$ is to changes in $\hat{\tau}$, and the greater the variability of $\hat{\tau}$, the greater will be the second term on the right-hand side of (3), and hence the greater will be the downward bias of $\mathrm{Var}(\hat{\beta}_1 \,|\, \hat{\tau})$ as an estimator of $\mathrm{Var}(\hat{\beta}_1)$. The larger this downward bias, the larger will be the discrepancy between the nominal and actual coverage of conditional confidence intervals based on the t distribution.

3. Bootstrapping

Despite recent progress, current techniques do not provide accurate small-sample interval estimates of $\hat{\tau}$ and $\hat{\beta}$. Bootstrap resampling techniques provide an attractive alternative for producing asymptotically correct confidence intervals and standard error estimates with superior small-sample properties. The basic insight of bootstrap resampling is that if a sample is a reasonable representation of the population, then repeated random resamples from that original sample are a reasonable representation of repeated samples from the population. Several techniques exist for bootstrapping regressions, discussed in Efron (1982), Freedman (1981), Wu (1986), LePage and Billard (1992), and Hall (1992). Bootstrapping the switching regression model requires a straightforward extension of these regression techniques.

First, estimate $\hat{\beta}_1$, $\hat{\beta}_2$, and $\hat{\tau}$ in the usual way, and then calculated the $\hat{\tau} \times 1$ residual vector e_1 from regime 1 and the $(T - \hat{\tau}) \times 1$ residual vector e_2 from regime 2. Then, to equate the variances of the residuals and true errors, rescale the residuals by multiplying e_1 by $\sqrt{\hat{\tau}/(\hat{\tau}-k)}$, and e_2 by $\sqrt{(T-\hat{\tau})/(T-\hat{\tau}-k)}$, where k is the number of regressors. Sample $\hat{\tau}$ times with replacement from the rescaled vector e_1 to get the $\hat{\tau} \times 1$ bootstrap sample vector e_1^{*1}, and sample $(T-\hat{\tau})$ times with replacement from the rescaled e_2 to get the bootstrap sample vector e_2^{*1}. Now create the bootstrap 'pseudodata' sample y^{*1}

$$y^{*1} = \begin{bmatrix} \hat{x}_1 & 0 \\ 0 & \hat{x}_2 \end{bmatrix} \begin{bmatrix} \hat{\beta}_1 \\ \hat{\beta}_2 \end{bmatrix} + \begin{bmatrix} e_1^{*1} \\ e_2^{*1} \end{bmatrix}$$
$$\equiv \hat{x}\hat{\beta} + e^{*1} .$$

In the first bootstrap iteration, re-estimate the model using the original matrix of regressors x, but with y^{*1} substituted for y, to get bootstrap estimates $\hat{\beta}_1^{*1}$, $\hat{\beta}_2^{*1}$, and $\hat{\tau}^{*1}$. Repeat the process some large number B times. On each iteration, resample from the original e_1 and e_2 some large number B times, and on each iteration recalculate a new pseudodata vector y^{*b}, $b = 1, \ldots, B$. Regress each y^{*b} on x to obtain B bootstrap estimates $\hat{\beta}_1^{*b}$, $\hat{\beta}_2^{*b}$, and $\hat{\tau}^{*b}$, $b = 1, \ldots, B$.[1]

Given the B bootstrap parameter estimates, the bootstrap standard error of an estimator $\hat{\theta}$ is

$$\mathrm{STD}^*(\hat{\theta}) = \sqrt{\frac{1}{B-1} \sum_{b=1}^{B} (\hat{\theta}^{*b} - \hat{\theta}^{*\cdot})^2} , \tag{4}$$

where $\hat{\theta}^{*\cdot}$ is the average of the B bootstrap estimates $\hat{\theta}^{*b}$. Of greater practical interest, however, is the bootstrap confidence interval. There are many alternative ways to calculate bootstrap confidence intervals (see Hall, 1992, for a summary of methods). The method with the strongest basis in statistical theory is the 'percentile' method and its various refinements. In general, a one-sided lower interval with confidence level α is bounded by $c_{\alpha L}$ such that

