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Economic evolution and the science of synergetics*

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Abstract. This paper deals with synergetic methods, which have developed as a sub-field of the self-organisation approach in the natural sciences. Such methods have been used successfully to model structural transitions in physio-chemical contexts. The synergetic approach is explained in a non-technical way and the main elements of the synergetic methodology are introduced. The extent to which such methods can be applied in the presence of historical time series data, which are subject to underlying processes of evolutionary economic change, is assessed. Proposals, concerning more appropriate synergetic methods for evolutionary economic application, are considered.

Key words: Synergetics – Self-organisation – Time-irreversibility – Evolution – Structural change

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I Introduction

Over the past decade, there has been a significant increase in interest in what may be termed 'evolutionary' approaches in economics. Several assessments of the

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associated literature have appeared in recent years, confirming that the field of evolutionary economics is now an established branch of the economics discipline.[1] Unlike older evolutionary approaches, to be found in the works of Joseph Schumpeter and in a number of American institutionalist contributions, modern evolutionary economics contains numerous attempts to provide formal representations of evolutionary change in economic systems.[2] Game theoretic approaches pioneered in evolutionary biology by John Maynard Smith and Robert Trivers and taken up in economics by, among others, Robert Axelrod and Kenneth Binmore have been popular.[3] The use of dynamical mathematics in economics pioneered by Paul Samuelson and extended into discrete, non-linear settings by, for example, Richard Day gave rise to attempts to provide formal representations of evolutionary processes. Faced with solution difficulties and verification problems, in the presence of historical data, simulation and calibration have become popular investigative procedures. Nelson and Winter (1982) initiated a tradition in evolutionary economics whereby insights were offered as to the selection mechanisms that could give rise to simulated non-linear paths. More recently, approaches to non-linear dynamics as developed at, for example, the Santa Fe Institute, following the seminal Anderson et al. (1988) volume have also proved increasingly popular as a way of examining, theoretically, the operation of evolutionary dynamics in economic settings.

What is viewed as 'intertemporal dynamics' provides a technical language within which theoretical representations such as evolutionary stable strategies, multiple equilibria and chaotic attractors can be expressed. Yet, despite the genuine advances made, particularly in demonstrating the poverty of much of past economic theory, the approaches are intended to increase the complexity of logical deduction. The usefulness of the resultant deductive mathematical models in interpreting historical time series data is strictly limited. There is no doubt that it is possible to discover a differential equation representation of timeless theory that can mimic a real process. Provided some clever 'storytelling' is introduced, it is possible to tempt listeners and readers into thinking the theory in question has some explanatory power in historical contexts and even some relevance to actual economic policy-making. However, in general, such theorising cannot address historical processes, except as an approximation in certain circumstances, because it is *timeless* theorising.

The existence of time subscripts on variables and the acknowledgment that unique equilibria are rare does not mitigate the timelessness of such theory. The old objections to timeless theorising, formalised through mathematical expression, offered by the unorthodox schools of economic thought, appear to be as valid as ever with respect to the new dynamical mathematics (see Blatt 1983a). However, this problem poses a great difficulty because to refrain from applying mathematics in attempting to understand economic processes unfolding in history is to reduce

[1] Hodgson (1993a) offers an institutionalist perspective and Witt (1993a) offers a neo-Austrian one. Andersen (1994) and Nelson (1995) present neo-Schumpeterian perspectives. Taken together, the bibliographies of these reviews offer a comprehensive record of the recent literature on evolutionary economics.

[2] An extensive literature also exists on the formalisation of structural change in economic systems from a broader perspective than evolutionary economics. See, for example, Baranzini and Scazzieri (1990) for a good survey of this literature.

[3] See Vromen (1995) for an assessment of the related 'new institutionalist' appraoch to economic evolution.

greatly the scientific relevance of economics. A century ago, Alfred Marshall was acutely aware of this problem and it led him to restrict the application of neoclassical economic principles to short-period and partial equilibrium settings, where it could be shown that static approximation is reasonable because of the near-absence of structural change. He tailored his approach to approximate situations where time irreversibility and system boundaries existed. Furthermore, he stressed that the logic of the neoclassical long-period did not hold in history and that economic analysis of an evolutionary type was necessary to deal with the long run (see Foster 1993). However, as Hodgson (1993b) stresses, Marshall never succeeded in creating a formal evolutionary representation of economic processes, and post-Marshallian neoclassical economics gradually espoused the position that the long period could be represented by mathematically tractable 'fixed structure' representations set in abstract time.

In the early 1990s, Costas Azariadis observed that "*... neoclassical growth theory and related dynamical approaches have spilled out of their traditional long-run domain and become routinely employed in exploring short-run phenomena.*' (Azariadis 1993 p.xi). Thus, the eclipse of Marshall's position was complete: timeless Walrasian representations of the long run had not only replaced his evolutionary vision but had also replaced his carefully constructed theory of the short run with 'disequilibrium dynamical' mathematics. Within this body of logic, evolutionary economics become no more than non-linear 'economies of scale' in the long period and/or a form of non-linear disequilibrium in the short period.

In postwar Europe, many evolutionary economists came to understand, as Marshall did, the severe limitations of mathematical representations of economic evolution. They saw such evolution as embedded in historical time and, as such, it came to be viewed as an endogenous and non-equilibrium process exhibiting three inter-connected characteristics: first, there is a degree of time irreversibility in all economic processes; second, there must be ongoing structural change since structural reversal is restricted in the face of exogenous shocks; and, third, true uncertainty must exist if structural change is to be an ever-present feature of systems. In such circumstances, equilibrium/disequilibrium dichotomies and deterministic mathematical representations of growth paths came to be viewed as, at best, incomplete and, at worst, inappropriate representations of evolutionary processes. As a result, contemporary evolutionary approaches to economics in Europe have placed much less emphasis on mathematical formalism.

Institutionalists have stressed the importance of time irreversibility and provided historical studies of the evolution, persistence and collapse of particular institutions and regulations. Neo-Schumpeterians have stressed the importance of ongoing structural change and offered numerous case studies of the process of innovation and structural change. Neo-Austrians have stressed the importance of 'true uncertainty' and offered conjectures as to the nature of novelty, variety and the emergence of 'spontaneous order'. However, despite the almost universal rejection of timeless mathematical deduction as the foundation of economic analysis in these strands of evolutionary economics, there has been acknowledgment that evolutionary trajectories in history have some kind of non-linear mathematical form, identifiable in particular phases of evolution. In recent literature, the language of non-linear dynamics has been employed frequently to explain why the outcomes of discontinuous structural transitions we observe in historical data are unpredictable. In this respect, mathematics has come to be used much more in a descriptive, rather than a deductive, way.

