The linearity of the U.S. hog–corn cycle has been questioned by Chavas and Holt (1991). Even so, attempts have not been made to model the potential nonlinear dynamics in the hog–corn cycle by using regime-switching models. One popular alternative is Teräsvirta’s smooth transition autoregressive (STAR) model, which assumes regime switching is endogenous and potentially smooth. In this article, we examine monthly data for the U.S. hog–corn cycle, 1910–2004. A member of the STAR family, the time-varying STAR, is fitted to the data and its properties examined. We find evidence of nonlinearity, regime-dependent behavior, and time-varying parameter change.

Key words: hog–corn cycle, nonlinearity, structural change, time-series models.

In recent years, there has been renewed interest in empirical business cycle research. While the motives for this resurgence may vary, there is little doubt that two fundamentally related reasons underlie much of this recent renaissance. One is that economists have long observed that contractionary and expansionary phases of the business cycle are qualitatively different. Keynes (1936), for example, provides observations on properties of business cycles that are consistent with notions of asymmetry, suggesting that contractions are shorter and more turbulent than expansions. An immediate implication is that the underlying process governing business cycle behavior possesses features that cannot be captured by linear models alone. But not until recently have economists developed tools, most typically in the form of regime switching models, capable of depicting asymmetric behavior in business cycles (Neftci, 1994; Falk, 1986). This is the second reason for renewed interest in this line of research.

Regime switching models are generally categorized as being one of two types. First, there are Markov-switching models wherein regimes are determined by an unobserved and exogenous state variable. Alternatively, there is a class of models for which it is explicitly assumed that the regime switch is endogenously determined by an observed state variable. Models belonging to this later category include Tsay’s (1989) self-exciting threshold autoregression (SETAR) and the smooth transition autoregression (STAR) proposed by Teräsvirta (1994). The STAR has several advantages over the SETAR including that several standard STAR models nest a SETAR representation. STAR models have been used to model nonlinear features of business cycles for developed countries (Öcal and Osborn, 2000; Skalin and Teräsvirta, 1999; Teräsvirta and Anderson, 1992; van Dijk and Franses, 1999).

Aside from potential nonlinearity, considerable research has also focused on structural change and time-varying parameters in time-series models (Stock and Watson, 1996). Structural breaks and parameter time variation may occur because of institutional change, an evolving policy environment, or technological innovation. Recently, nonlinear models have been combined with specifications that facilitate structural change and parameter time variation. Lundbergh, Teräsvirta, and...
van Dijk (2003); Skalin and Teräsvirta (2002); and van Dijk, Strikholm, and Teräsvirta (2003) for example, combine the time-varying autoregression (TVAR) of Lin and Teräsvirta with STAR models to obtain a time-varying STAR (TV-STAR) model.

In spite of the emerging popularity of nonlinear models in general and STAR models in particular for depicting aggregate business cycles, comparatively little research has focused on modeling similar attributes for primary commodity prices, a surprising result given that many commodity prices exhibit identifiable cyclical behavior (Labys, Kouassi, and Terraza, 2000) and, as well, may be associated with nonlinearity (Davidson, Labys, and Lesourd, 1998). There is a need then to investigate the potential of STAR models for capturing essential features of primary commodity price dynamics. Perhaps the longest and most widely recognized example of cyclic behavior in primary commodity prices is the hog–corn cycle. Beginning with Haas and Ezekiel (1926), Coase and Fowler (1937), and Ezekiel (1938), numerous studies have attempted to characterize the hog–corn relationship, typically by using linear models (Harlow, 1960; Jelavich, 1973; Larson, 1964; Shonkwiler and Spreen, 1986; Hayes and Schmitz, 1987). Alternatively, Chavas and Holt (1991) used quarterly data, 1910–84, to show that the U.S. hog–corn cycle might be associated with deterministic chaos.

In this article, we employ for the first time a STAR framework, and more specifically, a TV-STAR, to investigate fundamental aspects of the U.S. hog–corn relationship. Our working hypothesis is that the hog–corn cycle exhibits nonlinear features and time-varying parameters (technical change), and that these features may be adequately characterized by a smooth transition model. There are several reasons to believe a priori that a TV-STAR framework might prove fruitful. First, as already noted, prior research has found evidence of nonlinearities in the hog–corn cycle (e.g., Chavas and Holt, 1991). Second, due to the inherent biological nature of hog production, it is far easier to sell breeding stock when expected profits are low than it is to rebuild breeding herds when expected profits are large. Third, even if all agents in the pork market are fully rational, it is still possible to observe periodic behavior (Rosen, 1987; Rosen, Murphy, and Scheinkman, 1994), and perhaps even highly complex behavior (Brock and Hommes, 1997). Fourth, and aside from any price expectation issues, natural animal population dynamics are capable of giving rise to complex (i.e., nonlinear) behavior in livestock markets (Chavas and Holt, 1993). Finally, in the postwar period, there has been considerable technological innovation in hog production, including movement to total confinement operations, the advent of nearly continuous breeding-farrowing rotations, the now widespread use of antibiotics and growth hormones, and enhanced feed conversion and carcass quality through genetic improvements and dietary refinement. In the empirical analysis, we employ a dataset consisting of monthly observations for hog and corn prices, 1910–2004. Among other things, the large sample affords sufficient observations to isolate any potential nonlinear effects and, as well, to explore possibilities for structural change and parameter time variation.

The remainder of the article is organized as follows. In the next section, we provide a brief overview of the history of the U.S. hog–corn cycle. We then discuss the data and describe the STAR testing-modeling-evaluation cycle. Following this, we summarize model estimates when the STAR framework is applied to the hog–corn data and evaluate the results. In the penultimate section, the dynamics of the estimated nonlinear model are explored by using various techniques including sliced spectra and generalized impulse response function analysis. The final section concludes.1

Corn, Hogs, and the Emergence of the Hog–Corn Cycle2

In the economics literature, the expression “hog cycle” refers to the correlated—possibly lagged—component of the swings over time in the hog–corn price ratio. The combination of swine physiology, with its affinity for corn, and the inherent logic of supply and demand insured that, as long as markets existed for both corn and hogs beyond the farm gate, there would inevitably be hog cycles, and indeed, such cycles were recognized in American agriculture, albeit initially at the local level, as early as 1818 (Buley, 1980). The subsequent historiography of American agriculture owes much to the hog–corn nexus, and can be summarized by the observations of nineteenth-century British

1 In the article, a number of intermediate results have been omitted for space reasons; they are, however, in a technical appendix by Holt and Craig available at http://agecon.lib.umn.edu.
2 This section is an abbreviated version of the history of the hog–corn cycle found in Craig and Holt (2005).
journalist traveling in the United States: “The hog is regarded as the most compact form in which the Indian corn crop of the States can be transported to market,” as quoted in Cronon (1991) (pp. 208–09). By the end of the nineteenth century, the combination of the transportation revolution and the economic relationship between corn and hogs had generated something like a national hog cycle. Economists began to analyze the cycle in the twentieth century, and two classic articles were dedicated to the topic (Coase and Fowler, 1937; Ezekiel, 1938). Economists continue to analyze the cycle’s causes and consequence (see, most recently, the review in Chavas and Holt, 1991). The existence and importance of the hog cycle in American economic history stems from at least three related factors: the capacity of swine to convert corn into meat, the importance of swine in American agriculture, and the sheer size of the U.S. market.

Despite shortcomings as a staple in human diets—corn is low in glutenin and niacin (Collins, 1993; Brinkley, 1994)—corn has proved to be an ideal device for delivering carbohydrates to livestock, and hogs proved to be particularly efficient in converting carbohydrates into meat. With the rise of a national market in the United States, countless travelers and foreign observers noted the hog became corn incarnate (Craig and Holt, 2005). In addition to their ability to convert corn into meat, hogs possessed several biological characteristics that contributed to their importance in the early farm economy relative to, say, cattle. These included early onset of breeding (within one year of birth), short gestation periods (four months), and large litter size.