$$\Pr\{\theta \leq \hat{\theta} + c_{\alpha L}\} = \alpha . \tag{5}$$

To apply the percentile method, replace θ with $\hat{\theta}$, and replace $\hat{\theta}$ with the bootstrap estimate $\hat{\theta}^*$,

$$\Pr\{\hat{\theta} \leq \hat{\theta}^* + c_{\alpha L}\} = \alpha , \tag{6}$$

leading to the estimator $\hat{c}_{\alpha L}$, which is the distance from $\hat{\theta}$ to the $1-\alpha$ percentile of $\hat{\theta}^*$. The resulting one-sided lower confidence interval is

[1] Other corrections are necessary if ε is not i.i.d., or y follows an ARIMA process. For example, if ε follows an ARMA process, then various 'moving block' bootstrap methods may be used to calculate the $\hat{\beta}^*$ matrix (see Lahiri, 1992). Wu (1987), and comments thereafter, suggest corrections for heteroskedasticity. There is also a large literature, e.g. Rayner (1990), on bootstrapping when y follow an ARIMA process. See Jeong and Maddala (1993) for a summary.

$$I_{\alpha L} = (-\infty, \hat{\theta} + \hat{c}_{\alpha L}) .$$

The corresponding one-sided upper confidence interval would be $I_{\alpha U} = (\hat{\theta} - \hat{c}_{\alpha U}, \infty)$, where $\hat{c}_{\alpha U}$ is the distance from $\hat{\theta}$ to the α percentile of $\hat{\theta}^*$.

The literature describes several methods for creating two-sided percentile confidence intervals. One method, sometimes referred to as the 'equal tail percentile method', constructs the two-sided α confidence interval as the intersection of the upper and lower $(1+\alpha)/2$ confidence intervals:

$$I_{\alpha 2} = (\hat{\theta} - \hat{c}_{(1+\alpha)/2,U}, \hat{\theta} + \hat{c}_{(1+\alpha)/2,L}) . \tag{7}$$

This method produces intervals that are equally likely to lie above or below the true parameter.

Percentile-t bootstrap confidence intervals represent a refinement of the percentile method, achieved by transforming the estimator $\hat{\theta}$ into a pivotal statistic. A pivotal statistic is one whose distribution is (asymptotically, at least) not dependent upon unknown parameters. Under general conditions, percentile-t confidence intervals are more accurate than percentile confidence intervals for asymptotically normally distributed statistics such as $\hat{\beta}$.[2] To create a percentile-t confidence interval for a switching regression slope or intercept regression parameter j, regime k, β_{kj}, first 'studentize':

$$\frac{(\hat{\beta}^{*b}_{kj} - \hat{\beta}_{kj})}{\hat{\sigma}^{*b}_{\beta kj}} , \tag{8}$$

where $\hat{\beta}_{kj}$ is the estimate based on the original data set, and $\hat{\beta}^{*b}_{kj}$ and $\hat{\sigma}^{*b}_{\beta kj}$ are the bth bootstrap iteration estimate and its standard error. Now calculate confidence intervals using percentiles of (8), and with the intervals

$$J_{\alpha 2} = (\hat{\beta}_{kj} - \hat{\sigma}_{\beta_{kj}} \hat{c}_{(1+\alpha)/2,U}, \hat{\beta}_{kj} + \hat{\sigma}_{\beta_{kj}} \hat{c}_{(1+\alpha)/2,L}) , \tag{9}$$

where $\hat{\sigma}_{\beta_{kj}}$ is an estimate of the standard error of $\hat{\beta}_{kj}$, preferably the bootstrap estimator calculated as in (4). Note that $\hat{\sigma}^{*b}_{\beta_{kj}}$ (unlike $\hat{\sigma}_{\beta_{ij}}$) must be calculated separately for each bootstrap iteration b. The conditional LS standard error estimator may be used for $\hat{\sigma}^{*b}_{\beta_{kj}}$ despite its small-sample bias, since a statistic need only be asymptotically pivotal to have desirable properties.[3]

[2] In general, the percentile-t provides improvement over the percentile (reducing coverage error by a factor of $T^{-1/2}$) when the estimator $\hat{\theta}$ is a sufficiently smooth function, and when there is a stable estimator of the standard error. See Hall (1992) for assumptions used in proofs. Wu (1986) and Lahiri (1992) discuss properties for more general models.