Within all three identified strands of evolutionary economics we find economists who have explored the possibility that the self-organisation approach, now popular in the natural sciences, can offer an analytical framework within which these formalisable tendencies in economic history can be expressed (see Foster 1994b). The attraction of the self-organisation approach lies in the fact that the system dynamics it contains are very different to those in the conventional equilibrium/disequilibrium framework. Instead of a process beginning as a stable, stationary state, and which after being shocked into an unstable disequilibrium state (perhaps requiring a 'saddle path' restriction) returns to a new stable stationary state, the self-organisation approach envisages the stationary state itself as unstable, requiring a small shock to enact a relatively stable process of structuration, culminating in a new unstable stationary state. Instability is associated with non-linear structural transitions and it is the conditions which induce such change that have become the focus of attention.

History is rich with examples of both structural discontinuities and robust processes of structuration that follow logistic diffusion curves, the latter, of course, being mathematical representations of path-dependent growth processes that tend towards a boundary limit (see Foster 1995). Economic models of this type can be built upon modified population ecology models drawn from biology (see, for example, Murray 1989; Peschel and Mende 1986). The main difficulty with such models is that self-organisation involves endogenous, non-equilibrium processes, while mathematical representations are deterministic in nature. Thus, the reasons for the onset of structural instability and structural transition must reside outside such equations, either in higher, non-linear moments or in the properties of statistical residuals if we are dealing with an estimated logistic model.

In the natural sciences there is now a significant literature that examines these transitional properties in the presence of self-organisation. This is known as the 'science of synergetics'. The question arises as to whether this science offers evolutionary economists an analogy, a metaphor or simply an inspiration in understanding the structural transition from one non-linear path to another, in a different self-organisational context. Does synergetics offer a new way in which mathematics and statistics can be employed in economics to understand the dynamics of historical economic processes? A literature has already begun to emerge on this question, but it has been initiated, in the main, by non-economists. The purpose of the remainder of this paper is to approach this question from an explicit evolutionary economic perspective. In Section II, we explain, briefly, how the self-organisational perspective on evolutionary change led to the development of synergetic methods. Section III evaluates, critically, the synergetic methodology. In Section IV we evaluate the extent to which synergetics is applicable in economics and explain why the approach has to be modified before it can be used effectively in economic settings. Section V contains some concluding comments.

II The synergetic approach

Many mainstream economists appear to reject any approach that cannot be expressed in mathematical language. Natural scientists who apply the self-organisation approach also require mathematics both in their theoretical representations and in their empirical investigations. Indeed, many of the mathematical methods reviewed in, for example, Azariadis (1993) are used in specific self-organisational contexts, particularly in the physical sciences. However, a key methodological difference exists: although actual processes, which are viewed as evolutionary in character, are represented mathematically, the goal is the discovery of a *descriptive abstraction* of a process of structuration over time, rather than an a priori *deductive apparatus* set in a timeless and structurally-fixed dimension. It is well-understood that the structural change embodied in evolutionary processes in historical time often precludes the application of the usual rules of logical consistency. Instead of mathematics being an end in itself, dictating the restrictive form that structure must adopt for deductive convenience, mathematics becomes indispensable in describing the general character of a historical process.

Typically, formal economic theory removes time and homogenises, in some sense, space in order to be general in a mathematical sense. In a self-organisational context, mathematics can be employed to do the opposite. It can describe, in an abstract way, a path-dependent historical process and, also, it can be employed to provide a formal representation of the forces in space that affect a process at each point in history. The latter can be investigated through experimental controls or through statistical inference, if appropriate data exist. With the help of mathematical statistics, synergetic methods can be applied to assess the likelihood that a system will undergo structural transformation and, consequently, exhibit nonlinearity in its growth dynamics.

In what follows, however, we shall not be advocating the uncritical imitation of existing synergetic methods in economic contexts. Natural scientists have, to a significant extent, continued to construct non-linear mathematical models of processes upon the presumption that there are asymptotic tendencies towards deterministic stationary states. Conventional dynamic representations are overlaid with propositions that the process in question exhibits irreversible tendencies in structuration that will induce structural transition in a stationary state, despite the determinism embodied in the chosen mathematical model. Users of synergetic methods seek to identify such transition in the higher, non-linear moments of the process under consideration. Such a way of proceeding is sensible in systems which are only physio-chemical in nature and where repeated experimentation is feasible. We shall argue that economic systems, like many biological systems, exhibit more sophisticated forms of self-organisation, which are more open, more non-stationary and more endogenous in character. These differences require explicit recognition in the development of an appropriate synergetic approach. The time-irreversibility, structural change, and true uncertainty that exist in the historical data that economists employ do not mix well with strong notions of mathematical determinism and equilibrium.

The synergetic approach has emerged over the last two decades and broadly encompasses methods of analysis which are "... *capable of analysing the characteristics of non-linear dynamic economic systems.*" (Zhang 1991, p. 1). These methods are concerned with the investigation of "... *systems which are composed of many sub-systems which may have quite different natures.*" (Haken 1977, p. 1). Therefore, synergetics is concerned with the study of how cooperation and competition between these different sub-systems might bring about "... *spatial, temporal, or functional structures on macroscopic scales*" (Haken 1988, p. 1). In general, the application of these methods to evolutionary processes is concerned with the way in which these structures might arise in a self-organised manner. Specifically, synergetics seeks to determine principles which govern the process of self-organisation, irrespective of the nature of the sub-systems.[4]

A key characteristic of evolutionary systems with synergetic features is the definite hierarchical structure of those systems. In considering this hierachy in the context of economic systems, microeconomic and macroeconomic activity, as conventionally interpreted in the economic literature, can be distinguished clearly. While microeconomic structures correspond to microscopic structures, macroeconomic systems, as conventionally interpreted, correspond to the intermediate level of the synergetic hierarchy – the so-called mesoscopic level. The synergetic interpretation of macroscopic would then refer to an 'envelope' (or ensemble) of many different macroeconomic (mesoscopic) systems, which can, potentially, evolve over time.[5]

In addressing the issue of evolution on a macroscopic scale, the dynamic economic (mesoscopic) system will be typically "*... forced to undergo a hierarchy of instabilities between states with different spatio-temporal or behavioural patterns*" (Friedrich and Haken (1988), p. 109). Therefore, a direct link between structural instability and qualitative change is emphasised:

"*The qualitative changes occur due to the instabilities of a basic state of the system. A systematic theory of instabilities of stationary, time-periodic, and quasi-periodic states has been developed. It has found a unified formulation in the slaving principle of synergetics*" (Friedrich and Haken 1989, p. 109).

It is the identification of the potential importance of macroscopic behaviour in economic systems, i.e., the cumulation of a time irreversible hierarchical structure, which distinguishes the evolutionary approach of synergetics from the orthodox micro/macro/representative agent approach favoured in mainstream economics.