Once a sufficient combination of urbanization and transportation development occurred, farmers began producing pork for the market as opposed to on-farm consumption. Originally valued for its ability to forage, the hog’s subsequent lofty economic status required an off-farm market and transportation “revolution.” Urban consumers provided the ultimate demand for farm-produced fat and protein, but they could be supplied from the hinterland only so long as the cost of transporting the products did not itself consume their value. As the frontier moved west and the country urbanized back east, improvements in graded roads, followed by the emergence of canals and later railroads, farmers further out in the hinterlands not only had relatively low-cost access to urban consumers and world markets, but they also increasingly specialized in a relatively few products and increased their scale of production in those lines (Craig and Weiss, 1993).

Without low-cost transportation, early hog cycles were typically quite local in nature, usually centered on a nearby market town, which depending on its location, might occasionally be tied to a broader market, which itself reflected a trans-village cycle (Buley, 1980). Although Cincinnati was the original “Porkopolis,” the railroad helped shift the center of the trade to Chicago, where, by the early 1870s, packers slaughtered more than a million hogs annually (Cronon, 1991, pp. 230–31). Chicago’s rise marked the rise of a national and international market in meat. It was only with this transregional integration of commodity markets that the multitudinous local cycles became singular in the late nineteenth century. Integration itself resulted from an array of long-run economic changes that included, among other things, the railroad, urbanization, and, importantly, refrigeration. The national rail grid was in place at the end of the nineteenth century, and it was at that time that mechanical refrigeration came to play an important role in the process (Goodwin, Grennes, and Craig, 2002). This combination of economic changes, many components of which involved enormous fixed costs, was itself supported by improvements in U.S. financial markets, and these improvements also directly impacted farm production, facilitating expansion among other things (Craig, Goodwin, and Grennes, 2004; Craig and Holt, 2005).

Taken together, by the first decade of the twentieth century, these changes manifested themselves in a national hog–corn cycle that fundamentally differed from the older local and regional cycles. In particular, the adoption of mechanical refrigeration allowed farmers to expand or smooth production across the year. For meat and dairy products, in particular, refrigeration broke the tyranny of the seasons. In turn, processors of those agricultural products, themselves located in the urban areas to which the raw materials were initially shipped, could exploit economies of scale and scope and become relatively big businesses in their own right. The hog–corn nexus proved to be a crucial link in this chain.

Following the integration of the prairie with the east coast and from there the rest of the world, the hog farmer faced what he perceived to be an iron law of hog–corn economics, and it was this law that ultimately manifested itself in the hog cycle. The law was that a hog was
nothing more than “fifteen or twenty bushels of corn,” or that a bushel of corn could be converted into ten pounds (net) of hog. Thus, twenty bushels of corn spread over a year or so, depending on the breed, might reasonably yield 200 pounds of pork of various cuts—roughly the average for hogs slaughtered during the postbellum era (Cuff, 1992). The rule on the farm thus became that as long as the price of corn in bushels was less than ten times the price of hogs in pounds, it was profitable to feed corn to hogs.

This relationship created the hog–corn cycle. If the supply and demand for hogs were such that the price of corn was less than (roughly) ten times that of pork, farmers would feed all of their corn to, and if possible purchase more to be fed to, their maturing hogs, and breed more. In the absence of any other factors—such as weather shocks, that might improve or worsen the next corn crop—this behavior tended to put upward pressure on the price of corn, and productive resources that might have gone to other farm products went in search of more corn. In addition, as the number of hogs on the market increased, the price of hogs would decrease. As the price ratio fell below the magic number, farmers would cut back on hog production; corn inventories would begin to accumulate; and the cycle would begin again.

At this level of analysis, the hog–corn relationship appears to be a simple exercise in comparative statics: a decrease in the price of an input (corn) leads to a decrease in the marginal cost of production, and hence the average variable cost, of an output (hogs), and in a competitive (i.e., price-taking) market, firms increase production. And as each does so, market supply increases and the result is a decrease in market price. But with respect to the hog–corn relationship in particular and agricultural commodities more generally, especially in the past before technology divorced production from the antediluvian rhythm of the seasons, the decision to supply a product months into the future was made today based on yesterday’s price (and expectations about the future, of course). The result was not necessarily a new set of (assumed to be) stable equilibria, but rather a series of potentially unstable disequilibria.

To see how this might occur, consider that farmers necessarily had to make a decision about corn acreage in the spring. If this year’s crop proved to be in relatively short supply as a result of planting decisions which were made in response to last year’s price before hog producers began to bid it up, then that would tend to put more upward pressure on this year’s corn price. As the increase in hog production, which itself began before the run up in corn prices, simultaneously began to put downward pressure on hog prices, the hog–corn price ratio would fall below the crucial ten-to-one ratio. At that point, farmers would begin to slaughter increasingly younger hogs—even those well below the age of maturity—because the marginal cost of continuing to feed them would exceed the expected price. This step, however, only exacerbated the downward swing in the cycle, and so on it went. Graphing this behavior in price and quantity space yielded the famous diagram of a series of disequilibria, and because of the diagram’s shape the underlying theory came to be called the “cobweb theorem.” Depending on the behavior of the other factors influencing the hog and corn markets, the path of this divergence might be arrested as quickly as one or two years or it might continue for four or five years. Eyeballing the hog and corn price data, Shannon (1945) (p. 167) observed that as the national cycle emerged at the end of the nineteenth-century, the peak-to-peak duration typically lasted four to six years. It soon captured the attention of economists.

Early studies of the hog–corn cycle in the twentieth century were implicitly based on a linear model of the relationship between the two markets (Coase and Fowler, 1937; Ezekiel, 1938); however, statistical techniques at the time prohibited an explicit test of the markets’ dynamics. With the evolution of econometrics, it followed that subsequent efforts to do so employed linear models (Harlow, 1960; Jelavich, 1973; Larson, 1964). The fundamental problem associated with the use of such models in the hog–corn cycle pervades models of population dynamics more generally. Specifically, animals (porcine or otherwise) may be slaughtered, in response to market signals, literally overnight; but producing them, again in response to market signals, takes considerably longer. Therefore, one might logically expect this inherent asymmetry to be better represented by some nonlinear form.

Furthermore, the very nature of these relationships, linear or otherwise, which themselves are manifestations of the underlying structure of the corn and hog markets, would be expected to change through time. In addition, transportation improvements and refrigeration, public policies, and a host of organizational and technological innovations specific to corn and hog markets, have all been
observed over time. There is also a reason to suspect that structural change has been, at several junctures, a key feature of the hog–corn relationship, and as change has not been uniform in the two markets over time, one might expect the “magic ratio” to have changed as well. To obtain a better idea of the relationship between hog and corn prices over time, it is useful to examine in more detail the basic data, the topic of the next section.

Data Description

The data used in the empirical analysis consist of monthly prices of hogs relative to corn for the 1910—2004 period. Average prices received by farmers, U.S., for hogs (all grades) in dollars per hundredweight are available on a monthly basis, seasonally unadjusted, from the U.S. Department of Agriculture’s (USDA) National Agricultural Statistical Service (NASS) for the 1910–2004 period. Likewise, the average price of corn, in dollars per bushel, received by farmers, U.S. (all grades), is also available on a monthly basis, seasonally unadjusted, from NASS for the same period. Data through 1992 were obtained from the USDA-NASS data archive at Cornell University’s Mann library (http://usda.mannlib.cornell.edu/datasets/crops/92152/). Prices for 1993–2004 were obtained from NASS monthly prices received bulletins.

The hog–corn price ratio data, in log-levels form, are plotted in the left-hand panel figure 1. The plot is suggestive—there appears to be a substantial cyclical feature to these data. Indeed, it was exactly this observation that captured the attention of Coase and Fowler (1937), and Ezekiel (1938) in the 1930s. As observed from figure 1, there has been an upward trend in the ratio since the mid to late 1940s, and the ratio also appears to have become more variable since the early 1970s. Finally, although difficult to discern from the graph, there is a substantial seasonal component to the series. In the empirical application, the hog–corn ratio is converted to natural logarithms in an attempt to mitigate some of the observed heteroskedasticity in each model’s residuals.