[3] Alternatively, at considerable computational cost, $\hat{\sigma}^{*b}_{\beta_{kj}}$ may be estimated using the iterated bootstrap.

4. Sampling experiments

I tested the performance of the bootstrap using an extensive set of sampling experiments. Each experiment used 1000 Monte Carlo repetitions and 200 bootstrap repetitions, with regressors taken from a uniform distribution and errors from a normal distribution. The main performance criterion is the coverage ratio of the confidence intervals (i.e. the percentage of Monte Carlo repetitions in which the confidence interval contains the true parameters).

Table 1 shows specific results for the switch point and regime 1 slope estimators in six different sampling experiments. (Results for the other slope and intercept estimators are qualitatively identical, and are omitted for brevity.) The first four lines of the table detail the characteristics of each model, with the differences from model A shown in italics. Standard error estimates are low, but bootstrap estimates outperform conditional estimates, and slope estimates outperform switch point estimates. Bootstrap standard errors are more accurate where the number of observations T is large (experiment B) or the switch is well-defined owing to large parameter change (experiment D) or low error variance (experiment F).

Bootstrap slope parameter confidence intervals performed well across the board, both absolutely (i.e. their coverage is close to the nominal confidence interval of 95%), and in comparison with their conditional counterparts. Despite their theoretical advantage, the percentile-t confidence intervals did no better than the percentile intervals. Bootstrap switch point confidence intervals show consistently low coverage, however, and were outperformed

Table 1
Monte Carlo results for basic experiments

Experiment	A	mcstd[a]	B	mcstd	C	mcstd	D	mcstd	E	mcstd	F	mcstd
Number of observations	100		500		100		100		100		100	
τ/T	0.5		0.5		0.7		0.5		—		0.5	
β_2/β_1	1.5		1.5		1.5		2		1		1.5	
Std(ε)	5		5		5		5		5		1	
Switch point τ	50		250		70		50		50		50	
Benchmark[b] std	12.1		15.6		14.7		4.32		20.2		0.971	
Bootstrap std	10.3	*3.08*	13.0	*4.76*	11.2	*3.8*	4.04	*1.40*	16.1	*2.9*	0.940	*0.236*
τ Coverage ($\alpha = 0.95$)												
Percentile	0.745		0.947		0.687		0.904				0.862	
Normal[c]	0.884		0.930		0.850		0.915				0.930	
Regime 1 Slope β	8		8		8		8		8		8	
Benchmark[b] std	3.31		1.12		3.05		2.86		4.50		0.543	
Bootstrap std	3.11	*0.594*	1.10	*0.085*	2.88	*0.683*	2.69	*0.361*	3.69	*0.791*	0.520	*0.060*
Conditional std	2.62	*0.465*	1.09	*0.058*	2.37	*0.507*	2.57	*0.288*	2.82	*0.798*	0.516	*0.054*
β Coverage ($\alpha = 0.95$)												
BS percentile	0.942		0.950		0.939		0.923		0.904		0.936	
BS percentile-t	0.945		0.948		0.937		0.930		0.898		0.939	
Conditional	0.896		0.946		0.891		0.910		0.796		0.933	

[a] 'mcstd' is the standard error of the standard error estimate over the Monte Carlo experiment.
[b] The benchmark std is the standard error of the statistic over 10,000 independent Monte Carlo repetitions.
[c] 'Normal' confidence intervals for $\hat{\tau}$ were generated by multiplying the bootstrap standard error by 1.96.

by theoretically baseless 'normal' confidence intervals.[4] Again, the bootstrap performed better in the 'high information' experiments B, D, and F.