It should also be recognised that, in focusing upon the mesocopic level of the synergetic hierarchy, a degree of hierarchical aggregation is implied. This aggregative feature permits microscopic considerations to be dealt with, even though investigation is conducted at the mesoscopic level of inquiry. For example, Witt (1993b), who adopts the synergetic appraoch of Weidlich and Haag (1982) and Weidlich (1991), is able to conjecture about microeconomic behaviour that gives rise to, for example, 'critical mass' switches at the mesoscopic level. Two rationales are offered for such a microscopic focus. First, it has been observed that, in economic and social systems, in contrast to most physical systems, the constituent elements at the microscopic level are, themselves, intrinsically complex in nature. This is likely to lead to mutually overlapping time scales, with more densely-spaced hierarchical levels, and with both intelligible vertical (micro- mesoscopic) interactions and horizontal interactions at a mesoscopic level. Second, the major activating influence of random fluctuations is viewed by Witt as affecting the decisions and activities of microscopic entities (individual economic agents). Thus, the microscopic level is

[4] In the synergetics literature, self-organisation is defined as the evolution of highly-ordered structure in a system without specific interference from outside (Friedrich and Haken 1989, p. 104).

[5] For example, Aoki (1994) uses a synergetic approach which stresses the association between hierarchical structure and sluggishness of response in economic contexts. 'Mean-field' effects are used to represent global externalities. Multiple time scale dynamics, of the type used a century ago by Alfred Marshall in economics, are shown to exist. This type of evolution, in turn, occurs in response to endogenously or exogenously determined cooperative and competitive forces that are capable of instigating socioeconomic and institutional change.

appropriate for the investigation of the novel and creative behaviour that is the source of economic evolution.

However, it could also be argued that analysis based at the mesocopic level is likely to provide the most manageable basis for research. It is at the mesoscopic level of the synergetic hierarchy where evolutionary change is likely to be most effectively investigated and modelled. Moreover, in economic applications, it is at this level that the nexus between theoretical propositions and econometrics can be most fully and effectively utilised. Conjectural exercises, conducted at the microscopic level, are likely to provide useful insights for the mesoscopic modeller, but they cannot be more than conjectural given the kind of data that economists have at their disposal.

In broad terms, the synergetic interpretation of evolutionary economic processes, in contrast to conventional economic approaches, stresses the role of time-irreversibility. This, in turn, attaches importance to the formulation of 'theories of process', which are based upon an assessment of historical data, instead of timeless logical deductions of the type that currently permeate economic theory. Theories of process give prominence to structural change relative to permanence, nonlinearity relative to linearity, instability relative to stability, and discontinuity relative to continuity as manifestations of evolutionary change occurring in historical time, as opposed to abstract time, which can be viewed as the real number line $\mathbf{R}$ (i.e., the open interval $(-\infty, \infty)$).

III The synergetic methodology

In broad terms, synergetic investigations emphasise two inter-related areas of research.

These are:

- the identification of steady state (or singular) solutions, linearization about these solutions and an assessment of local stability, possibly about a range of control (or bifurcation) parameters;
- the reduction of complex multi-dimensional systems to simpler (lower) order systems.

At the outset, it should be made clear that, although the context is different, most of the mathematical analysis used is based upon an underlying deterministic framework, one which should be familiar to economic theorists and mathematical economists. In the emerging literature on the application of synergetics to economic systems, the mathematical representations correspond to quasi-deterministic descriptions of system dynamics that are, typically, defined in terms of mean values, with random fluctuating forces being neglected (Weidlich and Braun 1991, p. 5). Furthermore, the analysis is built upon autonomous dynamic systems, and, as such, stresses the importance of asymptotic properties of steady state (non-wandering) orbits. Investigations of the local stability of steady state trajectories enable a researcher to conclude whether any qualitative change in the system is likely. Specifically, if the steady state trajectory becomes locally unstable, then we can expect qualitative change in the steady state (static) properties and structure (dynamic properties) of the solution of the system, culminating in a new structural configuration at the macroscopic level. Points at which qualitative change is likely are termed critical points. In dealing with these two aspects, proponents of synergetics attempt to find the mechanisms by which these (macroscopic) structures evolve. As such, they seek an explanation of the way in which the transition to these new structures might occur. This implies that both quantitative and qualitative effects have, ultimately, to be accounted for.

The first area of research involves an assessment of the structural stability of the system about a singular point and is based generally upon standard linear (local) stability analysis. The concept of structural stability is linked intimately to the local qualitative theory of non-linear differential equations. In conjunction with this theory, much emphasis is placed upon the importance of hyperbolic flow and equilibria from the standpoint of obtaining a topologically-equivalent dynamic system.[6] It has been established that, under strict mathematical assumptions concerning the properties of the mapping and size of permitted perturbation, the (local) qualitative behaviour of a non-linear (hyperbolic) dynamic system can be approximated by a linear systems within a sufficiently-small neighbourhood of an (isolated) critical point. This result has been termed the *Hartman-Grobman Theorem* (Perko 1991, pp. 118–191, p. 30), and implies that the asymptotic properties of hyperbolic linearized system will be invariant to any small perturbation involving non-linear terms within some arbitrarily small neighbourhood of an isolated equilibrium point. Thus, the integral curves (trajectories) will be preserved, along with the direction of motion, along trajectories (Chiarella 1990, p. 168).

These results are not likely to hold for larger perturbations, larger neighbourhoods about the equilibrium point, or in quantitative terms (i.e. the magnitude of system eigenvalues will not necessarily be invariant). Specifically, the vector function depicting the mapping $\mathbf{f}:\dot{x} = \mathbf{f}(x)$, is assumed to have properties that ensure it does not oscillate very wildly around the initial point, and solutions will vary smoothly as initial conditions and parameters are varied. Furthermore, perturbations are constrained to be sufficiently small to ensure that the above mapping is homeomorphic – i.e., continuous and unique in both directions (Chiarella 1990, p. 12, 18).

Under these circumstances, the *linear* flow will characterise uniquely the original (non-linear) flow within the neighbourhood for the stationary point (Marek and Schreiber 1991, p. 18). The possibility of bifurcation and qualitative change have, therefore, been associated conventionally with the investigation of *non-hyperbolic* equilibria. A necessary condition for bifurcation is that $x^e = 0$, and $\mathbf{Df}(x^e) = \mathbf{A} = 0$, for the non-linear system $\dot{x} = \mathbf{f}(x)$, and linear system $\dot{x} = \mathbf{A}x$, where $\mathbf{A}x = \mathbf{Df}(x^e)x$ is the linear part of $\mathbf{f}$ at x^e (Perko 1991), pp. 101, 305–311). Stationary points which are not hyperbolic cannot be characterised unambiguously in terms of linear flow, and any small perturbations in the vector field will result in a qualitative re-arrangement of orbits – i.e., in the generation of bifurcating behaviour.[7]

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[6] A general class of dynamic systems is termed hyperbolic if the matrix of first partial derivatives of the linearized system contain eigenvalues with non-zero real parts (Perko 1991, p. 101). A system is said to be topologically (qualitatively) equivalent if two autonomous systems have the same qualitative behaviour within some arbitrarily-small neighborhood of the equilibrium state. This will be the case if a mapping (homeomorphism) exists which maps trajectories of the first system on to trajectories of the second autonomous system in such a way that the orientation (direction in time) of the mapping is preserved (Perko 1991, p. 118, 165; Zhang 1991, pp. 29–30).