Based on the plot in figure 1, there is some question as to whether the hog–corn series possesses a unit root. To further investigate this issue, several tests were performed. First, non-parametric bootstrap versions of augmented Dickey–Fuller tests were employed by using twelve lags of the hog–corn ratio. Results show that, with or without trend, the null hypothesis of a unit root is rejected at the 0.001 level.

Figure 1. Observed data and stochastic extrapolations of the TV-STAR and linear AR models of the U.S. hog–corn ratio (horizon thirty-six years)
Of course standard unit root tests are of questionable value when nonlinear STAR-type models are considered (Skalin and Teräsvirta, 2002). Therefore, bootstrap-based tests similar to those developed by Eklund (2003) were implemented, wherein the null is a linear model containing a unit root and the alternative is a first-order approximation of a STAR model specified in the levels. Again, using twelve lags both with and without a trend, the null hypothesis of a unit root is rejected at the 0.001 level in all instances. Additional results are available at the website. It is therefore reasonable to specify any statistical model of the hog–corn data, including a STAR model, in levels form, an issue to which we now turn.

**STAR Models and the STAR Modeling Cycle**

**The Basic STAR Model**

In this section, we describe the basic modeling framework used to examine the hog–corn cycle as might be applied to a time series of monthly observations. The STAR model of Teräsvirta (1994) is used throughout. Accordingly, a STAR model of order $p$ and augmented with (monthly) seasonal dummies is specified as

$$
\Delta y_t = \varphi'_1 x_t (1 - G(\Delta_{12} y_{t-1}; \gamma, c)) + \varphi'_2 x_t G(\Delta_{12} y_{t-1}; \gamma, c) + \varepsilon_t
$$

or, alternatively,

$$
\Delta y_t = \Phi'_1 x_t + \Phi'_2 x_t G(\Delta_{12} y_{t-1}; \gamma, c) + \varepsilon_t
$$

where $y_t$ is the log-level of the hog–corn price ratio; $\Delta$ is a first difference operator; $x_t = (1, \bar{x}_t, D)'$, where $\bar{x}_t = (\Delta y_{t-1}, \ldots, \Delta y_{t-p}, y_{t-1})'$; $D_t = (D_{1t}^*, D_{2t}^*, \ldots, D_{12t}^*, \bar{D}_t)$, where $D_{iat}$, $i = 1, \ldots, 12$, are seasonal dummy variables with $D_{i0} = 1$ when time $t$ corresponds to month $i$ and zero otherwise; $\Phi_i = (\varphi_{i0}, \varphi_{i1}, \ldots, \varphi_{ip})'$, $i = 1,2$ are parameter vectors, and $\Phi = (\varphi_1, \varphi_2) = (\varphi_2 - \varphi_1)$; and $\varepsilon_t$ is a white noise process, $\varepsilon_t \sim IID(0, \sigma^2)$. Based on the unit root tests reported above, we follow Skalin and Teräsvirta (2002) by including the lagged level term $y_{t-1}$ as an additional explanatory variable, and thereby allow for the possibility of a moving equilibrium (i.e., a reparameterized model in levels form). In (1) $G(\Delta_{12} y_{t-1}; \gamma, c)$ is the so-called transition function; by construction it is bounded between zero and one, and therefore allows the structure of the model to change, in a possibly smooth manner, with the value of $\Delta_{12} y_{t-1}$, that is, lagged annual differences of the hog–corn ratio. In other words, the model’s structure will vary depending on whether the hog–corn cycle is in approaching a peak (i.e., $\Delta_{12} y_{t-1} > c$) or a trough (i.e., $\Delta_{12} y_{t-1} < c$) regime.

In what follows, we specify transition function $G(\Delta_{12} y_{t-1}; \gamma, c)$ to be a logistic function of $\Delta_{12} y_{t-1} = y_{t-1} - y_{t-12}$ of the form

$$
G(\Delta_{12} y_{t-1}; \gamma, c) = \left[1 + \exp\{-\gamma(\Delta_{12} y_{t-1} - c)/\sigma(\Delta_{12} y_{t-1})\}\right]^{-1}, \quad \gamma > 0
$$

where $\sigma(\Delta_{12} y_{t-1})$ is the sample standard deviation of $\Delta_{12} y_{t-1}$. Here $d$ is referred to as the delay parameter. The combination of (1) and (2) leads to a logistic STAR (LSTAR) model. In (2), $\Delta_{12} y_{t-1}$ is the transition variable and $\gamma$ and $c$ are, respectively, slope and location parameters. For the LSTAR, $c$ is interpreted as the threshold between two regimes in that $G(c; \gamma, c) = 0.5$, with $G(\cdot)$ changing smoothly from zero to one (i.e., from one regime to another) as $\Delta_{12} y_{t-1}$ increases. Here $\gamma$ is called the smoothness parameter. As $\gamma \to \infty$, $G(\Delta_{12} y_{t-1}; \gamma, c)$ approaches a Heaviside indicator function $I_t = (\Delta_{12} y_{t-1} > c)$, defined as $I_t = |(A) - 1|$ if $A$ is true and $I_t = |(A) = 0$ otherwise. In other words, as $\gamma \to \infty$, the regime switch becomes instantaneous. Therefore, when $\gamma$ is very large, the LSTAR given by (1) and (2) becomes a SETAR. Also, as $\gamma \to 0$, the LSTAR converges to an autoregression (AR) model of order $p$, or AR($p$). Finally, in specifying (2) another possibility is to assume, as in Lin and Teräsvirta (1994), that, in lieu of $\Delta_{12} y_{t-1}$, $t, t = 1, \ldots, T$ is the transition variable. Replacing $\Delta_{12} y_{t-1}$ with $t$ in (2) results in an AR model with parameters that time vary in a potentially smooth manner, that is, the TVAR model.

**Testing Linearity and Parameter Constancy**

As the foregoing discussion makes clear, linear AR models are nested within the LSTAR framework. It is therefore desirable to test the LSTAR against an AR specification. A fundamental problem with using the LSTAR to
test linearity is that an AR model may be achieved in one of two ways: an AR model obtains if \( \gamma = 0 \) or, alternatively, if the restrictions \( \varphi_1^i = \varphi_2^i \) are imposed on (1) (Teräsvirta, 1994). The problem, therefore, is that testing \( H_0: \gamma = 0 \) against \( H_1: \gamma \neq 0 \) within the LSTAR results in a nonstandard test, that is, a test with unidentified nuisance parameters under the null (i.e., the autoregressive coefficients and the location parameter). One approach to dealing with this problem, proposed by Luukkonen, Saikkonen, and Teräsvirta (1988), is to replace transition function \( G(\Delta_{12}y_{t-\tau}; \gamma, c) \) in (1) with a suitable Taylor series approximation. The reparameterized model is no longer associated with an identification problem, and linearity testing proceeds by using standard Lagrange multiplier (LM) tests.

