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The effect of linear filters on dynamic time series with structural change

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Abstract

Quite often, when parametric models are tested for structural change, they are fitted to filtered series instead of raw data. Many filters, like those associated with the X-11 seasonal adjustment program, have smoothing properties. Hence, they have a tendency to disguise structural instability. The paper analyzes, both theoretically and via Monte Carlo simulations, the effect of linear filtering on the statistical properties of several tests involving structural change. Historical series of economic activity covering the Great Depression are used to study and illustrate the sensitivity of some tests to the application of seasonal adjustment filters.

Key words: Unit roots; Structural change; Asymptotic bias; Seasonal adjustment; Census X-11 filter
JEL classification: C1; C12; C22

1. Introduction

The analysis of structural change has occupied an important place in econometrics to assess the adequacy of particular models and to characterize the temporal behavior of economic time series. Typically, a parametric model is not

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fitted to raw data but instead to filtered series, such as seasonally adjusted data. As these filters entail smoothing of data, they may conceal a structural change in the unadjusted Data Generating Process (DGP). The widely used Census X-11 seasonal adjustment program, for instance, leaves a constant and linear trend unaffected, as noted in Ghysels and Perron (1993), but has no such invariance property with respect to breaking trends and level shifts. This observation, discussed in Section 2, makes seasonal adjustment with X-11, not an innocuous operation with regard to tests involving aspects of structural instability.

The paper analyzes, both theoretically and via simulations, the effect of linear filtering on the statistical properties of various classes of tests in the presence of structural change. While our discussion focuses on a general class of two-sided linear filters, satisfying certain regularity conditions, specific attention is given to the linear approximation of the X-11 procedure. Three classes of tests are considered, namely, (1) tests for a unit root allowing for the presence of a change in the trend function, as discussed in Perron (1989, 1994), Banerjee, Lumsdaine, and Stock (1992), Zivot and Andrews (1992), and others, (2) tests for changes in a polynomial trend function for a dynamic time series model, proposed by Gardner (1969), MacNeill (1978), and Perron (1991b), and (3) tests for parameter instability with unknown change point, as discussed by Andrews (1993).

A general theoretical treatment of filtering effects on the asymptotic properties of the tests, particularly those belonging to the second and third classes, is not presented. Certain simplifications are made to obtain tractable analytical results. For instance, in some developments it is assumed that the DGP is a simple level shift model without seasonals. While there is no point in seasonally adjusting such series, one can interpret our analysis as focusing on a particular component of interest which is part of a more complex time series. Monte Carlo simulations complement our theoretical findings and show that the qualitative effects uncovered by the asymptotic results extend to more general models.

The outline of the paper is as follows. Section 2 presents a preliminary analysis of the effects caused by seasonal adjustment procedures on purely deterministic components when structural breaks are present. Section 3 discusses in more detail the models and statistics involved, while Section 4 elaborates on the large sample behavior of tests with filtered data. On the other hand, Section 5 reports simulation experiments which allow us to better assess these effects in small samples and extends some of the large sample results to more complex time series models. Finally, Section 6 concludes with empirical examples. Historical series covering the Great Depression are used to illustrate the adverse effects seasonal adjustment filters may have on tests involving structural changes.

2. Filtering and breaking trends

For the purpose of motivating the discussion, let us consider two purely deterministic time series, namely:

$$y_t = \mu + \beta t + \theta DU_t, \tag{2.1}$$

$$y_t = \mu + \beta t + \theta DU_t + \gamma DT_t^*, \tag{2.2}$$

where $DU_t = 1$, $DT_t^* = t - T_b$ if $t > T_b$ and 0 otherwise, with T_b representing a breakpoint. In (2.1), a level shift is present in the DGP with intercept μ for $t < T_b$ and $\mu + \theta$ thereafter. In (2.2), a change in both the intercept and the slope occurs after T_b. Let us consider now the effect of 'seasonally adjusting' these processes. Of course, there is no point in seasonally adjusting such series since they exhibit no seasonal behavior. Yet, as they may be a component of a time series which is being seasonally adjusted, it is useful to consider the effect of a filter like X-11 on these trend components.[1] To simplify the discussion, we consider the linear approximation of the X-11 filter rather than the actual procedure and focus on the monthly filter denoted by $\nu_{\text{X-11}}^M(L)$.[2] It is a two-sided symmetric filter spanning 65 observations on each side with weights that add to 1.[3]

The 'seasonally adjusted' series $y_t^{sa} \equiv \nu_{\text{X-11}}^M(L) y_t$ are plotted in Figs. 1a and 1b which contain the original series (panel A) as well as their filtered counterparts (panel B). For purpose of comparison, panel C presents a graph of $y_t - y_t^{sa}$ in both cases. The first example, appearing in Fig. 1a, is one where a level shift occurs at $T_b = 150$ and the sample size T is 300 (though not the entire sample is plotted on the graphs). To simplify the presentation, we set $\beta = 0$ in (2.1) and choose $\mu = -0.5$ and $\theta = 1$. Hence, at $t = T_b = 150$, a level jump equal in magnitude to one occurs. Such an abrupt level shift is obviously difficult to smooth. Two things happen when a level shift is filtered with $\nu_{\text{X-11}}^M(L)$. First, the magnitude of the discrete jump at $t = T_b$ is reduced by approximately 10% (this

[1] Several researchers have proposed a set of desirable properties that any seasonal adjustment procedure should have (e.g., Granger, 1978; Hylleberg, 1986, Ch. 2). One of them, sometimes referred to as idempotency, is that adjustment filters should leave already adjusted and/or nonseasonal time series unaffected. In that spirit, a desirable seasonal adjustment procedure would leave Eqs. (2.1) and (2.2) unaffected.

[2] For a more detailed discussion of the linear approximation, see, for instance, Bell (1992) and Ghysels and Perron (1993). We will not repeat the details here, and the reader should refer to these papers. By focusing on this linear approximation, we abstract from the modus operandi of the X-11 procedure in practice. At the end of this section, we briefly discuss issues which make the actual X-11 procedure different from its linear filter approximation and to what extent these differences are relevant with respect to analyzing structural changes.

[3] The filter weights appear in Ghysels and Perron (1993, Table A.1).

feature is more explicit in panel C where the difference between the two, i.e., $y_t - y_t^{sa}$, is plotted). Second, a saw-toothed pattern appears before and after the actual break. The pattern, in fact, looks seasonal. The source of this pattern is relatively easy to understand, considering the filter weights of $\nu_{\text{X-11}}^{M}(L)$. As the filter is two-sided, it starts picking up the break at $t = T_b - 65$ when the most extreme lead term of the linear approximation 'hits' T_b. Moreover, the break still affects y_t^{sa} at $T_b + 65$ due to the most extreme lag term. Moreover, the saw-toothed pattern is a consequence of the design of the filter weights. Consider next the case corresponding to (2.2), where the slope and the intercept change at time T_b. Here again, we observe the two effects of passing y_t through $\nu_{\text{X-11}}^{M}(L)$, namely, the level shift is reduced while the change in slope zig-zags through time.

Before turning our attention to the test procedures, we make two observations about the use of the linear X-11 filter. First note that the smoothing produced by the *actual* X-11 program is probably *greater* than that resulting from the application of the linear filter $\nu_{\text{X-11}}^{M}(L)$. Indeed, two features of the actual procedure have a smoothing effect not captured by the linear approximation. First, the detrended series, obtained using the so-called Henderson filter, is

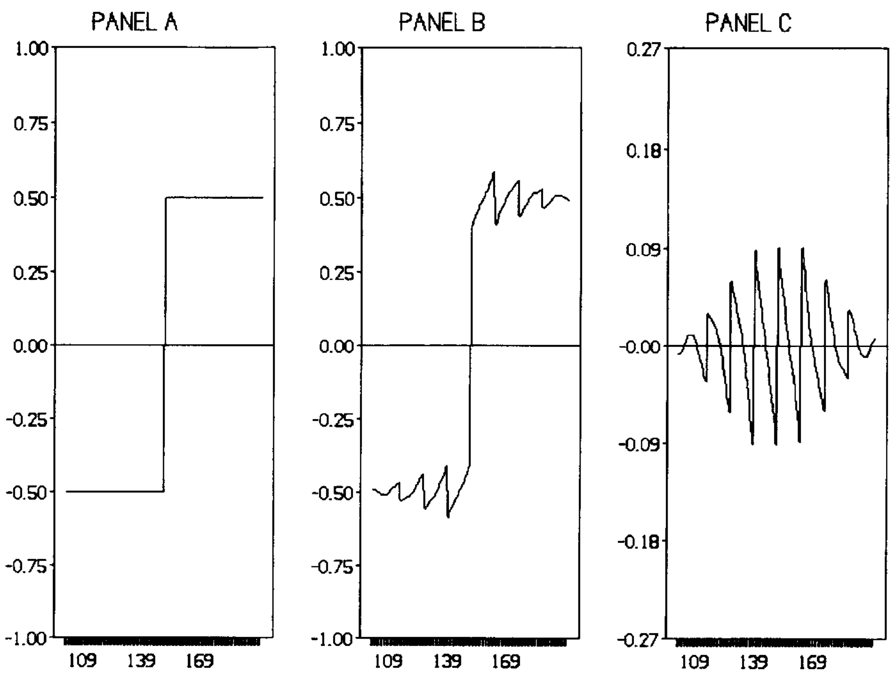

Fig. 1a. Level shift before and after filtering.