[7] As a part of these investigations, unstable and stable modes are typically identified which may be of crucial importance to the successful implementation of the second area of research.

The second area of research refers to methods that can be used to reduce the dimension of the system at these critical points. Conventionally, dimension-reductions and assessments of structural stability have been based upon the employment of both static and dynamic bifurcation theory. In general bifurcation theory is used to achieve two objectives. The first objective is to determine and group 'equivalent' classes of solutions according to criteria based usually upon a Taylor Series or amplitude-based expansion about a singular point. As part of this procedure, information is obtained as to whether it is possible to finitely-truncate the series expansion and still obtain the generic equivalence implied in the definition of the underlying solution set. This problem is commonly termed the *finite determinancy problem* (Golubitsky and Schaeffer 1984). The second objective is to assess the qualitative change in solution sets *in the small* as control (bifurcation) parameters are varied. This area of research has been termed the *recognition problem* (Golubitsky and Schaeffer 1984).

The *static* theory of bifurcation refers to bifurcations about the origin (the equilibrium solution) of *algebraic* systems of the form $g(x, \pi) = 0$ as the bifurcation parameter π is varied. Numerous methods of analysis can be used to investigate static bifurcation, with perhaps the two most notable methods being the implicit function theorem and singularity theory. For example, Golubitsky and Schaeffer (1984) effect a reduction in dimension through the use of the Liapunov-Schmidt method.

The *dynamic* theory of bifurcation is concerned with changes in the structure of solutions of differential equation systems as parameters in the vector field are varied. In examining this type of bifurcation problem, attention is focused upon the asymptotic properties of the differential equation system that is determined by employing linear stability analysis.[8] Analytically, the theory of dynamic bifurcation is based upon the *Centre Manifold Theorem.* This approach relies on the Hartman-Grobman Theorem because near a singular point, any nonlinear behaviour of the system will be strongly connected to eigenvalues that are marginally stable (i.e. eigenvalues that have zero real parts). This is the conventional approach adopted in the literature. However, it implies that interest is only extended to the bifurcation of stable phenomena, although this approach is not necessary to results which are obtained (Carr 1981, p. 11, 13).[9]

The equation on the centre manifold will determine the asymptotic behaviour of solutions of the full differential equation system. The importance of this latter result cannot be overstated because it permits, at least in principle, a reduction in the dimension of the problem to one corresponding to the number of marginally stable modes. It is also evident from the viewpoint of the asymptotic properties of orbits of the semi-flow, which is implied by the presence of time irreversibility, that the effect of unstable modes will dominate the effect of stable and purely imaginary modes.

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[8] Consult Carr (1981); Hale (1981), Iooss and Joseph (1990); Zhang (1991), p. 34.

[9] The centre manifold theorem, in essence, addresses the asymptotic behviour of the system for small perturbations. If the system has marginally stable modes, then such a system is viewed, once again, as being 'near critical'. In such cases, the invariant manifold is termed a centre manifold. If a function $y = h(x)$ is an invariant manifold for the system $\dot{x} = N(x)$, and h is smooth, then it will be a centre manifold if $h(0) = h'(0) = 0$. An invariant manifold, for the differential equation $\dot{x} = N(x)$ s.t. $x \in \mathbf{R}^n$, is defined as the set $\mathbf{S} \subset \mathbf{R}^n$, if for $x_0 \in \mathbf{S}$, the solution of $x(t)$, with $x(0) = x_0$, is in $\mathbf{S}$ for $t \leq T$ where $T > 0$. If we can always choose $T = \infty$, then we can view $\mathbf{S}$ as an invariant manifold (see Carr 1981, p. 3 and Perko 1991, p. 107, 115).

The implication of this is that, in a non-linear setting, instability about singular points will be a factor capable of instigating qualitative change as time proceeds and motions leave the range of validity associated with the linear approximation. For example, it has been noted that:

"··· *It is well known that many dynamical systems have such complicated orbits that their effective description requires a statistical approach. This results from the internal instability of the system, reflected in the fact that two orbits starting at two neighbouring points can diverge exponentially fast and after a finite time become practically uncorrelated*". (Marek and Schreiber 1991, p. 13).

This implies that constraints, such as capacity limits or the requirement of non-negativity will be approached. This, in turn, will generate behaviour that is essentially non-linear, and therefore, incapable of being modelled by linear equations. Thus, we cannot presume that motions are going to become unbounded and escape to infinity; they may approach capacity and other constraints where qualitatively different behaviour will emerge. In fact, in the presence of evolutionary change, in particular, it would seem to be hard to justify a priori, any connection between dynamic behaviour about unstable critical points on the one hand, and the possibility of *unbounded* motions on the other. Indeed, the conception of limit cycles and chaotic motion is based on this type of reasoning (see Blatt 1983b, pp. 149–152). Therefore, it is for the above reasons that instability has been given such a prominent role in explaining structural change in the synergetics literature.

Perhaps the most pre-eminent method identified in the synergetic literature is Haken's 'slaving principle'. The slaving principle can be used to eliminate the stable (damped) variables when the system is near a critical point because the behaviour of the system near such a point will be determined by the unstable variables (modes). In other words, the unstable variables will *dominate* the stable variables (modes) at such points. At these critical points, the fast damping (stable) variables are assumed to have reached, in a momentary sense, their equilibrium values, i.e., $dx_s/d_t = 0$ (by the "adiabatic" approximation principle). If the adiabatic approximation principle can be validly employed, then solving the equations for x_s will allow one to express the stable variables as functions of the unstable variables. The adiabatic approximation will be valid if:

- the 'damping' constants of the stable variables x_s are large relative to the damping constants associated with the unstable variables x_i and *amplitude* of marginally stable modes;
- the moduli of both x_s and x_i are small so that we can validly employ linear approximation techniques.

These conditions will generally ensure that the newly evolving state will stay close to the unstable state (Haken 1977, pp. 198–200; 1983, p. 35). Moreover, the evolving state will be *observable* in the sense that the evolutionary process will be determined by non-linearities embedded in amplitude-based expansions of the powers of the unstable modes (Friedrich and Haken 1989, p. 110, 112; Weidlich and Braun 1991, pp. 23–24). Provided the slaving principle holds, the values for stable variables will be a function of the unstable variables. This implies, further, that the macroscopic behaviour of the system will be governed by the (few) unstable variables at critical points.