Let \( s_t \) denote either \( \Delta_{12}y_{t-\tau} \) or \( \tau \) in (2), that is, let \( G(s_t; \gamma, c) \) denote the transition function. Then, assuming delay parameter \( d \) is known, one linearity test is obtained by replacing \( G(s_t; \gamma, c) \) in (1b) by a first-order Taylor series approximation, which yields the following artificial regression

\[
(3) \quad \Delta y_t = \beta_0^i x_t + \beta_1^i x_t s_t + v_t
\]

where parameters \( \beta_i, i = 0, 1 \) are functions of original parameters in (1b) such that when \( \gamma = 0, \beta_0 \neq 0 \) and \( \beta_1 = 0 \). In this case, a linearity test involves testing \( H_{01}: \beta_1 = 0 \) against the alternative that \( H_{01} \) is not true. This nonlinear test, called the LM1 test, may be conducted by using either an asymptotic \( \chi^2 \) test with \( (p + 2 + 11) \) degrees of freedom or an appropriate \( F \) version of the test.\(^*\)

As Luukkonen, Saikkonen, and Teräsvirta (1988) note, the LM1 statistic has low power in cases where only the intercept varies across regimes. A test that apparently does have power in this situation involves a third-order Taylor series approximation for \( G(s_t; \gamma, c) \) in (1b). The following artificial regression obtains

\[
(4) \quad \Delta y_t = \beta_0 x_t + \beta_1 x_t s_t + \beta_2 x_t s_t^2 + \beta_3 x_t s_t^3 + v_t
\]

Now a test of linearity involves testing \( H_{03}: \beta_1 = \beta_2 = \beta_3 = 0 \) against the alternative that \( H_{03} \) is not true. This test, denoted the LM3 test, may be conducted by using either an asymptotic \( \chi^2 \) test with \( 3(p + 2 + 11) \) degrees of freedom or its \( F \) test counterpart. An “economy” version of the LM3 statistic is derived by including only \( s_t^2 \) and \( s_t^3 \) as additional regressors in (3). The artificial regression in this case is

\[
(5) \quad \Delta y_t = \beta_0^i x_t + \beta_1^i x_t s_t + \beta_2 x_t s_t^2 + \beta_3 s_t^3 + v_t.
\]

A test of the null hypothesis \( H_{03}: \beta_1 = 0, \beta_2 = 0 \) yields the LM3 test.

In practice, when \( s_t \) is taken to be \( \Delta_{12}y_{t-\tau} \) (as opposed to \( \tau \), delay parameter \( d \) is unknown, and therefore must also be determined as part of the testing procedure. As in Teräsvirta (1994), \( d \) is determined by repeating the LM1 and LM3 tests for all values of \( d \) such that \( 1 \leq d \leq D_{\max} \), where \( D_{\max} \) being the maximal lag length considered. If \( H_{00} \) (\( H_{03} \)) is rejected for more than one value of \( d \), then \( \hat{d} \) may be determined by choosing the value associated with the smallest overall \( p \)-value. On the other hand, if none of the \( p \)-values for LM1 (LM3) indicate rejection of \( H_{01} \) (\( H_{03} \)), then the linear AR model is not rejected.

Model Diagnostics—Autocorrelation

Once a candidate LSTAR model is chosen, parameter estimates are obtained by using standard nonlinear estimation techniques.\(^*\)

And once the model has been estimated, its ability to adequately characterize the data should be evaluated by employing a battery of diagnostic tests. Of particular interest are tests of the hypothesis of no remaining autocorrelation in the model’s residuals and tests of hypotheses of no remaining nonlinearity or of no parameter nonconstancy.

To illustrate, consider a test of the hypothesis of no remaining autocorrelation. As such, let

\[
F(x_t; \theta) = \varphi_1' x_t (1 - G(s_t; \gamma, c)) + \varphi_2' x_t G(s_t; \gamma, c)
\]

denote the skeleton of the model, where \( \theta = (\varphi_1', \varphi_2', \gamma, c) \). Eitrheim and Teräsvirta (1996)

\(^*\) While estimation of an LSTAR model involves, in principle, a straightforward application of nonlinear least squares, certain issues do require additional consideration. For example, reasonable estimates of starting values may be obtained by doing a two-dimensional grid search over the \( \gamma \) and \( c \) parameters. As well, estimates may be obtained by concentrating the sum of squares function. Finally, the \( \gamma \) parameter is generally not estimated with precision, especially when the true value of \( \gamma \) is large. Of course such a result does not necessarily militate against nonlinearity, as the asymptotic distribution of the speed of adjustment parameter \( \gamma \) is, in any event, nonstandard under the hypothesis that \( \gamma = 0 \). Regarding these issues and more, see van Dijk, Terásvirta, and Franses (2002) (pp. 19–21).
propose testing the hypothesis of no remaining autocorrelation up to and including order \( q \) by estimating the auxiliary regression

\[
\hat{\epsilon}_t = \pi_1 \nabla F(x_t; \hat{\theta}) + \sum_{i=1}^{q} \alpha_i \hat{\epsilon}_{t-i} + \hat{d}_t
\]

where \( \nabla F(x_t; \hat{\theta}) = (\partial F(x_t; \hat{\theta})/\partial \theta) \). The LM test statistic is computed in the usual fashion as \( TR^2 \), where \( R^2 \) is the r-squared coefficient from the auxiliary regression in (6). Under the null hypothesis of no remaining autocorrelation, that is, under \( H_0: \alpha_1 = \cdots = \alpha_q = 0 \), the resulting test statistic has an asymptotic \( \chi^2 \) distribution with \( q \) degrees of freedom. An \( F \)-version of the test may also be constructed.

**TV-STAR, MRSTAR, and Additive STAR Models**

As already suggested, there may be occasions where the LSTAR’s parameters are not constant through time, due perhaps to institutional or technological change, a possibility that is plausible for the hog–corn ratio. In this case, it may be better to specify a model for \( \Delta y_t \) that includes both regime switching and nonconstant parameters, a TV-STAR model.

The TV-STAR is expressed as

\[
\Delta y_t = \begin{bmatrix} \varphi_1 x_t, (1-G_1(\Delta_{12} y_{t-d}; \gamma_1, c_1)) \\ \varphi_2 x_t G_1(\Delta_{12} y_{t-d}; \gamma_1, c_1) \end{bmatrix} + [\varphi_3 x_t (1-G_1(\Delta_{12} y_{t-d}; \gamma_1, c_1))] \\
\times G_2(t; \gamma_2, c_2) + \epsilon_t
\]

where \( G_1(\cdot) \) and \( G_2(\cdot) \) are logistic functions as in (2). Clearly if \( \gamma_2 = 0 \), or if \( \varphi_1 = \varphi_3 \) and \( \varphi_2 = \varphi_4 \), the two-regime LSTAR obtains. If \( t \) in transition function \( G_2(\cdot) \) in (7) is replaced by a second endogenous transition variable, \( \Delta_{12} y_{t-d} \), the Multiple Regime STAR, or MRSTAR model of van Dijk and Franses (1999) obtains.

Several strategies may be used to test an LSTAR versus a TV-STAR. First, once a candidate STAR model has been estimated, \( G_2(t; \gamma_2, c_2) \) in (7) may be replaced by a suitable Taylor series expansion. For example, if a third-order Taylor series is used the approximation to (7) is

\[
\Delta y_t = \theta_1 x_t + \theta_2 x_t G_1(\Delta_{12} y_{t-d}; \gamma_1, c_1) \\
+ \theta_3 x_t^2 + \theta_4 x_t^3 + \theta_5 x_t^3 G_1(\Delta_{12} y_{t-d}; \gamma_1, c_1) + \epsilon_t
\]

The null hypothesis of no time variation is \( H_0: \beta_1 = \cdots = \beta_6 = 0 \), with the LM test constructed by running a regression similar to that in (6) wherein the residuals from the estimated LSTAR (TVAR) are regressed on the gradient vector \( \nabla F(x_t; \hat{\theta}) \) and additional regressors \( x_t, x_t^2, x_t^3, x_t G_1, x_t^2 G_1, x_t^3 G_1, \dot{G}_1 = G_1(\Delta_{12} y_{t-d}; \gamma_1, c_1) \) (van Dijk and Franses, 1999). The LM test statistic is then constructed either as an asymptotic \( \chi^2 \) test with \( 6(p + 2 + 11) \) degrees of freedom or as a comparably defined \( F \) test. This testing strategy is the “Specific-to-General” procedure outlined by Lundbergh, Teräsvirta, and van Dijk (2003).

An alternative approach, also suggested by Lundbergh, Teräsvirta, and van Dijk (2003), is to test a TV-STAR directly against a linear model, the “Specific-to-General-to-Specific” approach. In this case, the transition functions in (7) are approximated directly, by say, a first-order Taylor series expansion. Doing so gives

\[
\Delta y_t = \theta_0 x_t + \theta_1 x_t \Delta_{12} y_{t-d} \\
\times \theta_2 x_t + \theta_3 x_t \Delta_{12} y_{t-d} + \nu_t
\]

and testing the null hypothesis \( H_0^{TV-STAR}: \theta_1 = \theta_2 = \theta_3 = 0 \) yields the LM TV-STAR test, which may be conducted by using either an asymptotic \( \chi^2 \) test with \( 3(p + 2 + 11) \) degrees of freedom or an \( F \) test.