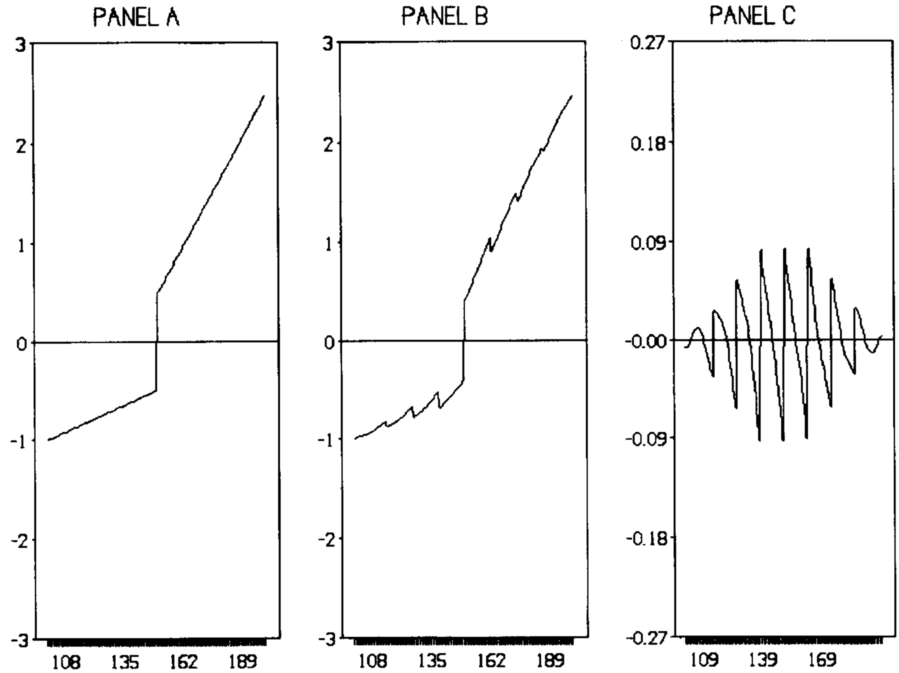

Fig. 1b. Slope and level shift before and after filtering.

rescaled once outliers are detected.[4] This part of the procedure replaces actual observations by refitted values based on a rescaled and nearest neighbor smoothing scheme. Secondly, the Henderson filter can be replaced by a longer moving average with 23 terms instead of the default value of 9. In the remainder of this paper, we continue to work with the linear approximation, as any theoretical development would be difficult with any of the real-time complications associated with the procedure. One should keep in mind though that the nonlinearities of the X-11 program not taken into account in fact exacerbate the undesirable effects of seasonal adjustment on procedures involving structural breaks. Finally, note that we consider only two-sided filters while in practice one-sided filters are often used when all the data required to apply the two-sided filters are not available. Such is the case at either end of the sample or whenever preliminary data releases are studied. We do not pursue any analysis of

[4] Hylleberg (1986, p. 90) provides a reasonably nontechnical description and summary of this feature of the X-11 program. Ghysels, Granger, and Siklos (1995) discuss in detail and provide simulation evidence about the nonlinear features of the X-11 program.

one-sided filters primarily for two reasons. First, regularity conditions required for a linear filter not to affect a linear trend rule out one-sided filters (see Ghysels and Perron, 1993). Second, there are a multitude of one-sided linear filters, in principle 131 for the monthly X-11 case. Choosing a specific one could only be justified using some arbitrary criterion.

3. The models and statistics

In this section, we briefly review the models and statistical procedures and, when necessary, extend them to a seasonal context. Three different classes of tests are investigated, namely, (1) tests for a unit root allowing for the presence of a change in the trend function, (2) tests for changes in a polynomial trend function for a dynamic time series, and (3) tests for general parameter instability with unknown change point.

3.1. Unit root tests

A detailed discussion of tests for a unit root allowing for the presence of a change in trend function appears, for instance, in Perron (1989, 1994). A first model is one where only a change in the intercept of the trend function is allowed under both the null and alternative hypotheses. The 'innovational outlier' version generalized to allow for seasonal components leads to the following regression to compute the relevant unit root test:

$$y_t = \mu + \theta DU_t + \beta t + \delta D(T_b)_t + \alpha y_{t-1} + \sum_{i=1}^{k} c_i \Delta y_{t-i} + \sum_{s=1}^{S-1} b_s d_{st} + e_t, \tag{3.1}$$

where $D(Tb)_t = 1$ if $t = T_b + 1$ and zero otherwise, and d_{st} is a set of $S - 1$ seasonal dummies with corresponding mean shifts denoted by b_s.

Before turning to the second and third models, a brief discussion about the appearance of seasonal dummies in (3.1) is in order. First note that in all the models considered seasonal mean shifts remain fixed under both the null and the alternative hypotheses. This assumption avoids the complication of changing seasonal patterns discussed in Ghysels (1990) and Canova and Ghysels (1994). As all auxiliary regressions include a constant, we know from results in Hylleberg et al. (1990) that the asymptotic distributions of test statistics will not be affected. Hence, the presence of seasonal dummies in (3.1) and other regressions below does not entail any change in the asymptotic critical values to be used.

Under the second model, both a change in the intercept and a change in the slope of the trend function are allowed at time T_b and the appropriate regression is

$$y_t = \mu + \theta DU_t + \beta t + \gamma DT_t^* + \delta D(T_b)_t + \alpha y_{t-1} + \sum_{i=1}^{k} c_i \Delta y_{t-i} + \sum_{s=1}^{S-1} b_s d_{st} + e_t. \tag{3.2}$$

In the third model, a change in the slope of the trend function is allowed but both segments are joined at the break. Hence, the change is presumed to occur rapidly and corresponds to the 'additive outlier model', as discussed in Perron (1989). The null hypothesis of a unit root can be tested using the following two regressions:

$$y_t = \mu + \beta t + \gamma DT_t^* + \tilde{y}_t + \sum_{s=1}^{S-1} b_s d_{st}, \tag{3.3a}$$

$$\tilde{y}_t = \alpha \tilde{y}_{t-1} + \sum_{i=1}^{k} c_i \Delta \tilde{y}_{t-i} + e_t. \tag{3.3b}$$

We denote by $t_{\hat{\alpha}}(i, T_b, k)$ $(i = 1, 2, 3)$ the t-statistic for $\alpha = 1$ under model i with a break date T_b and truncation lag parameter k. In the simulation experiments to be reported in Section 5, we consider both cases where T_b is assumed known and unknown. In the former case, the proper critical values to be used are those in Perron (1989) for all three models (see also Perron and Vogelsang, 1993). When the breakpoint is treated as unknown, we follow Zivot and Andrews (1992) and consider the statistics $t_{\alpha}^*(i) = \min_{T_b \in (k+2, T)} t_{\hat{\alpha}}(i, T_b, k)$ $(i = 1, 2, 3)$, whereby T_b is chosen such that the t-statistic for $\alpha = 1$ is minimized over all possible breakpoints. In this case, the appropriate asymptotic critical values to be used are those reported in Zivot and Andrews (1992) for models 1 and 2 and in Perron and Vogelsang (1993) for model 3.

To select the truncation lag we consider, in both the simulations and the empirical applications, a data-dependent method based on a general to specific recursive strategy using the value of the t-statistic on the coefficient associated with the last lag in the estimated autoregressions.[5]

[5] More specifically, the procedure selects that value of k, say k^*, such that the coefficient on the last lag in an autoregression of order k^* is significant and that the coefficient on the last lag in an autoregression of order greater than k^* is insignificant, up to some maximum order k_{max} selected a priori. We use a two-sided 10% test based on the asymptotic normal distribution to assess the significance of the last lags. See Ng and Perron (1995) for further discussion on the theoretical justification for this procedure and Perron and Vogelsang (1992) for simulation results in the context of unit root tests with breaks.

3.2. Tests for changes in a polynomial trend function

We now consider tests for structural change in a polynomial trend function. The basic process has three components, namely: (1) a polynomial trend function of order p denoted N_t, (2) a stationary AR(k) process denoted X_t, and (3) a set of seasonal deterministic mean shifts. Except for the third component, the setup is similar to that in Gardner (1969), MacNeill (1978), and Perron (1991b). The process y_t is then characterized as

$$y_t = N_t + X_t + \sum_{s=1}^{S-1} b_s d_{st}, \tag{3.4a}$$

$$N_t = \sum_{i=0}^{p} \beta_{i,t} t^i, \tag{3.4b}$$

$$X_t = \sum_{j=1}^{k} \alpha_j X_{t-j} + e_t, \tag{3.4c}$$

where e_t is i.i.d. $N(0, \sigma_e^2)$. Under the null hypothesis, $\beta_{i,t} = \beta_i$ for all i. Under the alternative, some of the $\beta_{i,t}$ change at least once over time. Again, the seasonal pattern is assumed to be fixed under both the null and the alternative hypotheses. A one-time change in the coefficients at a given date T_b will be the alternative hypothesis of interest. To describe the test statistics, consider first the following regression estimated by OLS:

$$y_t = \sum_{i=0}^{p} \hat{\beta}_i t^i + \sum_{s=1}^{S-1} \hat{b}_s d_{st} + \sum_{j=1}^{k} \hat{\alpha}_j y_{t-j} + \hat{e}_{p,t}^S, \qquad t = 1, \dots, T, \tag{3.5}$$

where we denote the estimated residuals by $\hat{e}_{p,t}^S$ to highlight the fact that they are obtained from a regression involving a polynomial trend of order p and a set of seasonal dummies. We shall denote the residuals by $\hat{e}_{p,t}$ when the dummies are not present in the regression. The test statistic, denoted $QD_T^S(p)$, is given by

$$QD_T^S(p) = T^{-2} \hat{\sigma}^{-2} \sum_{t=1}^{T-1} \left(\sum_{j=1}^{t} \hat{e}_{p,j}^S \right)^2, \tag{3.6}$$

where $\hat{\sigma}^2 = T^{-1} \sum_{t=1}^{T} (\hat{e}_{p,t}^S)^2$. A similar test statistic for the nonseasonal case will be denoted $QD_T(p)$ when $\hat{e}_{p,t}^S$ is replaced by $\hat{e}_{p,t}$. The asymptotic distribution of this test was derived in Perron (1991b). It depends on p and is tabulated in MacNeill (1978) for the case where the noise component is a stationary process.