The slaving principle rests upon the identification of sets of unstable and stable modes. In describing the behaviour of the system close to criticality, the non-linear nature of the system will play an essential role, with these non-linear properties being generally modelled by amplitude-based expansion about the unstable modes. Initially, this expansion is based upon the complete set of modes defined from linear stability analysis. It is then possible to use the dynamic equation of the model to determine a set of ordinary differential equations for the amplitudes of the *complete* set of modes. However, near the point of criticality, the time-scale of variation of the amplitude of the stable modes will be very small when compared to the unstable modes. With the neglect of non-linearities in the amplitudes of the stable modes, these modes must then have a small relaxation time when compared to the time-scale of variation of the amplitude of the unstable modes. It then follows that the motion associated with the stable modes is able to 'follow' legitimately the 'slow' motion associated with the amplitudes of the unstable modes, thus becoming slaved to these latter modes. It is for this reason that we are able to obtain the invariant manifold $\varepsilon_s(t) = \varepsilon_s(\varepsilon_u(t))$, where the subscripts "u" and "s" denote unstable and stable respectively, required for the successful application of the adiabatic approximation method. This allows us to make the assumption that $dx_s/d_t = 0$. It is then possible to use a well-established iteration method to generate an asymptotic expansion of $\varepsilon_s(\varepsilon_u(t))$, as a specific power series expansion about $\varepsilon_u(t)$.[10] Thus, in the case of a basic stationary state, the slaving principle establishes that the temporal behaviour of the system close to instability will be determined by the dynamics of the amplitudes of the *unstable* modes.

The methods mentioned above do not constitute an exhaustive listing of those which can be used to reduce the dimension of the problem about the critical points. However, they constitute the most commonly cited methods in the literature. The resulting equations of the lower order systems generally contain quadratic, cubic, and even higher order terms to allow for the non-linearities implied in amplitude-based expansions. These equations permit an assessment of the qualitative nature of the transition between old and new structures. The non-linear expansion about the unstable modes is of crucial importance in defining the cooperative and competitive pressure which will ultimately govern the transition from one macroscopic structure to another. These cooperative and competitive pressures will generally be non-linear functions. This transition refers to the degree of *growth* or *decay* between collections of modes, which will ultimately determine the 'selection of the fittest mode or collection of modes. This will, in turn, determine *which* new macroscopic structure will evolve and *how* this occurs.

In many synergetic investigations, certain parameters have a crucial effect on the system. Such parameters have been termed *control* or *bifurcation* parameters, and have been used principally to account for external (exogenous) influences. In many physical and biological science applications, external influences are modelled as constant (or white noise) processes. The control parameter is the main mechanism through which the sensitivity of the system to changes in exogenous influences is examined. This is generally accomplished by deriving laws of motion for the system in the neighbourhood of a steady state that contain, explicitly, the control parameter. In this way, the first partial derivatives (eigenvalues) of the system can be determined analytically or numerically for a given range of values of the control variable. This procedure, in turn, permits an assessment of stability of the system

[10] The amplitudes $\varepsilon_u(t)$ are termed order parameters (Friedrich and Haken 1989, pp. 112–113).

about the steady state for a range of control parameter values. From this analysis, we can then determine the critical values for the control parameters.[11]

IV The adaptation of synergetics for economic application

The slaving principle constitutes the centrepiece of the synergetic approach to modelling transitions. However, a survey of the economic literature makes it clear that there has been very limited application of this principle even among those who are interested in synergetics. For example, Zhang (1991) was not able to provide one example of an economic investigation that utilised the principle. In fact, the only economic study found by the authors at time of writing, prior to that of Aoki (1994), was undertaken by *physicists* (Weidlich and Braun 1991). Aside from the fact that there is widespread unfamiliarity with the technical details of synergetic methods amongst economists, this lack of studies may well be related to the prominence given to unstable modes and time irreversible ('symmetry breaking') processes in synergetics. It is unlikely that technical considerations present a problem because, as Zhang (1991) makes clear, there is not much difference, in a technical sense, between the non-linear mathematics of synergetics and those to be found in modern dynamic economics.

It is evident, from a theoretical standpoint, that the treatment of qualitative change in synergetics is based upon the existence of structural instability, and the generation of bifurcation behaviour. This latter theory, in turn, is based upon the existence of non-hyperbolic equilibrium states and the violation of the property of topological equivalence. Furthermore, the existence of qualitative change is linked inextricably with equilibrium states that lose their local stability. This contrasts with the conventional definition of a dynamic system and hyperbolic flow. These are based on concepts such as topological equivalence and structural stability, which are built upon underlying mathematical structures that are unique and time-reversible. Therefore, it is evident that qualitative change induces symmetry breaking tendencies and, thus, time-irreversibility is closely linked to a loss of uniqueness. Given that this is the case, one can question the actual existence of a unique continuous inverse function (mapping), which is central to the definitions of dynamic systems and associated concepts, such as topological equivalence and structural stability.

In particular, the preference for stationary conditions when autonomous dynamic system specifications are applied leads to doubts about the efficacy of the

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[11] It has also been implied, in the discussion of macroscopic behaviour, that synergetics is concerned primarily with qualitative changes at critical points that often generate dramatic changes in the macroscopic behaviour of the system. This type of behaviour will arise if the system is structurally unstable about the critical point. Structural instability implies that a one-to-one mapping is not possible, implying a loss of uniqueness, and the existence of multiple equilibria (Haken 1983, p. 32). Given the autonomous framework emphasising asymptotic properties of steady state trajectories, it follows that in many cases, the loss of local (linear) stability will be a necessary condition for a bifurcation. However, additional techniques such as the slaving principle might be needed to determine fully and efficiently the nature of multiple equilibria and qualitative properties implied in all potential phase-transitions at the macroscopic level (Haken 1977, Ch. 8; Haken 1983, pp. 34–35). At another level, the existence of unstable modes will also be central to both quantitative and qualitative change as these modes will combine to drive the evolutionary system to its 'current' hierarchical boundary.

underlying description, from the point of view of modelling actual evolutionary processes. Specifically, it is well known that a key property of autonomous systems is that structural characteristics are held fixed (Azariadis 1993, p. 9)). It would appear that this is not conducive, by definition, to the modelling of evolutionary processes. Although it gives prominence to the concept of time-independent (timeless) equilibria and qualitative change about such equilibrium sets, it understates the potential link between the dynamic path of the system and the notion of a time-independent attractor. Furthermore, in emphasising the role of asymptotic tendencies, the potential for describing evolutionary change in terms of the confluence of factors having a common property of a short-time scale of variaton is overlooked. There is no strong independent empirical evidence supporting the superiority of asymptotic explanations of evolutionary change over alternative explanations based on transitory-type behaviour.