Lundbergh, Teräsvirta, and van Dijk (2003) describe several additional tests of interest nested within (9) when \( H_0^{TV-STAR} \) is rejected. Specifically, in this case test \( H_0^{TVAR}: \theta_0 = \theta_2 = \theta_3 = 0 \), the LM TVAR test, in which case (9) reduces to a TVAR under \( H_0^{STAR} \). Also, test \( H_0^{TVAR}: \theta_1 = \theta_2 = 0 \), the LM TVAR test, wherein (9) reduces to a STAR model under \( H_0^{TVAR} \). If both \( H_0^{STAR} \) and \( H_0^{TVAR} \) are rejected, then the TV-STAR model is retained. Alternatively, if \( H_0^{STAR} \) is rejected but \( H_0^{TVAR} \) is not,
then a STAR model is indicated. The opposite conclusion obtains (i.e., a TVAR is selected) if \( H_0^{TVAR} \) is rejected but \( H_0^{STAR} \) is not. Lundbergh, Teräsvirta, and van Dijk (2003) report simulation evidence assessing the relative merits of the two testing strategies (i.e., Specific-to-General versus Specific-to-General-to-Specific), and suggest that it may be desirable to employ both in the model selection stage.

Finally, Eitrheim and Teräsvirta propose an alternative to the TV-STAR (or MRSTAR): the additive STAR. In this case, (1b) is modified by appending a second additive STAR component. That is,

\[
\Delta y_t = \phi'_1 x_t + \phi'_2 x_t G_1(\Delta_{12} y_{t-d}; \gamma_1, c_1) + \phi'_3 x_t G_2(s_{2t}; \gamma_2, c_2) + \epsilon_t
\]

is an additive STAR model where either \( s_{2t} = \Delta_{12} y_{t-d} \) or \( s_{2t} = t \). The foregoing tests for remaining nonlinearity of the TV-STAR/MRSTAR type may be modified to test additive STAR effects by simply excluding regressors in artificial regression (5) involving \( \hat{G}_1 \).

**Heteroskedasticity Robust Tests**

When performing LM tests of remaining residual autocorrelation, unspecified heteroskedasticity may result in spurious rejection of the null hypothesis. Ignored heteroskedasticity in LM tests of linearity, parameter constancy, and model misspecification may have similar effects. It is therefore desirable to have test statistics that are robust in the presence of heteroskedasticity. Wooldridge (1990) has developed a simple set of procedures for obtaining heteroskedasticity robust LM tests in a general setting. Details on implementing heteroskedasticity robust tests in a STAR-type framework are provided in van Dijk, Teräsvirta, and Franses (2002).

While it seems advantageous to compute robust LM tests if there is evidence of heteroskedasticity, a note of caution is in order. Lundbergh and Teräsvirta (1998) provide simulation evidence showing that in certain instances robustification reduces the power of linearity tests. In other words, robustification may make it difficult to detect nonlinearity when in fact it truly exists. Here we simply present both standard and robustified versions of LM linearity tests. Final model specifications are then determined through careful evaluation of each candidate model’s properties at the estimation and misspecification testing stages.

**Modeling the Hog–Corn Ratio**

In this section, we present results on the estimation of a provisional linear model fitted to the hog–corn ratio data. We then present results of linearity tests, estimates of a candidate TVAR model, results of additional model misspecification tests, and finally estimates of a TV-STAR model. To conserve space, parameter estimates for the various models are not presented; they are, however, available in Holt and Craig (2005).

**Linear Model Results**

A linear AR model is first fitted to the data. To account for seasonality, we include eleven monthly dummy variables, as previously defined. The Akaike information criterion (AIC) is used to choose the lag length. Allowing up to 48 lags, the AIC is minimized at lag 11, implying a total of 1,103 usable observations. Several diagnostics for the best-fitting linear AR model are reported in the left-most column of table 1. LM test results show that even with 11 lags, the linear model apparently does not capture all of the residual autocorrelation. LM tests also reveal substantial evidence of ARCH effects. Based on the Lomnicki–Jarque–Bera (LJB) test (Lomnicki, 1961; Jarque and Bera, 1980), the residuals associated with the AR model fail to satisfy normality. As indicated by the excess kurtosis measure reported in table 1, the error distribution for the linear model has thicker tails than that implied by normality.

**Linearity, Parameter Constancy, and TV-STAR Test Results**

In testing nonlinearity, we use various lags of seasonal first differences of the hog–corn ratio, \( \Delta_{12} y_{t-d}, d = 1, \ldots, D_{\text{max}}, \) where \( D_{\text{max}} = 6 \). Of course to test parameter constancy, we use a linear trend.

---

\(^7\) In fact, the additive STAR model need not be viewed as a simple alternative to the TV-STAR (MRSTAR). As van Dijk, Strikholm, and Teräsvirta (2003) illustrate, it is possible to combine additional additive components with a TV-STAR. This later option may be especially useful if, say, observed seasonality has undergone several changes during the sample period.

\(^8\) Tests were performed initially by using \( D_{\text{max}} = 12 \), but all results following \( d = 6 \) were found to be statistically insignificant and are therefore not reported.
Table 1. Diagnostic Tests for Estimated Models for the U.S. Hog–Corn Ratio

<table>
<thead>
<tr>
<th>Measure</th>
<th>AR</th>
<th>TVAR</th>
<th>TV-STAR</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>1,103</td>
<td>1,103</td>
<td>1,103</td>
</tr>
<tr>
<td>No. of Parameters</td>
<td>24</td>
<td>50</td>
<td>100</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.144</td>
<td>0.263</td>
<td>0.345</td>
</tr>
<tr>
<td>$\hat{\sigma}_e$</td>
<td>0.082</td>
<td>0.076</td>
<td>0.072</td>
</tr>
<tr>
<td>$\hat{\sigma}<em>{e,NL}/\hat{\sigma}</em>{e,L}$</td>
<td>—</td>
<td>0.927</td>
<td>0.878</td>
</tr>
<tr>
<td>AIC</td>
<td>−4.961</td>
<td>−5.063</td>
<td>−5.091</td>
</tr>
<tr>
<td>SIC</td>
<td>−4.700</td>
<td>−4.519</td>
<td>−4.001</td>
</tr>
<tr>
<td>SK</td>
<td>0.113 (0.125)</td>
<td>0.074 (0.318)</td>
<td>0.018 (0.803)</td>
</tr>
<tr>
<td>EK</td>
<td>3.566 (0.000)</td>
<td>3.566 (0.000)</td>
<td>2.068 (0.000)</td>
</tr>
<tr>
<td>LJB</td>
<td>605.081 (0.000)</td>
<td>597.924 (0.000)</td>
<td>199.950 (0.000)</td>
</tr>
<tr>
<td>ARCH(4)</td>
<td>17.348 (0.000)</td>
<td>23.765 (0.000)</td>
<td>19.582 (0.000)</td>
</tr>
<tr>
<td>ARCH(6)</td>
<td>11.674 (0.000)</td>
<td>16.074 (0.000)</td>
<td>13.107 (0.000)</td>
</tr>
<tr>
<td>LM_{SC}(6) S</td>
<td>3.502 (0.002)</td>
<td>1.044 (0.395)</td>
<td>0.940 (0.466)</td>
</tr>
<tr>
<td>LM_{SC}(6) R</td>
<td>2.800 (0.010)</td>
<td>0.759 (0.602)</td>
<td>0.803 (0.567)</td>
</tr>
<tr>
<td>LM_{SC}(12) S</td>
<td>2.020 (0.020)</td>
<td>1.348 (0.185)</td>
<td>1.519 (0.111)</td>
</tr>
<tr>
<td>LM_{SC}(12) R</td>
<td>1.694 (0.063)</td>
<td>1.168 (0.301)</td>
<td>1.267 (0.232)</td>
</tr>
<tr>
<td>LM_{SC}(18) S</td>
<td>2.215 (0.002)</td>
<td>1.382 (0.131)</td>
<td>1.282 (0.191)</td>
</tr>
<tr>
<td>LM_{SC}(18) R</td>
<td>2.176 (0.003)</td>
<td>1.205 (0.249)</td>
<td>1.058 (0.391)</td>
</tr>
<tr>
<td>LM_{SC}(24) S</td>
<td>2.510 (0.000)</td>
<td>1.237 (0.199)</td>
<td>1.079 (0.361)</td>
</tr>
<tr>
<td>LM_{SC}(24) R</td>
<td>2.253 (0.001)</td>
<td>1.047 (0.401)</td>
<td>0.900 (0.603)</td>
</tr>
</tbody>
</table>