3.3. General tests for parameter instability with unknown change point

The last class of tests considered are special cases of the general framework considered in Andrews (1993). We again consider data generated by (3.4), with the restriction, however, that $p = 0$, yielding an AR(k) nonzero mean model, possibly with seasonal mean shifts. The null hypothesis H_0: $\beta_{0,t} = \beta_0$ is considered. This corresponds to what is termed by Andrews as a 'partial' structural change test, as it does not involve testing the time invariance of the parameters α_j and those associated with the seasonal mean shifts. We let $T_b = \pi T$ where π belongs to a subset of [0, 1]. We use $\pi \in [0.15, 0.85]$ in the simulations and empirical applications. For any given value of π, consider the regression

$$y_t = \beta_{01}(\pi)(1 - DU_t) + \beta_{02}(\pi) DU_t + \sum_{j=1}^{k} \alpha_j y_{t-j} + \sum_{s=1}^{S-1} b_s d_{st} + u_t^S(\pi). \quad (3.7)$$

From (3.7), it is relatively straightforward to construct Wald statistics over the range of possible breakpoints. Namely, for π given,

$$W_T^S(\pi) = T(\hat{\beta}_{01}(\pi) - \hat{\beta}_{02}(\pi))'(\hat{V}_1(\pi)/\pi + \hat{V}_2(\pi)/(1-\pi))^{-1} \times (\hat{\beta}_{01}(\pi) - \hat{\beta}_{02}(\pi)), \quad (3.8)$$

and compute $\sup_\pi W_T^S(\pi)$, denoted $\sup W_T^S$. The variances $\hat{V}_1(\pi)$ and $\hat{V}_2(\pi)$, for $\hat{\beta}_{01}(\pi)$ and $\hat{\beta}_{02}(\pi)$ respectively, are obtained from each of the subsamples and involve corrections for possible heteroskedasticity and autocorrelation as discussed, for instance, in Newey and West (1987). An equivalent statistic for the nonseasonal case will be denoted $\mathrm{Sup}\, W_T$. The asymptotic distribution of $\sup W_T^S$ and $\sup W_T$ is tabulated in Andrews (1993). Along the same lines, one can construct likelihood ratio tests denoted $\sup LR_T^S$ and $\sup LR_T$, this time involving the estimation of a constrained model.

4. Large-sample analysis

Our aim in this section is to discuss the qualitative features of the effect of seasonal adjustment filters on the behavior of some test statistics in large samples. In particular, we want qualitative results that will enable us to draw some conclusions about the likely direction of the biases in terms of size or power. As we shall see, things get complex quite quickly and, in view of keeping the exposition manageable, we consider only simple models and special cases of the statistics described above.

4.1. Unit root tests

For the unit root tests, we consider as DGP a special case of model 3 with a change in slope in the context of a known breakpoint T_b. The tests are constructed without the addition of seasonal dummies and without additional lags in the autoregression (3.3b). Under these restrictions, the two-step procedure for this model reduces to

$$y_t = \mu + \beta t + \gamma DT_t^* + \tilde{y}_t, \tag{4.1}$$

$$\tilde{y}_t = \alpha \tilde{y}_{t-1} + e_t, \tag{4.2}$$

estimated by OLS. Without loss of generality, we also set the true values $\mu = \beta = 0$. Consequently, the DGP considered is of the form:

$$y_t = \gamma DT_t^* + Z_t, \tag{4.3}$$

where Z_t is the noise component. If a unit root is present, we have $Z_t = Z_{t-1} + v_t$ where v_t is a stationary ARMA process of the form $A(L)v_t = B(L)e_t$ with $e_t \sim$ i.i.d. $(0, \sigma^2)$. For a trend-stationary process, Z_t is itself stationary.

We denote the seasonal adjustment filter by $v(L) = \sum_{-m}^{m} v_i L$, a two-sided polynomial with $2m + 1$ terms. The following analysis assumes this filter satisfies $v(L) = v(-L)$ and $v(1) = 1$ [the last condition being necessary to justify our elimination of the intercept and the slope in (4.3)]. This framework covers the case of the linear approximation to X-11. Let y_t^f denote the filtered data. As is well-known, the normalized least-squares estimator of α in (4.2) using filtered data is given by $T(\hat{\alpha}^f - 1) = T^{-1}\sum_{t=2}^{T} \tilde{y}_t^f(\tilde{y}_t^f - \tilde{y}_{t-1}^f)/T^{-2}\sum_{t=2}^{T}(\tilde{y}_{t-1}^f)^2$. Our aim is to study the limiting distribution of $T(\hat{\alpha}^f - 1)$ under the null hypothesis of a unit root and the probability limit of $\hat{\alpha}^f$ when considering the alternative hypothesis of a stationary noise component. The filtered data is given by

$$y_t^f = v(L)y_t = \gamma v(L)DT_t^* + v(L)Z_t. \tag{4.4}$$

Note first that the unit root property is preserved by the application of the filter. Indeed, if a unit root is present, $v(L)Z_t \equiv Z_t^f = Z_{t-1}^f + v(L)v_t \equiv Z_{t-1}^f + \eta_t$ where $\eta_t = v(L)A(L)^{-1}B(L)e_t$. Since $v(L)$ does not contain a root on the unit circle, η_t is itself a stationary process having a different variance from v_t though an identical spectral density function at the origin. The effect of the filter on the trend properties of the data is such that

$$\begin{aligned} v(L)DT_t^* &= 0 && \text{if} \quad t \leqslant T_b - m, \\ &= t - T_b && \text{if} \quad t \geqslant T_b + m, \\ &= \chi_{m,t} \equiv \sum_{i=T_b-m+1}^{t} (t+1-i)v_{T_b+1-i} && \text{if} \quad T_b - m < t < T_b + m. \end{aligned} \tag{4.5}$$

It is shown in the Appendix that the asymptotic distribution of $T(\hat{\alpha}^f - 1)$ is the same as that stated in Perron and Vogelsang (1993) for the case where the data is not filtered except for the fact that the nuisance parameter $\delta = (\sigma_\eta^2 - s_\eta^2)/2\sigma_\eta^2$ is now defined in terms of $s_\eta^2 = \lim_{T\to\infty} T^{-1}\sum_{t=1}^{T} E(\eta_t^2)$ instead of

$s_v^2 = \lim_{T\to\infty} T^{-1}\sum_{t=1}^T E(v_t^2)$ where we recall that $\eta_t = v(L)v_t$ [note that $\sigma_\eta^2 = \lim_{T\to\infty} T^{-1}(\sum_{t=1}^T \eta_t)^2 = \sigma_v^2 = \lim_{T\to\infty} T^{-1}(\sum_{t=1}^T v_t)^2$ since $v(1) = 1$]. Since usual tests for a unit root do not depend asymptotically on nuisance parameters, the tests will have an identical limiting distribution under the null hypothesis whether the data are filtered or not.

Consider now the limiting behavior of $\hat{\alpha}^f$ under the alternative hypothesis of a stationary noise component. Tedious algebra yields:[6]

$$T^{-1}\sum_{t=2}^T \tilde{y}_t^f \tilde{y}_{t-1}^f = T^{-1}\sum_{t=2}^T Z_t^f Z_{t-1}^f + o_p(1), \tag{4.6}$$

$$T^{-1}\sum_{t=2}^T (\tilde{y}_{t-1}^f)^2 = T^{-1}\sum_{t=2}^T (Z_{t-1}^f)^2 + o_p(1). \tag{4.7}$$

Hence, the limiting bias of $\hat{\alpha}^f$ is the same as in the case where no break in the trend function is present. This case was analyzed in detail in Ghysels and Perron (1993) who showed that the probability limit of $\hat{\alpha}^f$ depends on the underlying process and is, in almost all cases, greater than the true first-order autocorrelation coefficient when $v(L)$ is the X-11 filter. This last fact, which still prevails here, implies a loss of asymptotic power for tests of unit roots. Note that, as shown in Ghysels and Perron (1993), this asymptotic bias still prevails if the tests are based on augmented autoregressions.

The basic reason for the fact that filtering the data in the presence of a break in the trend function does not add a further element of bias to the test asymptotically is that, even though the filter does not leave the trend function unchanged, it affects it for a finite number of periods only, related to the length of the filter (m). An alternative asymptotic framework would let this number of leads and lags increase as the sample size increases. The idea here is akin to a continuous time asymptotic framework where the sampling interval decreases to zero as the sample size increases to infinity. Indeed, it is well-known that seasonal filters, such as the linear approximation to X-11, incorporate more lags the finer the sampling interval.[7] Though we do not analyze explicitly a continuous-time approximation, such an asymptotic framework with m increasing can yield additional insights into the qualitative properties of the tests in the presence of filtering.

To that effect, we first need to specify the framework relating the behavior of the filter weights as the sample size increases. We specify the sequence of weights:

$$T v_T([Ts]/T) \to v(s) \quad \text{as} \quad T \to \infty. \tag{4.8}$$

[6] Using (A.2) in the Appendix and especially the fact that c_3 and c_4 are $O_p(T^{-3/2})$ when Z_t is stationary (as well as the fact that m is fixed as $T \to \infty$).

[7] The quarterly X-11 filter involves 27 leads and lags whereas the monthly one has 65.