Another issue concerns the degree of openness of evolutionary economic processes when compared to those dealt with in the natural sciences. This issue was addressed, in part, in Weidlich and Braun (1991), as a requirement of their modelling methodology:

"··· *it is unavoidable to make a separation between the sector of endogenous variables treated by the model and exogenous parameters treated as given constant or variables.*" (*Weidlich and Braun 1991, p. 5*).

This recognition led them to highlight the need, within their modelling methodology, to allow for the inclusion of exogenous variables. However, this recommendation, while correct and commendable, goes only part of the way in dealing with openness. For example, given the absence of fundamental laws in economics and the resultant uncertainty concerning the temporal evolution of the vector structure of the process under investigation, it is necessary to extend the sets of both endogenous and exogenous variables to include, in some future time period, certain endogenous variables not currently included. Moreover, given the dependence of the process on the evolving environment surrounding it, these variables may not yet exist. Therefore, there is no prospect of engaging in a priori reasoning to select these variables in advance. Furthermore, many variables will enter as distributed lags that can only be chosen on the basis of tests of goodness of fit, based on current information. These lag structures are likely to change as the process evolves in a way that cannot be determined by theoretical reasoning. Therefore, it appears that the notion of evolutionary change, the associated time-scale of variation, the time frame of investigation and the degree of openness can affect crucially the method of investigation adopted.

When the context is, for example, light beam transitions in laser research, the conventional approach to synergetics appears to work very well because experimental design can ensure that a system can be closed in certain desired respects. Furthermore, self-organisation manifests itself in transitions between conditions that satisfy the definition of a stationary state. However, when we enter the socio-economic domain (or the biological one), we typically confront systems that are open and non-stationary most of the time. If we view this as evolutionary change, then much depends upon the view of evolution taken. If the 'punctuated equilibrium' view of evolution (Eldredge 1989) is favoured, as in Growdy (1992) in an economic context, then it may be possible to employ the synergetic approach without much qualification.

However, in economic systems it is not at all clear that we can carry over the biological metaphor of punctuated equilibrium. For example, the evidence we have concerning most macroeconomic time series is that they represent non-stationary processes. Although, it is often concluded that the non-stationarity observed can be approximated by a random walk, it is argued in Foster (1992, 1994a) that there is evidence to suggest that in many instances, the non-stationary process in question is an evolutionary one that can be represented by a logistic growth trajectory. As has been shown in a number of neo-Schumpeterian studies of growth, in the presence of technical innovation such transitions are remarkably stable. It is only when conditions of quasi-stationarity are approached – in the saturation phase of the logistic growth trajectory – that structural instability and associated short phases of structural discontinuity emerge. Thus, in economic evolution, we often appear to have processes that are strongly non-equilibrium in character, manifesting continual structural change, which can be differentiated into stable (slow) and unstable (fast) transition phases.

Whether or not a structural transition occurs in a stationary or non-stationary context is important for synergetics. In particular, while the conversion of a non-autonomous system to an autonomous one is fairly straightforward in terms of the requisite mathematical analysis, the validity of such a conversion, in the presence of evolutionary change, is not a trivial issue. Evolutionary processes are inherently endogenous and non-equilibrium in character, or they involve structural change. By definition they must be non-autonomous and display non-stationary behaviour. These properties, together with the sensitivity of evolutionary development to the presence of hierarchical boundaries, random fluctuations, and initial conditions, will be responsible for generating processes that are intrinsically non-unique and time-irreversible.

However, if time-series data are non-stationarity, they can be examined to test for the presence of time-irreversibility (see Lawrance 1991). Empirical meaning can then be given to Prigogine's notion of an "arrow of time" (Prigogine 1984, pp. 22, 27–28). Thus, it is not necessary to define time-irreversibility in terms of symmetry-breaking tendencies, generated by non-linear behaviour, as is the case in the conventional approach to the synergetics of qualitative change. In dealing with synergetics in evolutionary economic contexts, it is necessary to develop methods that can be used to investigate 'transient-like' behaviour associated with quantitative and qualitative change. The methods that currently permeate the synergetic literature do not do this but, instead, emphasis the asymptotic properties of equilibrium sets.

In this latter respect, it should also be noted that, in the synergetics literature, most prominence is given to *qualitative* change based upon asymptotic considerations. The potential importance of quantitative change is almost entirely discounted. This reflects the fact that the most crucial parameter in investigations of the underlying structural stability of hyperbolic equilibria is the *sign* of system eigenvalues. In particular, the Hartman-Grobman Theorem implies that the dimension of the unstable and stable manifolds is invariant to small perturbations of the system. However, in evolving economic systems, significant *shifts* in the magnitude of system eigenvalues could occur during the saturation phase of a period of evolutionary development. Furthermore, in an evolutionary process, 'critical slowing down' is likely to occur.[12] This will generally pre-empt (or even trigger) qualitative change in

[12] See Friedrich and Haken (1989), p. 113.

conditions of path-dependency where competitive and cooperative factors, as well as random fluctuations, become relatively more important, as the boundary constraints of the system are approached in historical time.[13]

The role that fluctuation in the growth path plays, in a time-irreversible, evolutionary context, is also important. Specifically, it is essential that such a process exhibits unstable modes that can drive the system towards its boundary constraints. Furthermore, it is the existence of unstable modes during the saturation phase of a period of evolutionary development which is most likely to induce critical interactions between the endogenous system and its boundary constraints. These interactions are most likely to generate both quantitative and qualitative change in the underlying evolutionary process.

The existence of time-irreversibility and structural change in historical data has important implications for the derivation of probabilistic information. First, because most economic time-series are *not* generated from controlled experiments, one cannot replicate experiments a large number of times to generate *objective* information about the probabilistic properties of a model. Second, most statistical tests assume that the model has been correctly specified, which seems dubious given the complexity and uncertainty surrounding most economic processes. Unlike the physical sciences, there are no fundamental equations of motion in economics. Finally, practitioners often have to rely on significant simplifications (i.e. linearizations) in order to obtain the distributional properties for many conventional test statistics (see Kalaba and Tesfatsion 1989a, p. 1216). Ultimately, investigators often end up using an *ad hoc* stochastic framework containing a probabilistic structure that they either have little knowledge about or little faith in. Furthermore, economists working in the field of chaos theory have demonstrated that deterministic non-linear models are capable of producing outcomes which mimic the type of behaviour typically associated with stochastic processes (see Kalaba and Tesfatsion 1989b, p. 435).