Note: $T$ denotes sample size, $R^2$ the unadjusted $R^2$, and $\hat{\sigma}_e$ the residual standard error. $\hat{\sigma}_{e,NL}/\hat{\sigma}_{e,L}$ is the ratio of the residual standard error from the respective nonlinear (STAR) model relative to the linear (AR) model. SK is skewness, EK is excess kurtosis, and LJB is the Lomnicki–Jarque–Bera test of normality of the residuals. ARCH is the LM test of no autoregressive conditional heteroskedasticity (ARCH), and LMSC($\tau$) denotes the $F$ variant of standard (S) and heteroskedasticity robust (R) versions of the LM test of no remaining autocorrelation in the residuals up to and including lag $\tau$. Numbers in parentheses after values of the test statistics are $p$-values.

Results for the LM3 and LM1 “Specific-to-General” linearity tests applied to the AR model, both standard and robustified, are presented in table 2, along with comparable results for parameter constancy. Tests were performed by using 11 lags of the hog–corn ratio along with eleven monthly dummy variables. As well, linearity (parameter constancy) tests are performed using only monthly dummy variables and only lagged dependent variables. While there is some evidence in favor of STAR-type nonlinearity for several values of delay parameter $d$, the most striking test results in table 2 are those for parameter constancy. Regardless of the test used, the null hypothesis of parameter constancy is soundly rejected when all regressors are included. An essentially identical result is obtained when only seasonal dummy variables or only lagged dependent variables are included.

The overall picture that emerges from table 2 then is that of some support for STAR-type nonlinearity, but overwhelming support for the notion that the model’s parameters have not remained constant through time. For many of the reasons mentioned in earlier sections, including institutional and technological change, this result is not surprising. For example, technological change has occurred in hog production (i.e., multiple farrowings per year, total confinement operations, improved genetics, etc.) that has caused seasonality in prices (production) to be less pronounced over time. Similarly, corn yields have risen dramatically over the century along with the ability to dry and store large quantities of grain. As well, since the 1930s various government programs have, at times, substantially impacted corn prices and production. All of these factors, and more, have likely contributed to the observed parameter instability in the linear model of the hog–corn ratio.

Results for the “Specific-to-General-to-Specific” testing sequence are presented in table 3. In this case results for the LM_{TV-STAR} test indicate that linearity is overwhelmingly rejected for all values of $d$ considered, with the minimum $p$ value occurring at $d = 1$. Furthermore, there is clear evidence for $d = 1$ and 6 that both the LM_{STAR} and LM_{TVAR} statistics may be rejected at conventional levels for both the standard and robustified tests; in other words, for these values of $d$ a TV-STAR model is retained.

A Provisional TVAR Model

Based on the combined results in tables 2 and 3, we first fit a TVAR model to the data by using nonlinear least squares. Details of various
Table 2. Results of Standard and Heteroskedasticity Robust LM Tests for Nonlinearity, Specific-to-General Procedure, for Monthly Hog–Corn Ratio

<table>
<thead>
<tr>
<th></th>
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</thead>
<tbody>
<tr>
<td></td>
<td>LM3</td>
<td>LM1</td>
<td>LM3</td>
</tr>
<tr>
<td>yt−1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>yt−2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>yt−3</td>
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<td></td>
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<tr>
<td>yt−4</td>
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</tr>
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<td>yt−5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>yt−6</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>yt−7</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>yt−8</td>
<td></td>
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<td>yt−9</td>
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<tr>
<td>yt−11</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>yt−12</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: Numbers are p-values of F-variants the LM-type tests for specification of STAR-type models described by Ter"asvirta (1994) applied to the U.S. hog–corn ratio, 1913:02–2004.12. The tests are applied to an AR model with 11 lags of first differences and seasonal dummies. LM3 denotes the linearity test based on the third-order Taylor series in (4), while LM1 is the linearity test based on the first-order Taylor series in (3).

Results show there is an improvement in fit for the TVAR model relative to the linear AR specification: the standard deviation of the residuals from the TVAR is over 7% smaller than that of the AR model. Moreover, unlike for the AR model, the residuals of the TVAR model show no evidence of remaining serial correlation. Overall, the TVAR model fits the data better than does the constant parameter AR.

Diagnostic tests for remaining nonlinearity (d = 1, . . . , 6) and for parameter constancy for the TVAR, notably the LM3 and LM1 tests, were obtained for the TVAR. While these results are not reported to save space (they are available in Holt and Craig), they indicate that the TVAR is rejected against the TV-STAR for d = 1 and 6, a result consistent with Specific-to-General-to-Specific testing results presented in table 3. The results also show that there is no evidence of remaining parameter nonconstancy. Based on these additional tests and the evidence in table 3, we next fit a TV-STAR to the hog–corn data.

A TV-STAR Model

Results in table 3 and those just discussed for the TVAR suggest several possibilities for the delay parameter in a TV-STAR, most notably d = 1 or 6. To this end TV-STAR models with both d = 1 and 6 were fitted. Preliminary evidence, including model fit and diagnostic statistics and a post-sample forecasting exercise, indicated that the TV-STAR with d = 1 was preferred. We therefore focus our remaining attention on results for the TV-STAR with transition variable $\Delta_{12}y_{t-1}$. Consequently, the TV-STAR model fitted to the hog–corn data is specified as

$$
(11) \quad \Delta y_t = [\varphi_1^* x_t (1 - G_1 (\Delta_{12} y_{t-1}; \gamma_1, c_1))
+ \varphi_2^* x_t G_1 (\Delta_{12} y_{t-1}; \gamma_1, c_1)]
\times (1 - G_2 (t^*; \gamma_2, c_2))
+ [\varphi_3^* x (1 - G_1 (\Delta_{12} y_{t-1}; \gamma_1, c_1))
+ \varphi_4^* x_t G_1 (\Delta_{12} y_{t-1}; \gamma_1, c_1)]
\times G_2 (t^*; \gamma_2, c_2) + \varepsilon_t
$$

where $x_t = (1, \tilde{x}_t, D_t)'$, $\tilde{x}_t = (\Delta y_{t-1}, \ldots, \Delta_{12} y_{t-1})'$, $D_t$ is a vector of seasonal dummies, and $t^* = t/T, t = 1, \ldots, T$. Results of several misspecification tests for the estimated
TV-STAR are recorded in Table 1. Based on the relative standard error and the AIC, the TV-STAR represents an improvement over both the AR and TVAR specifications. The LJB statistic implies that this model also fails the normality assumption; however, excess kurtosis has now been reduced substantially relative to the other models. Evidence of significant ARCH effects remains. As well, the TV-STAR is associated with no significant autocorrelation at any lag considered. Diagnostic tests for remaining additive nonlinearity and parameter constancy, although not presented to conserve space (they are available in Holt and Craig), indicate there is no evidence of remaining nonlinearity of the additive type. The estimated TV-STAR model therefore appears to do an adequate job of capturing the nonlinearity and time variation in the hog–corn series.