Condition (4.8) is reasonable in the sense that it lets the weights on distant leads and lags decrease to zero at a fast enough rate. Let the number of lags on each sides of the filter be such that $m/T \to \kappa$ as $T \to \infty$ (we also specify $T_b/T \to \lambda$). Using (4.8), we have

$$\begin{aligned} T^{-1}\chi_{m,[Tr]} &= T^{-1} \sum_{i=T_b-m+1}^{[Tr]} ([Tr] + 1 - i)\, v_{T_b+1-i} \\ &= T^{-1} \sum_{i=1}^{[Tr]-T_b+m} ([Tr] - T_b + m + 1 - i)\, v_{m+1-i} \\ &= \int_0^{r-\lambda+\kappa} [r - \lambda + \kappa - s]\, T\, v_T(\kappa - s)\,\mathrm{d}s \qquad (4.9) \\ &\Rightarrow \int_0^{r-\lambda+\kappa} [r - \lambda + \kappa - s]\, v(\kappa - s)\,\mathrm{d}s \\ &= \int_{-\kappa}^{r-\lambda} [r - \lambda - s]\, v(-s)\,\mathrm{d}s. \end{aligned}$$

Under this alternative asymptotic framework, we obtain a rather different characterization. From (A.2) in the Appendix, we can verify that the term $\chi_{m,t}$ is $\mathrm{O}_p(T)$ and dominates all others under the alternative hypothesis of a stationary noise component. Hence, we deduce that

$$\begin{aligned} \hat{\alpha}^f &= T^{-3}\sum_2^T y_t^f y_{t-1}^f \Big/ T^{-3}\sum_2^T (y_t^f)^2 \\ &= T^{-3} \sum_{t=T_b-m}^{T_b+m} \chi_{m,t}\chi_{m,t-1} \Big/ T^{-3} \sum_{t=T_b-m}^{T_b+m} \chi_{m,t}^2 + \mathrm{o}_p(1). \qquad (4.10) \end{aligned}$$

Considering first the numerator of $\hat{\alpha}^f$, we have

$$T^{-3} \sum_{t=T_b-m}^{T_b+m} \chi_{m,t}\chi_{m,t-1} = T^{-3} \sum_{t=T_b-m}^{T_b+m} [\chi_{m,t-1}^2 + v_{T_b+1-t}\,\chi_{m,t-1}].$$

Using (4.8) and (4.9), we have the limiting results

$$\begin{aligned} T^{-3} \sum_{t=T_b-m}^{T_b+m} \chi_{m,t-1}^2 &= \int_{\lambda-\kappa}^{\lambda+\kappa} (T^{-1}\chi_{m,[Tr]})^2\,\mathrm{d}r \\ &\Rightarrow \int_{\lambda-\kappa}^{\lambda+\kappa} \left(\int_{-\kappa}^{r-\lambda} (r - \lambda - s)\, v(-s)\,\mathrm{d}s \right)^2 \mathrm{d}r. \qquad (4.11) \end{aligned}$$

Using (4.10), (4.11), and the fact that $T^{-2}\sum_{t=T_b-m}^{T_b+m} v_{T_b+1-t}\,\chi_{m,t-1} = \mathrm{O}_p(1)$, it is readily seen that $\hat{\alpha}^f \to 1$ under the alternative asymptotic framework where

m increases to infinity as T increases. Our argument is not that this alternative limiting result provides a better approximation to the finite-sample distribution. Rather, we view the fact that $\hat{\alpha}^f \to 1$ under the alternative hypothesis as suggesting the presence of an additional bias, caused by seasonal adjustment filters, that will reduce the power of the tests in finite samples.

To summarize, our results, though obtained from a special model and test statistic, suggest the following features to be expected in finite samples about unit root tests that allow for the possibility of a break in the trend function: (1) seasonal adjustment filters have little effect on the size of the tests; (2) they, however, create a bias towards nonrejection of the unit root. This bias is caused by two components: the upward bias on $\hat{\alpha}^f$ that would occur without breaks (as analyzed in Ghysels and Perron, 1993) and a further bias caused by the distortionary effect of filtering on the trend function itself.

4.2. Tests for structural change

To keep the theoretical derivations analytically tractable again, while still aiming for qualitative results about distortions to size and power caused by filtering, we consider the simple case of a change in mean in an i.i.d. sequence whereby the statistic $QD_T(0)$ is applied. The data-generating process is given by

$$y_t = (\delta/\sqrt{T})DU_t + e_t, \tag{4.12}$$

where $e_t \sim$ i.i.d. $(0, \sigma^2)$ and DU_t is defined in Eq. (2.1). Under the null hypothesis, $\delta = 0$. By specifying the process with change in mean $\delta/\sqrt{T}$, our goal is to provide a comparison of the local asymptotic power of the statistic constructed with and without filtered data. This derivation is obtained using the asymptotic framework whereby m increases to infinity as T increases and (4.8) is specified for the sequence of filter weights.

We recall from the definition of $QD_T(0)$ that we can write $QD_T(0) = T^{-2}\hat{\sigma}^{-2}\sum_{t=1}^{T-1}\{\sum_{j=1}^{t}(y_j - \bar{Y})\}^2$, where $\hat{\sigma}^2 = T^{-1}\sum_1^T(y_t - \bar{Y})^2$. The statistic constructed with filtered data, $QD_T^f(0)$, is defined analogously with y_t replaced by $y_t^f = \nu(L)y_t$. We first discuss the limit of the statistics under the null hypothesis. From MacNeill (1978), we have

$$QD_T(0) \Rightarrow \int_0^1 B(s)^2\,\mathrm{d}s, \tag{4.13}$$

where $B(s) = W(s) - \int_0^1 W(s)\mathrm{d}s$ is a demeaned Wiener process. It is straightforward, using arguments similar to those in Perron (1991b), to show that

$$QD_T^f(0) \Rightarrow \psi^{-1}\int_0^1 B(s)^2\,\mathrm{d}s, \tag{4.14}$$

where $\psi = \int_{-\kappa}^{\kappa} v(s)^2\,\mathrm{d}s = \lim_{T\to\infty} T^{-1}\sum_1^T (v(L)e_t)^2/\sigma_e^2$. Comparing (4.13) and (4.14), we see that filtering will induce size distortions in the limit if $\psi \neq 1$. Note that we have the approximation $\psi \approx \sum_{-m}^{m} v_i^2$. For the linear approximation to the X-11 seasonal adjustment procedure, ψ is 0.826 for the quarterly version while 0.785 for the monthly version. In both cases, the application of the seasonal adjustment filter will induce tests that are liberal (oversized) even in the limit (slightly more so in the monthly case).

We now consider the local asymptotic power function of the tests. Consider first the case where the unfiltered series are used. From Perron (1991b, Thm. 6), we deduce:

$$QD_T(0) \Rightarrow \int_0^1 B_{0,\delta}^*(r)^2\,\mathrm{d}r,$$

where $B_{0,\delta}^* = W(r) - rW(1) - (\delta/\sigma_e)\ (1-\lambda)r + 1(r>\lambda)(\delta/\sigma_e)(r-\lambda)$. To simplify, we analyze the case where δ is 'large'. In that case, we have the approximate relation

$$QD_T(0) \approx (\delta^2/\sigma_e^2)\lambda^2(1-\lambda)^2/3. \tag{4.15}$$

We now consider the case where filtered data are used. It is shown in the Appendix that for large δ we have the approximation:

$$\begin{aligned} QD_T^f(0) \approx (\delta^2/\sigma_e^2)\Big\{&[\Gamma(\lambda+\kappa) + (1-\lambda-\kappa)]^2(\lambda-\kappa)^3/3 \\ &+ [\Gamma(\lambda+\kappa) + (\lambda-\kappa)]^2(1-\lambda-\kappa)^3/3 \\ &+ \int_{-\kappa}^{\kappa} [\Gamma(r+\lambda) - (r+\lambda)\Gamma(\lambda+\kappa) - (r+\lambda)(1-\lambda-\kappa)]^2\,\mathrm{d}r\Big\}, \end{aligned} \tag{4.16}$$

where $\Gamma(r+\lambda) = \int_{\lambda-\kappa}^{r+\lambda}[\int_{-\kappa}^{j-\lambda} v(-s)\,\mathrm{d}s]\,\mathrm{d}j = \int_{-\kappa}^{r}\int_{-\kappa}^{j} v(-s)\,\mathrm{d}s$. The relative asymptotic power function of $QD_T^f(0)$ and $QD_T(0)$ in the filtered and unfiltered case is given by the ratio of (4.16) to (4.15). This ratio can be evaluated using the following approximations:

$$\Gamma_i \equiv \Gamma(r+\lambda) = T^{-1}\sum_{j=-m}^{i}\left(\sum_{s=-m}^{j} v_{-s}\right) \qquad \frac{i-1}{T} \leqslant [Tr] < \frac{i}{T},$$

$$i = -m+1, \ldots, m,$$

and the approximation for the integral in (4.16) is given by

$$T^{-1}\sum_{i=-m}^{m} [\Gamma_i - (i/T+\lambda)\Gamma_m - (i/T+\lambda)(1-\lambda-\kappa)]^2.$$

Table 1
Relative local asymptotic power of $QD_T^f(0)$ and $QD_T(0)$

T		$\lambda = 0.1$	$\lambda = 0.2$	$\lambda = 0.3$	$\lambda = 0.4$	$\lambda = 0.5$	$\lambda = 0.6$	$\lambda = 0.7$	$\lambda = 0.8$	$\lambda = 0.9$
100	Quarterly	0.857	0.932	0.969	0.991	1.008	1.025	1.046	1.084	1.215
	Monthly	—	—	—	—	—	—	—	—	—
200	Quarterly	0.918	0.969	0.989	1.000	1.008	1.017	1.027	1.045	1.097
	Monthly	0.951	0.966	0.982	0.933	1.002	1.011	1.023	1.046	1.137
500	Quarterly	0.968	0.989	0.997	1.002	1.005	1.008	1.012	1.020	1.040
	Monthly	0.965	0.987	0.995	1.000	1.003	1.007	1.011	1.019	1.041
1000	Quarterly	0.985	0.995	0.999	1.001	1.003	1.004	1.007	1.010	1.020
	Monthly	0.983	0.994	0.998	1.001	1.002	1.004	1.006	1.010	1.021

Using these approximations, we evaluated the relative asymptotic power function of the $QD_T^f(0)$ and $QD_T(0)$ tests when the data were filtered using the linear approximation to X-11. Note that for the quarterly case $m = 27$, and for the monthly case $m = 65$. We set $\kappa = m/T$ and considered a range of T between 100 and 1,000 and $\lambda = 0.1, 0.2, \ldots, 0.9$. The results are presented in Table 1.