These difficulties raise questions concerning the applicability of employing the *master* and *Fokker-Planck equation* approaches identified in the synergetics literature. Specifically, these techniques appear to have been drawn directly from the physical sciences and emphasise a strict probability-based methodology. The key probabilistic quantities are transition probabilities. However, it is not clear how the transition probabilities, which are pivotal in synergetic methods, can be determined in economic settings. Given the legitimate doubt surrounding the formulation of objective probabilistic measures, they would, presumably, have to be imposed on the basis of either subjective opinion or *a priori* reasoning. Moreover, most analytically-tractable versions of these equations are based on representations that contain the key assumptions of Gaussianity and stationarity – the justification of the latter assumption is often based on appeal to the 'principle of detailed balance' and its role in the derivation of the master equation can be found in Haken (1977, 1983), Weidlich and Braun (1991) and Weidlich and Haag (1992). However, actual evolutionary processes, even at the microscopic level, are likely to contain both non-Gaussian and time dependent (non-stationary) tendencies. Although open coefficients ('trend parameters') in the transition probabilities can be introduced, through comparisons of the solutions of the macro-equations of evolution with the

[13] See Foster (1995) for a discussion of co-operative, competitive and predator-prey interactions embodied in logistic (or EVOLON) processes in economic contexts.

real situation,[14] the possibility of deriving analytic solutions almost disappears completely, and with it the insight into system behaviour which, typically, accompanies such solutions. Recourse has to be made to numeric-based computer solutions with associated dependence on and sensitivity to whatever parameterisation is adopted. Thus, numerical simulation becomes unavoidable.

In the face of these difficulties, economists would do well to consider approaches to synergetics that do not rely so much on strong probabilistic assumptions and associated mathematical statistics. In the presence of economic self-organisation, there exist long, relatively stable, phases of structural transition. Although we are dealing with endogenous, non-equilibrium processes, it is possible to specify a range of logistic diffusion equations that can capture this type of transition. Statistical and econometric methods have already been devised to deal with a context where historical, rather than experimental, data exist. These methods are built upon statistical approaches to the modelling of time-series data that are descriptive in nature. Conventionally, deductive economic theorising enters such models, first, in the choice of economic variables in multi-variate specifications and, second, in sets of restrictions imposed on such models. The latter restrictions usually involve some kind of convergence to an equilibrium state or states. However, this is not the only way to proceed: there is no reason why we cannot, instead, interpret a non-stationary the time-series process as a non-equilibrium, evolutionary one.

What this means is that the mathematical model cannot tell us the whole story because it is a closed and deterministic form, while a self-organisation process is endogenous and open. The endogeneity and openness is much more marked than in physico-chemical, or even biological, forms of self-organisation, rendering a deterministic mathematical representation a much less acceptable approximation. However, tractability of structuration in such open circumstances arises from the presence of offsetting forward-looking feedback loops, not evident in biological or physical systems. These have been rightly stressed in mainstream economics, albeit often in incorrect systemic contexts. The rational expectations hypothesis offers a good example in this regard. The upshot of such feedback is a coherence in the non-stationary growth measures of structuration that we do not see in natural science cases simply because the source of such coherence lies in stocks of acquired knowledge and flows of applicable information. Correspondingly, any deterministic mathematical model used will require augmentation with quasi-exogenous variables, deemed to affect the speed and extent of structuration.

However, this does not mean that the resultant specification is then adequate to understand fully the process of evolutionary change – it remains deterministic and, when fitted, it will not be capable of capturing the forces that give rise to fast structural transitions when critical slowing down is present. These forces will be buried in the residuals around the fitted model and, therefore, strong probabilistic inferences cannot be made, even though standard diagnostic tests may support the hypothesis that such residuals constitute Gaussian white noise. There are two possible modelling strategies which can be applied in such circumstances.

[14] These can be calibrated by a field of inquiry among an ensemble of economic decision makers acting under equivalent conditions. See Weidlich and Haag (1988) for an example of the use of open coefficients in the field of population dynamics. Witt (1993c) contains models designed to deal with economic dynamics.

First, given the importance of the nexus between quantitative and qualitative change, logistic-based models that have been identified satisfactorily in the presence of evolutionary economic process can be re-modelled using time-varying parameter approaches, based on time-indexed non-linear algebraic representations. Time varying estimation, in effect, absorbs the residuals, which register in a fixed parameter model, in parameter variation. Thus, the non-deterministic and non-equilibrium forces that constitute evolutionary change are encompassed in a model that, in effect, can deal with temporal non-linearities.

Such an approach contrasts with conventional non-linear representations employing differential equations, in which structural change is modelled by varying a bifurcation parameter *at a point in time* – typically the 'initial' point in time – and then computing trajectories based upon asymptotic (abstract) conceptions of time. We share the doubts of Allen (1993, pp. 6–7, 11) concerning the ability of economists to model evolutionary processes through the use of systems of differential equations, as expounded in the conventional literature, in an environment in which the underlying structure is not invariant. In fact, in the broader synergetic literature, all potential structure that can arise during the transition is latent. In contrast, the vector structure of differential equation systems is known. Structure evolves through the recombination of known elements. At no stage is there uncertainty over the constituent variables comprising the model. Thus, the incorporation of time-varying parameter estimates would appear to be very limited, if not impossible in systems of differential equations.

Instead, by estimating non-linear algebraic equations it is possible to examine the time variation of constituent parameters using, for example, Kalman Filter techniques. Thus, the importance of bifurcation (exogenous) parameters can be reflected in their time-varying impact upon the evolutionary system *through* time. This impact is, in turn, dependent upon values of exogenous (bifurcation) parameters in previous time periods. This modified synergetic approach, in principle, emphasises the importance of both relative shifts in the magnitude of eigenvalues, as well as changes in their signs, as explanations of structural change within an evolutionary process. Moreover, the identification of unstable modes is very important as they capture combinations of processes that generate time-irreversibility and drive the system to its boundary limits. An examination of how parameters vary over time and the properties of associated eigenvalues can provide indications as to when a slow phase of non-linear structural transition is about to change into a fast phase.

The second modelling strategy is to accept the fixed parameter model as an adequate representation of the deterministic component of the evolutionary process and to examine the generated residuals for evidence of evolutionary change. If any information is available concerning shifts from slow to fast transitions it should be contained in these residuals. However, although such information may be accessed to gauge the likelihood of a fast transition, modelling the transition itself, as is done in physics, would seem to be impossible given the absence of an experimental context in which large quantities of data over very short periods can be generated. In general, the fast transition path to a new structural configuration is likely to be highly non-linear and to depend crucially upon a pre-existing (inherited) structure and random fluctuations. Fluctuations in such circumstances are not likely to be governed by standard normality restrictions. This follows because small random fluctuations experienced by a system near criticality are capable of instigating qualitative change, thus affecting dramatically the mean path of the system. Therefore, unless normality is imposed upon stochastic influences by assumption, which would be a highly dubious exercise (by definition), the phase transition will be unpredictable.