The estimated transition functions for the TV-STAR are

\[
G_1(\Delta_{12} y_{t-1}; \hat{\gamma}_1, \hat{c}_1) = \left[ 1 + \exp \left\{ -500.0 \left( \Delta_{12} y_{t-1} + 0.081 \right) / \hat{\sigma}_{\Delta_{12} y_{t-1}} \right\} \right]^{-1}
\]

and

\[
G_2(t^*; \hat{\gamma}_2, \hat{c}_2)
= \left[ 1 + \exp \left\{ -2.364 \left( t^* - 0.449 \right) / (0.069) \right\} / \hat{\sigma}_t \right]^{-1}
\]

where heteroskedasticity consistent standard errors are reported in parentheses. The estimated location parameter \( c_1 \) in (12) is reasonably close to zero, implying that regimes where \( G_1(\Delta_{12} y_{t-1}) = 1 \) and \( G_1(\Delta_{12} y_{t-1}) = 0 \) are associated with positive and negative changes in the hog–corn ratio over the past twelve months. As illustrated in the upper panel of Figure 2, where each circle denotes at least one observation, the transition between the two regimes is rather abrupt, as would be suggested by the large estimate of \( \gamma_1 \) in (12). Because \( G_1(\Delta_{12} y_{t-1}) \) is simply a monotonic transformation of \( \Delta_{12} y_{t-1} \), it follows that periods for which \( G_1(\Delta_{12} y_{t-1}) = 1 \) or \( G_1(\Delta_{12} y_{t-1}) = 0 \) are roughly associated with peaks (troughs) in the hog cycle. In what follows, we refer to the \( G_1(\Delta_{12} y_{t-1} > c) \) regime as the “peak” regime and the \( G_1(\Delta_{12} y_{t-1} < c) \) regime as the “trough” regime.

As depicted in the lower panel of Figure 2, the structural change implied by the TV-STAR model is rather smooth. The estimate of location parameter \( c_2 \) suggests that structural change is centered on \( t^* = 0.45 \), which corresponds with May, 1954. This result corresponds with the long-run transformation of U.S. agriculture, as described above, which greatly accelerated in the postwar period.

To further explore the implications of the TV-STAR model, the time-varying and regime-dependent intercept terms are plotted in the top panel of Figure 3, along with the observed data. As expected, intercept terms are higher for the peak regime (approximately 0.51 at \( t^* = 0 \)) than for the trough regime.

### Table 3. Results of Standard and Heteroskedasticity Robust LM Tests for Nonlinearity, Specific-to-General-to-Specific Procedure, for Monthly Hog–Corn Ratio

<table>
<thead>
<tr>
<th>Transition Variable, ( s_t )</th>
<th>Standard Tests</th>
<th>Robust Tests</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Delta_{12} y_{t-1} )</td>
<td>LM_{TV-Star}</td>
<td>LM_{STAR}</td>
</tr>
<tr>
<td>8.19E−22</td>
<td>1.26E−06</td>
<td>4.90E−21</td>
</tr>
<tr>
<td>1.02E−21</td>
<td>1.51E−06</td>
<td>1.07E−21</td>
</tr>
<tr>
<td>5.87E−21</td>
<td>6.05E−06</td>
<td>9.35E−21</td>
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<tr>
<td>1.90E−18</td>
<td>4.48E−04</td>
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<td>2.61E−18</td>
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<tr>
<td>7.23E−21</td>
<td>7.11E−06</td>
<td>1.13E−17</td>
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<td></td>
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<td>3.88E−10</td>
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<tr>
<td></td>
<td>1.25E−07</td>
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</tr>
<tr>
<td></td>
<td>1.03E−08</td>
<td>0.030</td>
</tr>
</tbody>
</table>

Note: Numbers are \( p \)-values of \( F \) variants the LM-type tests for specification of TV-STAR-type models described by Lundbergh, Ter¨asvirta, and van Dijk (2003) applied to the U.S. hog–corn ratio, 1913:02–2004:12. The tests are applied to an AR model with 11 lags of first differences and seasonal dummies, and are based on the auxiliary regression in (10).

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9 This relative ranking was also maintained in a post-sample forecast evaluation. Specifically, the models were estimated initially by using data through 1989 and then reestimated recursively on a rolling window of data for each month through June, 2004. For each window, 1-step-ahead to 18-step-ahead forecasts for the level of the series were obtained, resulting in a total of 163 forecasts at each forecast horizon. At one and two month horizons, all models perform equally well in terms of root mean square forecast error. Beyond this horizon, however, the AR model exhibits inferior forecasting performance. And beginning with the nine-month horizon the TV-STAR model consistently outperforms the TVAR. Additional details are provided in Holt and Craig.
Figure 2. $G_1(\Delta_{12}y_{t-1})$, as a function of the transition variable, $\Delta_{12}y_{t-1}$ (top panel) and $G_2(t^*)$ over time (bottom panel)

(approximately 0.28 at $t^* = 0$). The variation from high-to-low intercept terms in the top panel of figure 3 therefore provides a graphical depiction of the hog–corn cycle over time. Because the transition variable is $\Delta_{12}y_{t-1}$, high (low) values of the ratio are immediately followed by peak (trough) periods in the hog–corn cycle. This result suggests, for example, that periods of relative scarcity in the corn market are followed by sell-offs in the hog market—a response to higher feed prices—which in turn induces a shift to a contractionary regime. The implied long-run (deterministic) equilibrium for the TV-STAR model is plotted in the lower panel of figure 3. The gradual increase in the long-run equilibrium values for regimes mirrors the perceptible increase in the hog–corn ratio over time, and therefore the slow departure from the historical “ten-to-one rule” for profitability in raising hogs, which characterized the market before the 1930s.\textsuperscript{10} The results indicate that, with the exception of early-to-mid 1930s (when the cycles were brief) and the late 1980s and early 1990s (when they were longer), there has been roughly a three-to-five year hog–corn cycle, a result consistent with previous research (e.g., Jelavich, 1973). There is also some evidence that the duration of the cycle, and especially troughs, has decreased since the late 1960s.

\textsuperscript{10} In part this result likely reflects the somewhat diminished importance of corn, a carbohydrate, as a dominant variable factor of production in raising hogs during the postwar period. Specifically, protein sources such as soybean meal have gained in relative import over this period.
Figure 3. Observed data and moving intercept (top panel) and observed data and moving long-run equilibrium (bottom panel)

Model Dynamics

As the foregoing makes clear, there are features of the hog market consistent with both nonlinear dynamics and structural change. It is therefore desirable to characterize the dynamic behavior of the estimated TV-STAR model in some consistent and reasonably transparent ways, the focus of this section.

Deterministic Extrapolation

We consider first a deterministic extrapolation of the model to obtain insights into its implied behavior. This is done by iterating the skeleton of the model, that is, the deterministic part of the model, ahead without introducing stochastic shocks. We start the extrapolation by using the final values of the sample data as initial values. Iterating the model ahead for thirty-six years, we find that the realizations converge to a unique seasonal pattern associated with the seasonal dummy variables. The results are plotted in the right-hand panel of figure 1. The long-run seasonal peak occurs in August, with a ratio of 23.2-to-1, and the long-run seasonal low occurs for April, with a ratio of 19.3-to-1.