Several interesting qualitative features emerge from these results. The most important being the fact that the relative efficiency of the test is lower with filtered data than with unfiltered data when λ is small, i.e., when the break occurs early in the sample. The reverse is true when λ is greater than 0.5. With λ around 0.5, the two versions are approximately as powerful. These qualitative results are little affected by different values of T. Finally, filtering the data induces a greater power loss, in general, with the quarterly filter compared to the monthly filter. In general, however, the power losses or gains are relatively small, within $\pm$ 5%.

Our results, though obtained from a simple model and test statistic, show important qualitative effects that are likely to extend to other models and tests. They show that size can be affected (overly liberal tests) as well as power. Unlike tests for unit roots, the effect on power can go either way, depending on the position of the break.

5. A simulation study of the finite-sample behavior

We now turn our attention to the finite-sample behavior of the various test statistics presented in Section 3. We first describe the Monte Carlo design followed by the results for each of the three classes of tests.

5.1. The Monte Carlo design

We focused exclusively on the monthly X-11 filter and studied that statistical properties of tests in samples of 100 and 200 filtered observations, hence a ten- to twenty-year span on monthly observations. In effect, data sets of 400 observations were drawn in each Monte Carlo iteration so that the two-sided X-11 filter could be applied. A sample with 200 filtered observations started with the 101st observation after the entire series of length 400 is adjusted with the $v^M_{X-11}(L)$ filter. Whenever seasonality was present in the DGP, the following monthly pattern was chosen: $b_1 = -0.05$, $b_2 = -0.03$, $b_3 = 0.03$, $b_4 = 0.05$, $b_5 = 0.05$, $b_6 = 0.02$, $b_7 = -0.05$, $b_8 = -0.02$, $b_9 = 0.02$, $b_{10} = 0.02$, and $b_{11} = 0.02$. Hence, the dummy shifts sum to zero and exhibit what can be viewed as a typical monthly seasonal pattern in economic time series (assuming b_1 corresponds to the second month).

For each model, we considered two scenarios regarding the treatment of a seasonal component in the DGP. One scenario consisted of generating data without a seasonal pattern. The other included seasonal dummies, with the mean shift values described above. With no seasonality in the data, we constructed the tests using regressions without seasonal dummies and compared their properties with and without filtering. Here, as seasonality is absent, we isolate the effect of linear filtering with $v^M_{X\text{-}11}(L)$ on the statistical properties of the different tests. In the second scenario, when the DGP contains seasonal patterns, filtering serves as an attempt to remove the seasonal mean shifts.

5.2. Unit root tests allowing for a breaking trend

We focus here on models 1 (a change in intercept) and 3 (a change in slope). Model 2 is not reported as the results were similar to those of model 3. The DGP's considered imposed an AR(1) structure so that, under the null hypothesis of a unit root, the process is a pure random walk. Under the alternative hypothesis, the noise component, denoted $\phi(L)e_t$, is an AR(1) such that $\phi(L) = (1 - \alpha L)^{-1}$. To analyze power, we set $\alpha = 0.85$. Moreover, it was assumed that $\mu = \beta = 0$ in Eqs. (3.1) and (3.3a). With seasonality in the DGP, three testing strategies are considered. The first consists of applying the tests to filtered series, while the second involves unfiltered data and adding seasonal dummies to the regressions used to calculate the tests. Finally, the third strategy consists also of using the unfiltered data and constructing the tests using standard augmented regressions without added seasonal dummies. The value of $k_{\max}$ for the data-dependent method to select the truncation lag was set equal to 4 except for the latter configuration, where it is 12. We report results for $T = 200$. In generating the data, the breakpoint T_b was set at half-sample, i.e., $T_b = 100$ (or the 200th observation generated). All simulations were done with 1000 replications.

Table 2
Size and power properties of the unit root tests against breaking trend alternatives, filtered and unfiltered series; $t_{\hat{\alpha}}(i, T_b, k)$ statistics, $i = 1, 3$, nominal size 5%; sample size $T = 200$, T_b midsample

	DGP without seasonal dummies				DGP with seasonal dummies					
	Unfiltered		Filtered		Unfiltered		Filtered		Unfiltered ($k_{max} = 12$)	
	Size	Power	Size	Power	Size	Power	Size	Power	Size	Power
Model 1 – Known breakpoint										
$\delta = 0.50$	0.058	0.621	0.053	0.583	0.059	0.601	0.061	0.720	0.048	0.761
$\delta = 1.00$	0.060	0.783	0.048	0.642	0.054	0.799	0.058	0.812	0.045	0.882
Model 1 – Unknown breakpoint										
$\delta = 0.50$	0.050	0.531	0.038	0.354	0.048	0.585	0.056	0.382	0.049	0.698
$\delta = 1.00$	0.052	0.573	0.041	0.418	0.053	0.773	0.054	0.611	0.051	0.833
Model 3 – Known breakpoint										
$\gamma = 0.05$	0.052	0.601	0.052	0.578	0.052	0.446	0.052	0.401	0.052	0.411
$\gamma = 0.10$	0.050	0.777	0.052	0.651	0.051	0.513	0.053	0.492	0.049	0.518
Model 3 – Unknown breakpoint										
$\gamma = 0.05$	0.049	0.333	0.041	0.282	0.041	0.379	0.045	0.361	0.048	0.378
$\gamma = 0.10$	0.042	0.411	0.041	0.351	0.051	0.448	0.047	0.489	0.049	0.452

Table 2 reports size and power of the unit root tests for models 1 and 3. In the first model (3.1), the parameters δ and θ were chosen such that $\delta = \theta/(1 - \alpha)$, hence for any value of δ, we have $\theta = \delta(1 - \alpha)$. We selected $\delta = 0.5$ and 1.0, measuring two different magnitudes of discrete jumps at time T_b. A level shift equal to 0.5 is small considering that its magnitude is half the standard error of the disturbance term. For model 3, we set $\gamma = 0.05$ and 0.10. The parameter γ determines a change in slope rather than a jump, hence the different order of magnitude. The top two panels of Table 2 display the size and power for tests related to model 1 under different configurations with the breakpoint assumed known or unknown and where the DGP lacks or exhibits seasonal mean shifts. The nominal and empirical sizes of the $t_{\hat{\alpha}}(1, T_b, k)$ statistics appear very close, indicating that size distortions are at most minor. This observation also applies to test statistics pertaining to model 3 (the bottom panels of Table 2), and, hence, we focus our attention exclusively on the power properties of the various tests. The fact that no size distortions occur in small samples agrees with the asymptotic results discussed in Section 4.1.

For the power properties, let us first turn our attention to cases where the DGP does not exhibit seasonal mean shifts. As the asymptotic development indicated, it clearly appears from the simulation experiments that tests are less powerful, irrespective of the assumption about knowing the breakpoint, whenever data series are passed through the linear X-11 filter. When the DGP exhibits seasonal mean shifts, this finding also generally holds with some exceptions when one compares the filtered and unfiltered simulation scenarios [the column labeled unfiltered ($k_{\max} = 12$) will be discussed later]. Indeed, for model 1 with a known breakpoint, it appears from Table 2 that tests applied to unfiltered data yield slightly better power, sometimes by a margin exceeding 10%.[8]

A third scenario for dealing with seasonality in the DGP consists of using unfiltered data combined with setting $k_{\max} = 12$ (since simulated data represent monthly series) but without including seasonal dummies in the regression. As the series are unfiltered, we no longer have the negative effect of X-11 on the power of tests. Moreover, as there are no seasonal dummies but only an autoregressive expansion of at most length 12, we may clearly expect to gain power relative to the first scenario which almost always involves more regressors. The power properties of the second scenario, i.e., filtering and $k_{\max} = 4$ versus the third one are *a priori* not easy to assess because the former usually involves less regressors.[9] Regarding model 1, the third scenario is the most powerful. Overall, the results for model 3 are quite similar, except for the fact that the three different scenarios in the seasonal case do not yield such marked differences in power.

Perhaps the most important conclusion to retain from these simulations is that introducing seasonal dummies in regressions, sometimes a natural thing to do with unadjusted data, does not seem to be as good compared to filtering either via AR lag augmentation or via a procedure like X-11. The difference between the latter two does not seem to be such a clear-cut case, though the third scenario seems to have an edge over standard seasonal adjustment filtering. It is also worth recalling at this point the fact that the actual implementation of the X-11 procedure entails most likely *more* smoothing than the linear filter induces. Taking this into account makes the edge of the third scenario all the more important in most practical circumstances.

5.3. *Tests for changes in a polynomial trend function and parameter instability with unknown change point*

We now study the finite-sample properties of the tests presented in Sections 3.2 and 3.3. Because the asymptotic derivations in Section 4.2 were restricted to

[8] There is an easy explanation for this. Two opposite effects on the overall power properties in finite samples must be taken into account. On the one hand, we know that filtering will reduce power; on the other hand, reducing the number of regressors implies increased power. Which of the two effects will dominate depends on the specific situation.

[9] Although, strictly speaking, one should correct for degrees of freedom lost due to filtering.

the analytically tractable simple case of the $QD_T(0)$ statistic, we focus first on a Monte Carlo design tailored towards the theoretical developments. Thereafter, we broaden the scope of the analysis by investigating cases which were not covered by the analytic local asymptotic developments. Moreover, we investigate, even in the simple case, not only the $QD_T(0)$ statistic but also the QD_T^S, $\sup W_T$, $\sup LR_T$, $\sup W_T^S$, and $\sup LR_T^S$ statistics as well. To conduct our first experiments, a data series of normally distributed N(0, 1) white noise was generated under the null hypothesis and a white noise process around a level shift under the alternative hypothesis. Such a design suits all classes of tests. Similar to the previous section, we distinguish data with and without seasonal means. Three scenarios were again considered when seasonal mean shifts were present in the data. Table 3 summarizes the results. The values of δ are the same as in Table 2. All tests now apply to cases where the breakpoint is assumed unknown and $T_b = 50$ with a sample size of 100.

