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# The ADF–KPSS test of the joint confirmation hypothesis of unit autoregressive root

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**Abstract**

This paper analyses properties of the ADF–KPSS test of the joint confirmation hypothesis of unit autoregressive root, when used in case of short samples with possible structural changes. Firstly, the critical values of the test are calculated for small samples, and secondly, the power of the test is investigated, including the case of structural change. The results lead to the conclusion that detecting potential structural breaks and inclusion of appropriate dummies in models of statistics can be treated as a successful strategy in inference on unit autoregressive root when the ADF–KPSS test is applied.


*Keywords:* Unit root; Dickey–Fuller; KPSS; Joint confirmation hypothesis; Joint distribution; Monte Carlo experiments

*JEL classification:* C12; C15; C16; C32

## 1. Introduction

Charemza and Syczewska (1998) proposed analysis of joint confirmation of the integration order, yielding new critical values calculated jointly for unit autoregressive root and unit moving average root tests, instead of conventionally used separate critical values. In the joint confirmation analysis one can assume under the null that either $H_d : y_t \sim \mathrm{I}(d)$ versus $H_{d-1} : y_t \sim \mathrm{I}(d-1)$, which is called joint confirmation hypothesis of unit autoregressive root and was proposed by Carrion-i-Silvestre et al. (2001), or $H_{d-1} : y_t \sim \mathrm{I}(d-1)$ versus $H_d : y_t \sim \mathrm{I}(d)$, which is called joint confirmation hypothesis of moving average root. Current analysis focuses on the former hypothesis, which ensures that the significance level is maintained, when the sequence of hypotheses is verified.

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Let $f^{\psi,T}$ $(z_D, z_K|H_i)$ be the joint density function of the Dickey–Fuller test statistics, $z_D$ (see Dickey and Fuller, 1979) and the Kwiatkowski–Phillips–Schmidt–Shin test statistics, $z_K$ (see Kwiatkowski et al., 1992) conditional on the null or the alternative hypothesis of the joint confirmation analysis. The vector of DGP parameters is denoted by $\psi$ and $T$ stands for the sample size. Probability of the joint confirmation ($PJC$) of unit autoregressive root in autoregressive-moving average representation of $y_t$ by the ADF–KPSS test, is defined as

$$\int_{z_D^{PJC}}^{\infty}\int_{z_K^{PJC}}^{\infty} f^{\psi,T}(z_D, z_K)\,|\,H_d)dz_D dz_K = PJC. \tag{1}$$

The above joint density function depends on two arguments, thus an additional condition is necessary to unambiguously determine pair of statistics $\left(z_D^{PJC};\ z_K^{PJC}\right)$ meeting Eq. (1). Charemza and Syczewska (1998) implied equality of the marginal density functions, namely

$$\int_{z_D^{PJC}}^{\infty} f^{\psi,T}(z_D\,|\,H_d)dz_D = \int_{z_K^{PJC}}^{\infty} f^{\psi,T}(z_K\,|\,H_d)dz_K, \tag{2}$$

which leads to a unique pair of critical values $\left(\tilde{z}_D^{PJC};\ \tilde{z}_K^{PJC}\right)$. It is readily verified from Eq. (1) that the rejection region of the ADF–KPSS test for the joint confirmation hypothesis of unit autoregressive root (the joint ADF–KPSS test throughout) is left-sided.

The critical values of the joint ADF–KPSS test computed so far concern rather large samples while in the empirical work samples are usually limited. The paper tries to fulfill this gap, which is consistent with contemporary tendency to use exact small sample distributions or corrected statistics for limit distributions (see Johansen, 2003). Some issues related with theory and application of joint confirmation analysis of the integration order have been stressed by Charemza and Syczewska (1999); Carrion-i-Silvestre et al. (2001). Extension of the joint confirmation analysis to the area of cointegration has been done by Gabriel (2003).

## 2. Critical values of the joint ADF–KPSS test

The critical values of the ADF–KPSS test were calculated for three distinct alternative hypotheses, assuming: (a) stationarity of $y_t$ with zero expectation, (b) stationarity of $y_t$ with non-zero expectation, (c) trendstationarity of $y_t$. The joint distributions of $z_D$ and $z_K$ statistics were approximated by Monte Carlo experiments.[1] The data generating process was specified as $y_t = \alpha_0 + y_{t-1} + \xi_t$, where $\xi_t$ is i.i.d., comes from standardized, normal distribution, $N(0;1)$, and $y_0 = 0$. The following values of coefficients were attributed to the above-mentioned alternatives, i.e. (a) $\alpha_0 = 0.1 \wedge y_1 = 1$, (b) $\alpha_0 = 0 \wedge y_1 = 1$ and (c) $\alpha_0 = 0 \wedge y_1 = 0$. In the second step, values of the statistics $z_D$, $z_K$ were calculated. The procedure was repeated a hundred thousand times, yielding the same number of replications of each statistic. The