Table 2 summarizes the performance of bootstrap and conditional confidence intervals in 163 Monte Carlo experiments. Column 2 lists the sample mean and standard deviation of coverage over the 163 experiments. Again, bootstrap confidence intervals for the slope parameter performed very well, with little difference between the percentile and percentile-t. Confidence intervals for τ and conditional confidence intervals for β, on the other hand, exhibited average coverage well below the nominal 95%, and high variability in coverage.

Columns 3–9 of Table 2 contain results of four regressions that use the coverage ratios as dependent variables. The coefficient estimates show how coverage of each estimator varies with the amount of parameter change β_2/β_1, the variance of the error term σ_ε, and first and second powers of the inverse of the square root of the number of observations T and the number of bootstrap repetitions B. (The response surface regression equation may be interpreted as a stochastic expansion of the estimates in powers of $T^{-1/2}$. As such, the coefficients of $T^{-1/2}$ and T^{-1} indicate skewness and kurtosis, respectively.) Because coverage in virtually all 163 experiments lay below 95%, a positive coefficient indicates that coverage error decreases as the value of the corresponding variable rises. The adjusted R^2 for the bootstrap β regressions is much lower than in the other two regressions, indicating that the bootstrap β intervals perform nearly as well in small samples and noisy data as they do under better conditions. Results for β_2/β_1 and σ_ε also indicate that both the τ and conditional β intervals improve as the switch is better defined, but the bootstrap intervals for β show less improvement, since they already take into account the additional variability of $\hat{\beta}$ induced by uncertainty of $\hat{\tau}$.

The performance of all estimators improves when the change in the regression parameters β_2/β_1 increases or the error variance σ_ε decreases. Performance also improves uniformly for all estimators as T increases. Somewhat surprisingly, the regression results show no significant

Table 2
Response surface results: Coverage of 95% nominal confidence intervals (CIs)

Dependent Variable:		AVg covg	Intrcpt	β_2/β_1	σ_ε	$T^{-1/2}$	T^{-1}	$B^{-1/2}$	B^{-1}
τ, Percentile CI coverage		0.793	0.974*	0.210*	−0.041*	−4.39*	7.30	0.940	−5.23
adj $R^2 = 0.81$	std	0.126	0.106	0.035	0.0022	1.31	6.70	1.33	8.83
β, Percentile CI coverage		0.940	0.940*	−0.017*	0.00007	1.27*	−8.83*	−0.266	1.43
adj $R^2 = 0.47$	std	0.017	0.024	0.008	0.0005	0.293	1.50	0.299	1.92
β, percentile-t CI coverage		0.941	0.959*	−0.0018	−0.002*	0.425	−3.71*	−0.350	2.14
adj $R^2 = 0.33$	std	0.013	0.022	0.007	0.0004	0.265	1.36	0.271	1.79
β, conditional CI coverage		0.905	0.981*	0.034*	−0.009*	−0.922*	−0.053		
adj $R^2 = 0.77$	std	0.032	0.27	0.011	0.0007	0.428	2.18		

"*" indicates significance levels better than 5%.

[4] Note that Bai et al.'s (1991) analytic confidence intervals for $\hat{\tau}$ also exhibit coverage ratios significantly smaller than their nominal confidence level.

effect of increasing the number of bootstrap repetitions B, indicating that the smoothness of the bootstrap approximation may not be a major issue. (The set of experiments included 24 runs of experiment A, with B varying between 50 and 1000.) A look at the raw data, however, shows improvement as the number of bootstrap repetitions increases from 50 to 250, but no consistent pattern as B rises from 275 to 1000. A conservative approach to estimation therefore would use at least 300 bootstrap repetitions.