We observe in Table 3 that in some cases minor size distortions appear due to filtering. This finding is in line with the asymptotic size distortions found in Section 4.2. We also observe that filtering may yield more powerful tests, yet taking into account the size distortions [see, for instance, $QD_T(0)$, filtered and unfiltered with seasonals] such increases in power are not very meaningful. For the QD statistic and the $\sup W$ and $\sup LR$ statistics, we also obtain a power loss due to filtering, though the loss is not as significant as in Table 2. It should also be observed that, when seasonals are present, it is advisable to include seasonal dummies instead of long lag expansions like $k = 12$. The use of the latter greatly reduces the power of the $QD_T(0)$ statistics compared to $QD_T^S(0)$, for instance. It should also be noted that the $\sup W$ and $\sup LR$ appear to be slightly less powerful than the tests for a change in a polynomial trend function. This may not be surprising, because the Monte Carlo design is specifically tailored to investigate QD-type statistics. Overall, we may conclude that we find a negative effect of filtering on power, though not as pronounced as in Table 2. As Table 3 covered the situation of a mid-sample break, it is not surprising, given the asymptotic results of Table 1 that filtering has a negligible impact on power. According to the computations based on asymptotic local power approximation we should find more impact of filtering in cases like $T_b = 20$ and $T_b = 80$. Though the simulation results using a sample of $T = 100$ do not display clearly such filtering effects, they hold in larger samples. For instance, with $T = 200$ and $T_b = 40$ the size corrected power was 8% higher with unfiltered series compared to filtered ones. With $T_b = 160$, using filtered series resulted in tests with 13% more power. These latter figures show the relevance of the qualitative results concerning the direction of the bias in the power function described in Section 4.2.

It was conjectured that the analytic asymptotic results, restricted to simple cases, would probably carry over to more complicated situations. We now consider a Monte Carlo design where the DGP is an AR(1) model, instead of

Table 3

Size and power properties of tests for changes in polynomial trend function and general tests for parametric instability with unknown change point; sample size $T = 100$

	DGP without seasonal dummies and with level shift and $T_b = 50$ under alternative		DGP with seasonal dummies and with level shift and $T_b = 50$ under alternative		
	$QD_T(0)$, $k = 0$ Unfiltered	$QD_T(0)$, $k = 0$ Filtered	$QD_T^S(0)$, $k = 0$ Unfiltered	$QD_T(0)$, $k = 0$ Unfiltered	$QD_T(0)$, $k = 0$ Filtered
Size					
$\delta = 0.50$	0.050	0.088	0.052	0.050	0.090
$\delta = 1.00$	0.051	0.090	0.051	0.047	0.089
Power					
$\delta = 0.50$	0.602	0.701	0.609	0.604	0.704
$\delta = 1.00$	0.992	0.986	0.994	0.995	0.996

	$QD_T(0)$ Unfiltered		$QD_T^S(0)$ Filtered		$QD_T^S(0)$ Unfiltered		$QD_T(0)$ Unfiltered			$QD_T(0)$ Filtered	
	$k = 1$	$k = 2$	$k = 1$	$k = 2$	$k = 1$	$k = 2$	$k = 1$	$k = 2$	$k = 12$	$k = 1$	$k = 2$
Size											
$\delta = 0.50$	0.049	0.048	0.079	0.065	0.047	0.048	0.048	0.047	0.034	0.077	0.070
$\delta = 1.00$	0.047	0.046	0.073	0.067	0.044	0.046	0.048	0.047	0.034	0.073	0.066
Power											
$\delta = 0.50$	0.557	0.516	0.639	0.573	0.559	0.520	0.555	0.514	0.059	0.641	0.576
$\delta = 1.00$	0.985	0.963	0.993	0.964	0.987	0.962	0.986	0.965	0.015	0.992	0.962

	sup $W_T(k = 1)$		sup $LR_T(k = 1)$		sup $W_T^S(k = 1)$	sup $LR_T^S(k = 1)$	sup $W_T(k = 1)$	sup $LR_T(k = 1)$
	Unfilterd	Filtered	Unfiltered	Filtered	Unfiltered	Unfiltered	Filtered	Filtered
Size								
$\delta = 0.50$	0.050	0.048	0.047	0.044	0.051	0.052	0.056	0.055
$\delta = 1.00$	0.048	0.048	0.053	0.049	0.050	0.053	0.059	0.057
Power								
$\delta = 0.50$	0.541	0.561	0.582	0.511	0.441	0.491	0.398	0.501
$\delta = 1.00$	0.887	0.818	0.919	0.920	0.810	0.888	0.783	0.843

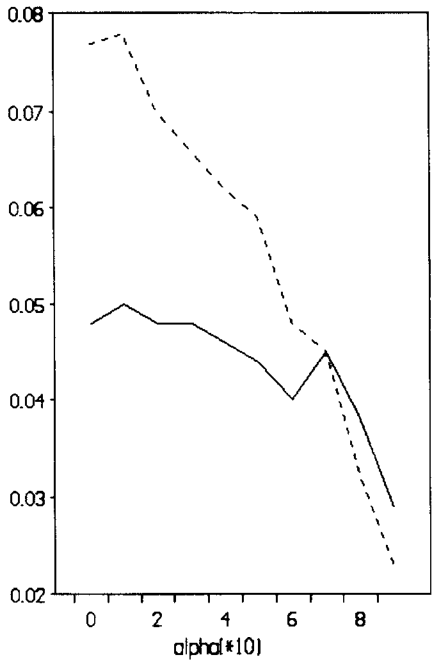

Fig. 2a. Size $QD(0)$, $k = 1$; unfiltered (——) and filtered (– – –).

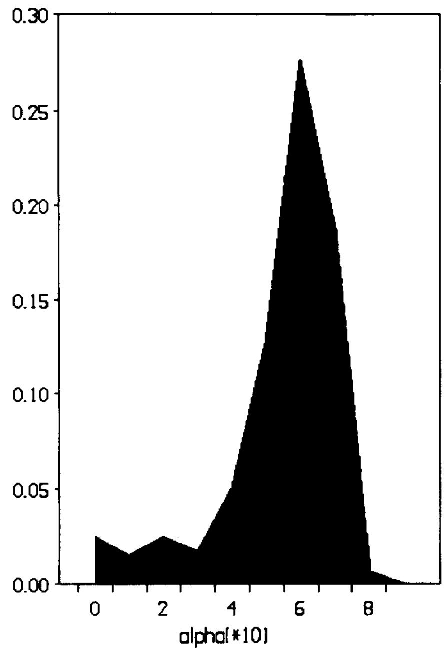

Fig. 2b. Size-corrected power; difference unfiltered vs filtered.

white noise, with or without a break in intercept at $T_b = 50$. To simplify the presentation, we focus exclusively on the $QD_T(0)$ statistic with $k = 1$ for an AR(1) with $\alpha = 0.0, 0.1, 0.2, \ldots, 0.9$. The size of the jump δ is set equal to 1. The results are reported in Figs. 2a and 2b. The first covers size and reveals that filtering induces size distortions which diminish, relative to the unfiltered case, as α increases towards 1. The next figure covers the difference in size-corrected power, unfiltered versus filtered. Here, we clearly see a remarkable and devastating effect on power produced by filtering data when values of α are in the range of 0.4 and 0.8. Indeed, up to almost 30% power is lost because of filtering. Hence, with $\alpha = 0$ and $T_b = 50$, we found little effect on filtering (cf. Table 1 and Table 3). In contrast, with AR(1) stationary models, we find quite strong filtering effects. Size distortions occur as well because of filtering, although they taper off as α increases.

6. Empirical examples

We now turn our attention to empirical examples, demonstrating the effect of filtering in practical applications. The examples relate to tests for unit roots discussed in Section 3.1. We analyze a set of monthly historical time series

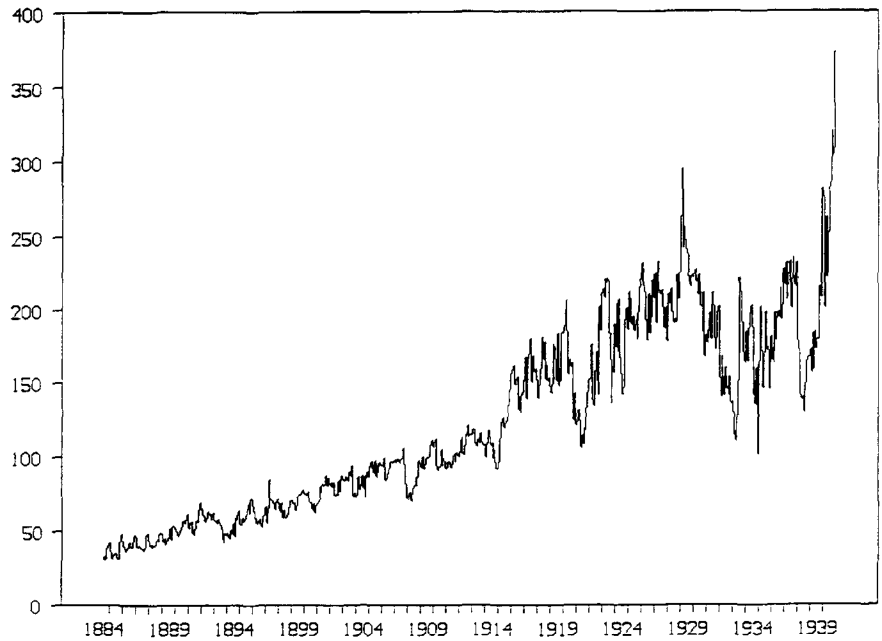

Fig. 3a. Monthly index of industrial production, 1884:1–1940:12.

measuring economic activity before WWII. More specifically, we consider an index of aggregate industrial production and an index of pig iron production both covering the period 1884:1–1940:12. The data are described in more detail in Miron and Romer (1990). This monthly data set covers a long span which is particularly desirable when testing for unit roots (see Perron, 1991a). From the monthly series, we also constructed quarterly indices covering 1884:Q1–1939:Q4. Figs. 3a and 3b display plots of the monthly series.