[1] Monte Carlo experiments were performed in GAUSS. We benefited from procedures prepared by E.M. Syczewska and M. Gambera, K. Strellec.

replications were put in ascending order to calculate quantiles, which divide two sets of replications into 250 equal parts. Each pair $\left(z_D^{PJC};\ z_K^{PJC}\right)$ was assigned to proper intervals indicated by the two next quantiles of each statistic. Then the frequency table was transformed into a cumulative frequency table. Finally, the pair $\left(\tilde{z}_D^{PJC};\ \tilde{z}_K^{PJC}\right)$ was chosen for which the ratio of accumulated frequencies located in diagonal to the number of replications was the smallest but not smaller than *PJC*. The experiment was repeated for each assumed sample size and each alternative hypothesis. The results are in Table 1. Results for a sample size larger than 45 are accessible upon request. However, since the DGP does not contain short-run dynamics, therefore high-order autoregressive and moving average process are not allowed.

The analysis of the joint ADF−KPSS test properties should be based on asymptotical, critical values. To approximate the critical values response surfaces were employed, which were amounted to

Table 1
Critical values of the joint ADF−KPSS test

| *T* | *PJC* | No deterministic term | | With constant | | With trend | |
|---|---|---|---|---|---|---|---|
| | | $\tilde{z}_D^{PJC}$ | $\tilde{z}_K^{PJC}$ | $\tilde{z}_D^{PJC}$ | $\tilde{z}_K^{PJC}$ | $\tilde{z}_D^{PJC}$ | $\tilde{z}_K^{PJC}$ |
| 20 | 0.99 | − 3.804 | 0.083 | − 5.498 | 0.100 | − 6.779 | 0.091 |
| | 0.975 | − 3.211 | 0.101 | − 4.711 | 0.109 | − 5.660 | 0.093 |
| | 0.95 | − 2.798 | 0.118 | − 4.239 | 0.118 | − 5.070 | 0.094 |
| | 0.9 | − 2.305 | 0.146 | − 3.749 | 0.133 | − 4.459 | 0.097 |
| | 0.85 | − 1.983 | 0.173 | − 3.469 | 0.143 | − 4.146 | 0.099 |
| 25 | 0.99 | − 3.484 | 0.083 | − 4.819 | 0.086 | − 5.669 | 0.072 |
| | 0.975 | − 2.948 | 0.103 | − 4.315 | 0.096 | − 5.039 | 0.075 |
| | 0.95 | − 2.586 | 0.123 | − 3.957 | 0.107 | − 4.627 | 0.078 |
| | 0.9 | − 2.151 | 0.156 | − 3.517 | 0.127 | − 4.200 | 0.082 |
| | 0.85 | − 1.870 | 0.188 | − 3.255 | 0.140 | − 3.948 | 0.085 |
| 30 | 0.99 | − 3.354 | 0.083 | − 4.592 | 0.079 | − 5.277 | 0.062 |
| | 0.975 | − 2.849 | 0.105 | − 4.116 | 0.090 | − 4.781 | 0.065 |
| | 0.95 | − 2.506 | 0.130 | − 3.789 | 0.102 | − 4.455 | 0.068 |
| | 0.9 | − 2.084 | 0.171 | − 3.377 | 0.125 | − 4.069 | 0.073 |
| | 0.85 | − 1.819 | 0.206 | − 3.097 | 0.144 | − 3.843 | 0.078 |
| 35 | 0.99 | − 3.325 | 0.083 | − 4.566 | 0.079 | − 5.185 | 0.062 |
| | 0.975 | − 2.831 | 0.105 | − 4.102 | 0.091 | − 4.718 | 0.065 |
| | 0.95 | − 2.491 | 0.128 | − 3.760 | 0.103 | − 4.411 | 0.068 |
| | 0.9 | − 2.085 | 0.169 | − 3.368 | 0.125 | − 4.048 | 0.073 |
| | 0.85 | − 1.817 | 0.206 | − 3.084 | 0.145 | − 3.828 | 0.077 |
| 40 | 0.99 | − 3.180 | 0.089 | − 4.371 | 0.075 | − 4.980 | 0.052 |
| | 0.975 | − 2.718 | 0.116 | − 3.932 | 0.090 | − 4.590 | 0.057 |
| | 0.95 | − 2.413 | 0.145 | − 3.612 | 0.105 | − 4.291 | 0.061 |
| | 0.9 | − 1.988 | 0.202 | − 3.187 | 0.136 | − 3.890 | 0.068 |
| | 0.85 | − 1.775 | 0.242 | − 2.961 | 0.157 | − 3.666 | 0.073 |
| 45 | 0.99 | − 3.279 | 0.083 | − 4.450 | 0.076 | − 5.049 | 0.056 |
| | 0.975 | − 2.774 | 0.108 | − 4.002 | 0.090 | − 4.614 | 0.060 |
| | 0.95 | − 2.447 | 0.135 | − 3.690 | 0.103 | − 4.322 | 0.063 |
| | 0.9 | − 2.015 | 0.187 | − 3.286 | 0.128 | − 3.956 | 0.069 |
| | 0.85 | − 1.796 | 0.223 | − 3.002 | 0.149 | − 3.714 | 0.075 |