In Foster and Wild (1995a), it is argued that historical growth data, generated from an evolutionary process of structuration, tend to conform to an underlying endogenous 'theory of historical process', which can be formalised in the logistic diffusion equation. It is shown that such a theory can be operationalised in the context of econometric modelling, provided that the influence of quasi-exogenous factors on the logistic diffusion coefficient and the capacity limit are allowed for. Thus, it is possible to test for the presence of economic evolution and the deterministic component of such a process can be estimated parametrically. Furthermore, the absence of cointegration between the variable in question and the variable, or variables, driving the capacity limit can offer further confirmation that we are dealing with a non-equilibrium process. However, discovery of a logistic diffusion model, given its deterministic form, can only be a necessary, but not sufficient, condition for economic evolution to be present. Indeed, in the past, logistic diffusion models have been viewed quite differently: as providing evidence of disequilibrium, following exogenous innovation 'shocks', rather than endogenous, non-equilibrium processes (see Dixon 1994).

In order to be confident that we are, indeed, modelling a process of evolutionary change, statistical methods are required to detect the existence of its three identifying characteristics: time irreversibility, structural change and true uncertainty. In Foster and Wild (1995b), appropriate methods are offered, focusing upon the oscillatory properties of the residuals of an estimated logistic diffusion model. Such properties can be compactly described by spectral methods. It is well-known that the spectrum of a sample of time series data must, at least, assume the properties associated with an $I(0)$ process before we can entertain the possibility that 'long-run equilibrium' tendencies are present in time series data. Equally, such methods can also establish the absence of a disequilibrium tendency towards equilibrium.[15]

The findings in Foster and Wild (1996) provide strong support to the hypothesis that the case dealt with, namely, the growth of Australian Building Society Deposits from 1966–1985, exhibits all three characteristics of an evolutionary process. Specifically, the three phases of this logistic diffusion process were found to have different spectral decompositions. First, power is concentrated on low-frequency components in the growth intensive stage. Second, in the middle phase – around the point of inflection – power is concentrated on middle-frequency components. Third, and importantly, the saturation phase exhibits dominant high-frequency components, indicating the presence of instability and uncertainty concerning 'long run' structural integrity. Furthermore, the 'moving' window' spectral evidence shows that the transition to this latter type of spectral decomposition emerges in a abrupt manner. Associated moving window measures of residual variance also confirm a path consistent with the presence of evolutionary change and the emergence of 'criticality' in the saturation phase.

These findings also suggest that structural change might reflect primarily the confluence of factors that have the basic characteristic of a short-time scale of variation. Moreover, our ability to predict the approach of discontinuous structural

[15] The approach adopted in Foster and Wild (1995b) is built upon recent developments in 'moving window' spectral methods (the 'evolutionary spectra'), which are designed to track changes in oscillatory properties over historical time.

transition is associated with such short-run (transitory) behaviour. This contrasts with large sample tests, such as the Chow test, which can identify structural change only after it has occurred, but cannot provide any basis for predicting its likelihood or enable analysts to come to some type of judgement about its likelihood before it actually arises. From the perspective of economic forecasting, the presence of hidden non-stationarity, of the type we have identified through shifting spectral decomposition, will cause problems for naive model extrapolation using estimated parameters. The onset of structural discontinuity will often induce large forecasting errors.

The suggested approach can enable a researcher to establish whether the process under consideration is evolutionary or disequilibrium in character. If it is the former, then the viability of extrapolative forecasting depends on the phase of the diffusion process. In the initial phase, extrapolative forecasting is viable. In the saturation phase, structural uncertainty precludes such extrapolation. Instead, the spectral evidence can provide an early warning of structural discontinuity, which could prove invaluable to policy-makers, allowing time for regulatory changes to avert system collapse and to promote a new phase of evolutionary development. However, this research constitutes only a beginning in the development of a synergetic approach for application in evolutionary economics. The spectral methods used cannot detect non-linearity in the residuals in any direct sense and it is very likely that the onset of criticality will be associated with increased dominance of non-linearities. Detection of such nonlinearity would require the application of poly-spectral and cross-spectral methods, but this remains a topic for further research.

V Conclusions

As evolutionary economists have begun to develop analytical foundations set within the self-organisation approach, it has become clear that a genuine alternative to conventional approaches to economic dynamics is emerging. The traditional dichotomy between theoretical abstraction and historical experience is in the process of breakdown as 'theories of historical process' are beginning to be considered in economics. These theories, which must always be set in the context of an observed historical process, are capable of mathematical formalisation, both in terms of their non-linear dynamic form and underlying structural content.[16] The logistic class of functions, already acknowledged as offering good characterisations of diffusion processes in economics,[17] also offers a sound basis for theorising about those economic processes that constitute slow evolutionary transitions in economic structure.

In the natural sciences, the self-organisational perspective has led to new ways of understanding stability and structural transition. Appropriate methods have already been developed and operationalised in experimental settings. However, we have argued that the uncritical imitation of synergetic science in economics would be as unfruitful as Zhang's (1991) attempt to weld synergetics on to

[16] See Burley and Foster (1994) for a number of contributions which explore the extent to which thermodynamics offers formal theoretical representations of irreversible processes in economic contexts.

[17] See, for example, Silverberg et al. (1988); Lippi and Reichlin (1994).

non-experimental economic theory of the conventional type. Dual development is required: economic analysis must be set in a self-organisational framework and the synergetics of structural stability, instability and transition must be modified to allow for the unique character of economic evolution. Economic self-organisation involves continual evolutionary transition that is non-linear but stable for significant periods of history. Non-stationarity is the norm with fast transitions occurring from one non-stationary process to another. Thus, it is the 'synergetics of saturation' that becomes of interest in economics, not in terms of pretending to be able to predict the timing and extent of discontinuity in such unstable circumstances, but rather in assessing whether a 'critical' condition has been reached in the evolutionary development phase of an economic process.

Economists today think it is still legitimate to construct timeless theories in an empirical vacuum and to pretend that these can provide insights into the workings of historical processes, simply because it has been the habit of economists to do so. This, in itself, constitutes a fine example of historical path-dependence and structural inertia. When we look back in the postwar era we can discern more easily that economics was, to a significant extent, affected by ideological priorities relating to the 'Cold War'. However, these have ceased to be relevant today and politicians now do not require a sophisticated version of free-market ideology to impress the populace. What they require is an economics that can deal with transition: a genuine science, not a pseudo-science, which can cope with processes such as deregulation and institutional change.

We feel that such an economic science is now within our grasp and that, contrary to the opinion of many critics of mainstream economics, this new science will incorporate mathematical and statistical techniques already familiar to economists and many interactions highlighted in neoclassical economics. However, neoclassical economic principles will have to be, as was stressed by Alfred Marshall a century ago, marginal to the main self-organisational thrust of historical processes of 'life and decay'. Then mathematics, important as it is in scientific endeavours, can return to its rightful Marshallian place: as a means rather than an end in economic science.

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