Table 4 contains empirical results for the quarterly and monthly IP series. For each series, three regressions were applied, namely two involving unadjusted data, once with and once without seasonal dummies.

In the sequential procedure to select the autoregressive order, we considered $k_{\max} = 12$ for the monthly data and $k_{\max} = 10$ for the quarterly series. In Table 4, we present tests for the unit root hypothesis using models 2 and 3, denoted $t_\alpha^*(2)$ and $t_\alpha^*(3)$ respectively.[10] Perhaps the most straightforward example in Table 4 is

[10] It has been assumed that the data generating process had not unit roots at some of the seasonal frequencies. A comment is in order, though, before turning to the empirical results. For the IP series, there are reasons to believe that there might very well be unit roots at seasonal frequencies. Although we will not provide a formal proof here, we can extend the arguments in Ghysels et al. (1994) to show that Dickey–Fuller type tests can still be used to test for a unit root at the zero frequency to the extent that the autoregression is appropriately augmented.

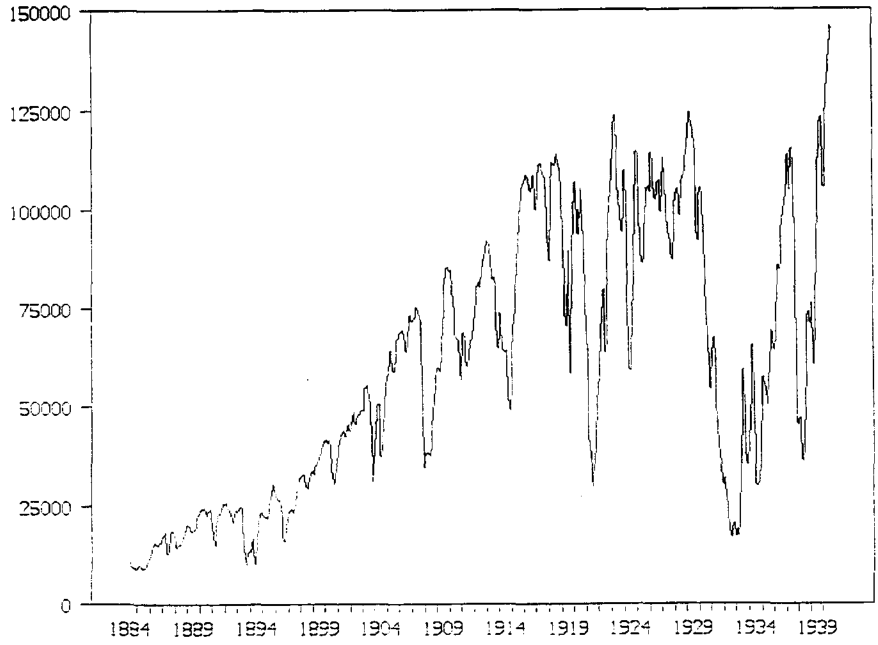

Fig. 3b. Monthly index of pig iron production, 1884:1–1940:12.

Industrial Production sampled at a monthly frequency as all model and data configurations agree on a rejection of the unit root hypothesis. The results with the other data sets and regression specifications are ambiguous and hence more interesting for our purpose.

Let us first discuss the quarterly IP series. While there is agreement among the results for model 3, there is a striking difference between using SA versus NSA series with the test statistic $t_\alpha^*(2)$. With NSA data and seasonal dummies, there is strong evidence against the unit root null hypothesis. With seasonally adjusted series, one cannot reject the null. We know from the theoretical discussions and the simulations that filtering entails a loss in power, to which nonrejection of the null with filtered series can be attributed. Yet, also using NSA data with a correction via an AR argumentation instead of using seasonal dummies also favors the null. It is important to note, however, that the AR augmentation involves ten lags, hence more coefficients than in the regression with seasonal dummies and its four-lag AR expansion. This comparison of both tests using NSA data tells us that the seasonal dummy scenario is probably the most striking. With quarterly Pig Iron production, we also find disagreement among the tests, this time for both models. Now, the scenario involving NSA data and the use of AR augmentations yields rejection. Note, however, that the AR expansions are more parsimonious and clearly should lead to the most powerful

Table 4
Empirical results – Historical time series evidence on unit roots against breaking trend alternatives using $t_\alpha^*(2)$ and $t_\alpha^*(3)$ test statistics

Seasonally unadjusted (NSA) versus adjusted (SA) data

Model	NSA/SA	Seas. dummies	k_{max}	k	T_b	p-value
Quarterly index of industrial production 1884 : 1 – 1940 : 4						
2	NSA	Yes	10	4	1931:3	0.00
	NSA	No	10	10	1931:3	0.38
	SA	No	10	8	1931:3	0.30
3	NSA	Yes	10	7	1925:2	0.35
	NSA	No	10	7	1925:2	0.36
	SA	No	10	7	1925:3	0.41
Quarterly index of pig iron production 1884 : 1 – 1940 : 4						
2	NSA	Yes	10	3	1920:4	0.27
	NSA	No	10	3	1930:1	0.02
	SA	No	10	8	1930:2	0.76
3	NSA	Yes	10	9	1914:1	0.41
	NSA	No	10	3	1914:3	0.03
	SA	No	10	8	1914:2	0.74
Monthly index of industrial production 1884 : 1 – 1940 : 12						
2	NSA	Yes	12	12	1931:11	0.00
	NSA	No	12	12	1931:11	0.00
	SA	No	12	11	1931:11	0.00
3	NSA	Yes	12	12	1925:5	0.01
	NSA	No	12	12	1925:5	0.02
	SA	No	12	11	1925:11	0.00
Monthly index of pig iron production 1884 : 1 – 1940 : 12						
2	NSA	Yes	12	12	1930:7	0.13
	NSA	No	12	12	1930:4	0.08
	SA	No	12	11	1930:7	0.06
3	NSA	Yes	12	12	1914:6	0.15
	NSA	No	12	12	1914:8	0.11
	SA	No	12	11	1914:6	0.09

T_b represents the estimated break point, k_{max} is the maximal lag in the selection procedure, and k is the selected order of the autoregression.

tests. This empirical example, like the former one, underlines the conclusions obtained from the theoretical developments and simulations. Indeed, filtering with X-11 has a strong effect in this case on the power of the tests, particularly when they also involve long AR expansions. The last remaining case is that of monthly Pig Iron series. Here, there do not appear to be significant differences between the tests.

Appendix

In this appendix, we derive the asymptotic distribution of $T(\hat{\alpha}^f - 1)$. Let $\tilde{y}_t^f$ be the residuals from a projection of y_t^f on$\{1, t, DT_t^*\}$ $(t = 1, \dots, T)$. Note first that, combining (4.4) and (4.5), we have

$$\begin{aligned} y_t^f &= Z_t^f && \text{if} \quad t \leqslant T_b - m, \\ &= \gamma(t - T_b) + Z_t^f && \text{if} \quad t \geqslant T_b - m, \\ &= \gamma\chi_{m,t} + Z_t^f && \text{if} \quad T_b - m < t < T_b + m. \end{aligned}$$

Straightforward algebra yields (see Perron and Vogelsang, 1993)

$$\begin{aligned} \tilde{y}_t^f &= y_t^f - \bar{Y}^f - (t - \bar{t})c_3^f + \bar{t}^* c_4^f, && t \leqslant T_b, \\ \tilde{y}_t^f &= y_t^f - \bar{Y}^f - (t - \bar{t})c_3^f - (t - T_b - \bar{t}^*)c_4^f, && t > T_b, \end{aligned} \tag{A.1}$$

where $\bar{Y}^f = T^{-1}\sum_{t=1}^{T} y_t^f$, $t = T^{-1}\sum_{t=1}^{T} t$, $\bar{t}^* = T^{-1}\sum_{t=1}^{T-T_b} t$. The variables c_3^f and c_4^f are defined by $[c_3^f, c_4^f]' = (W'W)^{-1} W'(Y^f - \bar{Y}^f)$, where $Y^f = (y_1^f, \dots, y_T^f)$ and

$$W = \begin{bmatrix} 1 - \bar{t} & -\bar{t}^* \\ \vdots & \vdots \\ \vdots & -\bar{t}^* \\ \vdots & 1 - \bar{t}^* \\ \vdots & \vdots \\ T - t & T - T_b - \bar{t}^* \end{bmatrix}.$$