5. Summary and conclusions

This paper describes a problem with estimators in the switching regression model, suggests a bootstrap alternative, and evaluates bootstrap confidence intervals using a large set of sampling experiments. By taking into account the effect of the uncertainty of $\hat{\tau}$ on the precision of the coefficient estimates, the bootstrap provides accurate confidence intervals for the regression parameter estimates. In particular, both established theory and the sampling experiments reported here suggest that percentile-t method provides accurate confidence intervals that represent a real improvement over previous methods.

For the switch point, the performance of the bootstrap confidence intervals is less impressive. One possible avenue for improved performance suggested by the literature is the iterated bootstrap, in which a second bootstrapping procedure is performed on the residuals from each bootstrap iteration. The second bootstrap can be used to prepivot switch point estimates, either by providing standard errors for studentization, or by the inverse distribution function method recommended by Beran (1987). Of course, the cost in terms of computation time is considerable, but that cost should continue to decline quickly over time. However, in a small set of Monte Carlo experiments (not reported here), the iterated bootstrap showed no promise. Beran's technique created confidence intervals with even lower coverage than the single-iteration method reported here, and studentization methods provided marginal or no improvement. The best alternatives at present appear to be the single bootstrap method or, if your model and data are appropriate, the asymptotic results in Bai et al. (1992). Results from either method should be treated with caution, and most likely will produce intervals that underestimate the variability of $\hat{\tau}$.

References

Andrews, D.W.K. 1993, Tests for parameter instability and structural change with unknown change point, Econometrica 61, 812–856.

Bai, J., R.L. Lumsdain and J.H. Stock, 1991, Testing for the dating breaks in integrated and cointegrated time series, Manuscript, Kennedy School of Government, Harvard University.

Beran, R., 1987, Prepivoting to reduce level error of confidence sets, Biometrika 74, 457–468.

Christiano, L.J., 1992, Searching for a break in GNP, Journal of Business and Economic Statistics 10, 237–250.

Douglas, S. and D.K. Guilkey, 1995, Using the bootstrap to improve standard error estimates in a switching regressions model, Communications in Statistics, in press.

Efron, B., 1982, The Jackknife, the bootstrap, and other resampling plans, Philadelphia: Society for Industrial and Applied Mathematics.

Freedman, D.A., 1981, Bootstrapping regression models, The Annals of Statistics 9, 1218–1228.
Hall, P., 1992, The bookstrap and Edgeworth expansion (Springer-Verlag, New York).
Hinkley, D.V. and E. Schechtman, 1987, Conditional bookstrap methods in the mean-shift model, Biometrika 74, 85–93.
Jeong, J. and G.S. Maddala, 1993, A perspective on application of bookstrap methods in econometrics, in: G.S. Maddala, C.R. Rao, and H.D. Vinod, eds., Handbook of statistics, Vol. 11 (North Holland, Amsterdam) 573–610.
Kim, H.-J. and D. Siegmund, 1989, The likelihood ratio test for a change-point in simple linear regression, Biometrika 76, 409–423.
Lahiri, S.N., 1992, Edgeworth correction by 'moving block' bootstrap for stationary and nonstationary data, in: R. LePage and L. Billard, eds., Exploring the limits of bootstrap (John Wiley and Sons, New York) 183–214.
LePage, R. and L. Billard (eds.), 1992, Exploring the Limits of Bootstrap (John Wiley and Sons, New York).
Picard, D., 1985, Testing and estimating change-points in time series, Advances in Applied Probability 176, 841–867.
Quandt, R.E. (1958), Estimation of the parameters of a linear regression system obeying two separate regimes, Journal of the American Statistical Association 53, 873–880.
Rayner, R.K., 1990, Bootstrap tests for generalized least squares regression models, Economics Letters 34, 261–265.
Vinod, H.D., 1993, Bootstrap methods: Applications in econometrics, in: eds., G.S. Maddala, C.R. Rao and H.D. Vinod, Handbook of statistics, Vol. 11, (North-Holland, Amsterdam) 629–662.
Wu, C.F.J., 1986, Jackknife, bootstrap, and other resampling methods in regression analysis, The Annals of Statistics 14, 1261–1295.