Table 4. Roots of Characteristic Polynomials for Select Values of Transition Functions $G_1(\Delta_{12}y_{t-1}) = 0$ and $G_2(t^*) = 0$

<table>
<thead>
<tr>
<th>Root</th>
<th>Modulus (Half-Life)</th>
<th>Period</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regime: $G_1(\Delta_{12}y_{t-1}) = 0$, $G_2(t^*) = 0$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.95 ± 0.15</td>
<td>0.96 (16.57)</td>
<td>38.82</td>
</tr>
<tr>
<td>-0.92 ± 0.15</td>
<td>0.93 (9.70)</td>
<td>2.11</td>
</tr>
<tr>
<td>Regime: $G_1(\Delta_{12}y_{t-1}) = 1$, $G_2(t^*) = 0$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.99 ± 0.13</td>
<td>1.00 (2330.78)</td>
<td>49.58</td>
</tr>
<tr>
<td>0.76 ± 0.52</td>
<td>0.92 (8.55)</td>
<td>10.57</td>
</tr>
<tr>
<td>-0.16 ± 0.88</td>
<td>0.90 (6.27)</td>
<td>3.59</td>
</tr>
<tr>
<td>Regime: $G_1(\Delta_{12}y_{t-1}) = 0$, $G_2(t^*) = 1$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-0.67 ± 0.39</td>
<td>0.90 (6.05)</td>
<td>2.59</td>
</tr>
<tr>
<td>Regime: $G_1(\Delta_{12}y_{t-1}) = 1$, $G_2(t^*) = 1$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.91 ± 0.12</td>
<td>0.92 (7.88)</td>
<td>48.26</td>
</tr>
</tbody>
</table>

Note: Only roots with modulus $\geq 0.90$ are reported. Period is denoted in months.
Figure 4. Sliced spectra, $G_1(\Delta_{12}y_{t-1}) = 0$ (solid line) and $G_1(\Delta_{12}y_{t-1}) = 1$ (dashed line), for select periods: (a) $G_2(t^*) = 0$, (b) $G_2(t^*) = 0.5$, and (c) $G_2(t^*) = 1$

**Characteristic Roots and Sliced Spectra**

It is possible at each point in the sample period and for various values of the transition functions $G_1(\Delta_{12}y_{t-1})$ and $G_2(t^*)$, to compute the roots of the characteristic polynomial of the model (Teräsvirta, 1994). Roots of the characteristic polynomial (with modulus $\geq 0.90$), along with period lengths and half-lives, are reported in table 4 for $G_1(\Delta_{12}y_{t-1}) = 0$ and $G_1(\Delta_{12}y_{t-1}) = 1$ for $G_2(t^*) = 0$, and likewise for $G_2(t^*) = 1$. Of interest is that, early in the period, the dominant root is associated with a complex pair and a modulus near one when $G_1(\Delta_{12}y_{t-1}) = 1$. Late in the sample data (i.e., $G_2(t^*) = 1$), both the dominant roots are represented by complex pairs, with moduli less than one and with relatively short half-lives. Therefore, late in the sample, there is a tendency for the model to return rather quickly to its long-run equilibrium level, as illustrated by the deterministic extrapolation in figure 1. This finding is consistent with accelerated information flows (i.e., market coordination), which in part have been induced by the rapid rise of vertical integration in the hog market (i.e., grower contracts).

A similar related picture emerges by considering the sliced spectra for the model at several different time periods (Skalin and Teräsvirta,
Figure 5. Mean paths for generalized impulse response functions of the TV-STAR model. (a) $G_2(t^*) = 0$, (b) $G_2(t^*) = 0.5$, and (c) $G_2(t^*) = 1$

Of interest is that the periodicity of the cycle in both peak and trough regimes has shortened throughout the sample period, and especially for troughs. This in part must reflect the fact that building livestock inventories is more readily accomplished later in the sample period than earlier, a result of the adoption of nearly continuous farrowing schedules and total confinement operations. In addition, increased spatial integration of commodity markets has facilitated greater market coordination, which has also reinforced a shortening of the periodicity of the cycle. In short, the flow of slaughtered hogs and information pertaining to them is more nearly instantaneous later in the sample period. Also of interest is that the contribution of the hog
cycle frequency becomes less prominent and the seasonal effect more prominent through time for peak regimes. Taken together, these results confirm the ability of the estimated TV-STAR model to capture fundamental aspects of the hog–corn series through time, and especially the asymmetric adjustments during peak and trough periods that are apparently a feature of the data.

Generalized Impulse Response Functions

To obtain additional information about the dynamic properties of the model, shock propagation is examined by computing generalized impulse response functions (GIs) proposed by Koop, Pesaran, and Potter (1996). Details on GI computation are available in Holt and Craig. Here we compute GIs for three STAR models: the one observed prior to the structural change \[[G_2(t^*) = 0]\]; the one observed when structural change is at the midway point \[[G_2(t^*) = 0.5]\], and the one obtained when structural change is complete \[[G_2(t^*) = 1]\]. Mean paths for the GIs, conditional on histories for \[G_1(\Delta_{12}y_{t-1}) > 0.5\] (peaks) and for \[G_1(\Delta_{12}y_{t-1}) \leq 0.5\] (troughs), and conditional on positive and negative shocks, are presented in figure 5.

Several features of interest are revealed in these plots. First, comparing panels (a) and (c), it is clear that shocks have a somewhat bigger effect at the end of the structural change than at the beginning. For example, following the structural change the largest effect of a positive shock in a peak period occurs at month 2, with an oscillatory decline toward zero thereafter. But before structural change takes place, the largest response occurs instantaneously. As well, the persistence seems more highly amplified toward the middle and the end of structural change (figures 5(b) and (c)) then at the beginning (figure 5(a)). While there are potentially many reasons why hog–corn markets are now more responsive to shocks than in previous periods, in part this must be related to (1) the quantity and speed with which information is now disseminated, processed, and acted upon; and (2) the fact that hog production in particular has become a more highly integrated process, implying that a smaller number of agents is coordinating production and marketing decisions. By the end of the structural change the responses are relatively symmetric to positive and negative shocks in both regimes, although this is not the case prior to the structural change being completed, and especially so for troughs. Finally, and most importantly, there is a distinct difference in shock transmission between the two regimes. Shocks during peaks are larger in magnitude and have more persistence, at least initially, than do shocks during troughs, a feature that remains even after the structural change. This effect is especially noticeable for negative shocks (figure 5), where there are distinct differences in the mean paths even at horizons of up to thirty months. Of course this result is expected because, as noted previously, it is easier to liquidate herds than it is to build them in response to changes in expected profits.

Conclusions

In this article, we have explicitly modeled potential nonlinear features of the U.S. hog–corn cycle in combination with structural change. While previous research has found evidence of nonlinearity in the hog–corn cycle, no prior attempts have been made to explicitly model the implied nonlinearity. We do so here by using a class of endogenous regime switching models belonging to the family of STARs. The time series of monthly observations on the hog–corn ratio used in the empirical analysis spans the 1910–2004 period. Not only does this period include a number of complete cycles, but also it encompasses many historical and institutional changes that might lead to structural instability. At the beginning of the sample period, a national cycle was just emerging as local and regional markets became more integrated. Moreover, breeding cycles were such that a new crop of pigs would typically be produced only once or at most twice a year. This situation began to change rather rapidly in the postwar period as producers switched to total confinement operations and to nearly continuous breeding-production cycles. Consolidation of this sort was especially prevalent from the 1950s onward. We might therefore suspect that these effects would have a substantial impact on seasonal production patterns, and therefore on seasonal price patterns.

In modeling the data, we followed the basic testing, estimation, and evaluation cycle proposed initially by Teräsvirta (1994). The results of various linearity tests suggested that a TV-STAR model is appropriate for modeling the hog–corn cycle. A TV-STAR model that uses \[\Delta_{12}y_{t-1}\] as a transition variable was subsequently fitted to the data, and found—based on comparisons with linear AR and TVAR
models and, as well, model diagnostics—to be a suitable specification in nearly every respect. We proceeded to analyze various features of this model.

A careful examination of the TV-STAR’s properties yielded several interesting features of the hog–corn cycle. First, the cycle appears to have occurred with a somewhat regular three-to-five year frequency during the sample period. The early 1930s emerged as a time of high activity, with cycles occurring much more frequently. Moreover, structural change has apparently occurred rapidly since the 1950s. As well, the role of the cycle itself seems to have diminished somewhat by the end of the sample period. Finally, calculation of generalized impulse response functions showed that the response of the model to a shock is quantitatively and qualitatively different in the two regimes. In the end, our research suggests that the hog–corn cycle itself is not a stationary process, but rather a feature of these markets that has, itself, evolved through time as dictated by institutional and technological change.

[Received December 2004; accepted May 2005.]

References