The expressions for the level of y_t^f are somewhat cumbersome. Tedious algebra [using, in particular, the fact that the detrended variables are invariant to γ except for values of t in the interval $(T_b - m < t < T_b + m)$] yields

$$\begin{aligned} \tilde{y}_t^f &= Z_t^f - \bar{Z}^f - (t - \bar{t})c_3 + \bar{t}^* c_4, && t \leqslant T_b - m, \\ \tilde{y}_t^f &= Z_t^f - \bar{Z}^f - (t - \bar{t})c_3 + \bar{t}^* c_4 + \gamma\chi_{m,t}, && T_b - m < t \leqslant T_b, \\ \tilde{y}_t^f &= Z_t^f - \bar{Z}^f - (t - \bar{t})c_3 - (t - T_b - \bar{t}^*)c_4 + \gamma\chi_{m,t}, && T_b < t < T_b + m, \\ \tilde{y}_t^f &= Z_t^f - \bar{Z}^f - (t - \bar{t})c_3 - (t - T_b - \bar{t}^*)c_4, && t \geqslant T_b + m, \end{aligned} \tag{A.2}$$

where $\bar{Z}^f = T^{-1}\sum_{t=1}^{T} Z_t^f$, and the variables c_3 and c_4 are defined by $[c_3, c_4]' = (W'W)^{-1} W'(Z^f - \bar{Z}^f)$, where $Z^f = (Z_1^f, \ldots, Z_T^f)$. Using (A.2), the first-differences are

$$
\begin{aligned}
\tilde{y}_t^f - \tilde{y}_{t-1}^f &= \eta_t - c_3, && t \leqslant T_b - m,\\
&= \eta_t - c_3 + \gamma\chi_{m,t}, && t = T_b - m + 1,\\
&= \eta_t - c_3 + \gamma(\chi_{m,t} - \chi_{m,t-1}), && T_b - m + 1 < t \leqslant T_b,\\
&= \eta_t - (c_3 + c_4) + \gamma(\chi_{m,t} - \chi_{m,t-1}), && T_b < t < T_b + m, \qquad \text{(A.3)}\\
&= \eta_t - (c_3 + c_4) - \gamma\chi_{m,t-1}, && t = T_b + m + 1,\\
&= \eta_t - (c_3 + c_4), && t > T_b + m + 1.
\end{aligned}
$$

Consider the numerator of $T(\tilde{\alpha}^f - 1)$. We have

$$
\begin{aligned}
T^{-1}\sum_{t=2}^{T} \tilde{y}_t^f(\tilde{y}_t^f - \tilde{y}_{t-1}^f) &= T^{-1}\sum_{t=2}^{T}(Z_{t-1}^f - \bar{Z}^f - (t-1-\bar{t})c_3)\eta_t\\
&\quad + T^{-1}\sum_{2}^{T_b}(\bar{t}^* c_4\eta_t) + T^{-1}\sum_{T_b+1}^{T}(t-1-T_b)c_4\eta_t\\
&\quad + T^{-1}\sum_{T_b-m+1}^{T_b+m-1}\gamma\chi_{m,t-1}\eta_t + \mathrm{o}_p(1), \qquad \text{(A.4)}
\end{aligned}
$$

where the terms subsumed under $\mathrm{o}_p(1)$ correspond to some elements associated with the observations at $t = T_b - m + 1$ and $t = T_b + m + 1$. We note the following asymptotic results: $T^{-1}\bar{t} \Rightarrow 1/2$, $T^{-1}\bar{t}^* \Rightarrow (1-\lambda)^2/2$, and under the null hypothesis of a unit root, $T^{-1/2}\bar{Z}^f \Rightarrow \sigma_\eta\int_0^1 w(r)\mathrm{d}r$, $T^{1/2}c_3 \Rightarrow -\sigma_\eta\psi_3/g_B$, and $T^{1/2}c_4 \Rightarrow -\sigma_\eta\psi_4/g_B$ with g_B, ψ_3, ψ_4 as defined in Perron (1989, Thm. 2), and $\sigma_\eta^2 = \lim_{T\to\infty} T^{-1}E[S_{T,\eta}^2]$ with $S_{T,\eta} = \sum_{t=1}^{T}\eta_t$. Consider the last term in (A.4), we have

$$
\begin{aligned}
T^{-1}\sum_{T_b-m+1}^{T_b+m-1}\gamma\chi_{m,t-1}\eta_t &= T^{-1}\sum_{T_b-m+2}^{T_b+m}\gamma\chi_{m,t}\eta_{t+1}\\
&= T^{-1}\sum_{T_b-m+2}^{T_b+m}\ \sum_{i=T_b-m+1}^{t}(t+1-i)v_{T_b+1-i}\,\eta_{t+1} \qquad \text{(A.5)}\\
&= T^{-1}\sum_{j=1}^{2m-1}\sum_{i=0}^{j} j v_{m+i+1}\,\eta_{T_b-m+i+1}\,.
\end{aligned}
$$

The whole expression converges to zero as $T \to \infty$. Hence, the limit of (A.4) is given by the limit of the first three terms and using results in Perron and

Vogelsang (1993), we have

$$T^{-1}\sum_{t=2}^{T}\tilde{y}_t^f(\tilde{y}_t^f-\tilde{y}_{t-1}^f)\Rightarrow\sigma_\eta^2\left\{H_B/g_B+(\psi_4/g_B)\int_\lambda^1 w_B(r)\,\mathrm{d}r\right\}$$

$$\equiv\sigma_\eta^2\left\{\int_0^1 w_B(r)\,\mathrm{d}w(r)+\delta+(\psi_4/g_B)\int_\lambda^1 w_B(r)\,\mathrm{d}r\right\},\qquad\text{(A.6)}$$

where $\delta=(\sigma_\eta^2-s_\eta^2)/2\sigma_\eta^2$ with $s_\eta^2=\lim_{T\to\infty}T^{-1}\sum_{t=1}^T E(\eta_t^2)$. Also $w_B(r)$ is the residual from a continuous-time projection of the Wiener process $W(r)$ on the function $\{1,r,\mathrm{d}r^*(\lambda)\}$ with $\mathrm{d}r^*(\lambda)=r-\lambda$ if $r>\lambda$ and 0 otherwise. Similar arguments hold for the denominator of $T(\hat{\alpha}^f-1)$ and we have $T^{-2}\sum_{t=2}^T\tilde{y}_{t-1}^f\Rightarrow\sigma_\eta^2K_B/g_B\equiv\sigma_\eta^2\int_0^1 w_B(r)^2\,\mathrm{d}r$.

Proof of (4.16). We first note that

$$\begin{aligned} y_t^f=v(L)y_t&=v(L)e_t, && t<T_b-m,\\ &=(\delta/\sqrt{T})+v(L)e_t, && t>T_b+m,\\ &=(\delta/\sqrt{T})\bar{\psi}_{m,t}+v(L)e_t, && T_b-m\leqslant t\leqslant T_b+m,\end{aligned}$$

where $\bar{\psi}_{m,t}=\sum_{i=T_b-m+1}^{t}v_{T_b+1-i}$, and we note at the outset the limiting result

$$\bar{\psi}_{m,[Tr]}=\sum_{t=1}^{[Tr]-T_b+m}v_{m+1-i}=T^{-1}\sum_{i=1}^{[Tr]-T_b+m}Tv([m+1-i]/T)$$

$$\Rightarrow\int_0^{r-\lambda+\kappa}v(k-s)\,\mathrm{d}s\quad\text{as}\quad T\to\infty,\qquad\text{(A.7)}$$

for $\lambda-\kappa<r<\lambda+\kappa$. Using this result and the fact that $T^{-1/2}\sum_{j=1}^{[Tr]}v(L)e_t\Rightarrow\sigma_eW(r)$ [since $v(1)=1$] and

$$T^{-1}\sum_{j=T_b-m+1}^{[Tr]}\bar{\psi}_{m,j}\Rightarrow\int_{\lambda-\kappa}^{r}\left[\int_{-\kappa}^{j-\lambda}v(-s)\,\mathrm{d}s\right]\mathrm{d}j,$$

we obtain

$$\begin{aligned} T^{-1/2}\sum_{j=1}^{[Tr]}y_j^f&\Rightarrow\sigma_eW(r), && r<\lambda-\kappa,\\ &\Rightarrow\sigma_eW(r)+\delta\int_{\lambda-\kappa}^{r}\left[\int_{-\kappa}^{j-\lambda}v(-s)\,\mathrm{d}s\right]\mathrm{d}j, && \lambda-\kappa\leqslant r\leqslant\lambda+\kappa,\\ &\Rightarrow\sigma_eW(r)+\delta\int_{\lambda-\kappa}^{\lambda+\kappa}\left[\int_{-\kappa}^{j-\lambda}v(-s)\,\mathrm{d}s\right]\mathrm{d}j\\ &\quad+\delta[r-\lambda-\kappa], && r>\lambda+\kappa.\end{aligned}\qquad\text{(A.8)}$$

Using (A.8), the numerator of $QD_T^f(0)$ has the following limit:

$$
\begin{aligned}
&T^{-2}\sum_{t=1}^{T-1}\left[\sum_{j=1}^{t}(y_j^f-\bar{Y}^f)\right]^2 \\
&\Rightarrow \int_0^{\lambda-\kappa}\left\{\sigma_e[W(r)-rW(1)]-r\delta\int_{\lambda-\kappa}^{\lambda+\kappa}\left[\int_{-\kappa}^{j-\lambda}v(-s)\,\mathrm{d}s\right]\mathrm{d}j-r\delta(1-\lambda-\kappa)\right\}^2\mathrm{d}r \\
&\quad+\int_{\lambda-\kappa}^{\lambda+\kappa}\left\{\sigma_e[W(r)-rW(1)]+\delta\int_{\lambda-\kappa}^{r}\left[\int_{-\kappa}^{j-\lambda}v(-s)\,\mathrm{d}s\right]\mathrm{d}j\right. \\
&\quad\left.-r\delta\int_{\lambda-\kappa}^{\lambda+\kappa}\left[\int_{-\kappa}^{j-\lambda}v(-s)\,\mathrm{d}s\right]\mathrm{d}j-r\delta(1-\lambda-\kappa)\right\}^2\mathrm{d}r \\
&\quad+\int_{\lambda+\kappa}^{1}\left\{\sigma_e[W(r)-rW(1)]+(1-r)\delta\int_{\lambda-\kappa}^{\lambda+\kappa}\left[\int_{-\kappa}^{j-\lambda}v(-s)\,\mathrm{d}s\right]\mathrm{d}j\right. \\
&\quad\left.-\delta(\lambda-\kappa)(1-r)\right\}^2\mathrm{d}r. \qquad \text{(A.9)}
\end{aligned}
$$

Note that the denominator of $QD_T(0)$ has the following limit:

$$\hat{\sigma}^2\rightarrow\psi\sigma_e^2. \qquad \text{(A.10)}$$

The result (4.16) follows after some manipulations combining (A.9) and (A.10) and considering again the case where δ is large.

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