Table 2
Approximations of asymptotical, critical values

| *PJC* | No deterministic term | | With constant | | With trend | |
|---|---|---|---|---|---|---|
| | $\tilde{z}_D^{PJC}$ | $\tilde{z}_K^{PJC}$ | $\tilde{z}_D^{PJC}$ | $\tilde{z}_K^{PJC}$ | $\tilde{z}_D^{PJC}$ | $\tilde{z}_K^{PJC}$ |
| 0.99 | −2.847 | 0.391 | −3.735 | 0.236 | −4.224 | 0.102 |
| 0.975 | −2.502 | 0.558 | −3.370 | 0.320 | −3.896 | 0.130 |
| 0.95 | −2.242 | 0.746 | −3.100 | 0.420 | −3.604 | 0.162 |
| 0.9 | −1.891 | 1.097 | −2.822 | 0.572 | −3.340 | 0.201 |
| 0.85 | −1.699 | 1.374 | −2.629 | 0.709 | −3.160 | 0.234 |

regression of the critical values repeatedly calculated through the Monte Carlo experiments on the sample sizes:

$$z_{S,i}^{PJC} = \sum_{k=0}^{2} \lambda_k t_i^{-k} + \xi_i, \tag{3}$$

where $i$, denotes observations ($i=1, 2,\ldots, N{\cdot}M$); $n$, denotes critical values computed for the same data generating process and sample size ($n=1, 2,\ldots, N$); $m$, numerates distinct sample sizes the critical values were evaluated for ($m=1, 2,\ldots, M$); $k$, numerates consecutive regressors ($k=0, 1, 2$); $S$, denotes distinct statistics, $S=D$ for Dickey–Fuller test, $S=K$ for Kwiatkowski–Phillips–Schmidt–Shin test; $\mathrm{z}_{S(i)}^{PJC} = \left[\tilde{\mathrm{z}}_{S,1,(n)}^{PJC} | \tilde{\mathrm{z}}_{S,2,(n)}^{PJC} | \ldots | \tilde{\mathrm{z}}_{S,M,(n)}^{PJC}\right]_{(N\cdot M)}^{T}$ – $N \cdot M$-element column vector consisting of $M$ subvectors, each containing the $N$ critical values computed for single assumed sample size; $\mathrm{t}_{(i)} = [\mathrm{t}_{1(n)} | \mathrm{t}_{2(n)} | \ldots | \mathrm{t}_{m(n)}]_{(N\cdot M)}^{T}$; $\lambda_{(k)} = [\,\lambda_0 \quad \ldots \quad \lambda_K\,]^T$; $\xi_i$, error term.

Intercept $\lambda_0$ in Eq. (3) is directly interpretable as an approximation of the asymptotical, critical values (see Ericsson and MacKinnon, 2001).

In order to calculate the approximations, the Monte Carlo experiment was repeated for $T$ equal: 50, 100, 250, 500 and each alternative hypothesis. Twelve experiments were conducted, each repeated 20 times, yielding 240 pairs of $\left(\tilde{z}_D^{PJC};\ \tilde{z}_K^{PJC}\right)$ for each assumed *PJC*, equal: 0.99, 0.975, 0.95, 0.9, 0.85. Then vectors $\mathbf{z}_{(i)}$ were used to estimate the asymptotical, critical values. The results are presented in Table 2.

The critical values depending on sample size converge to the approximations of the asymptotical, critical values of the ADF–KPSS test, but the more complex the models are, the slower the convergence is. Comparison of the critical values with the approximation of asymptotical, critical values impose the general conclusion that the convergence of statistics $\tilde{z}_D^{PJC}$ is faster than the convergence of statistics $\tilde{z}_K^{PJC}$, regardless of alternative hypothesis.

## 3. Power of the joint ADF–KPSS test

The power of the joint ADF–KPSS test can be expressed as

$$\int_{-\infty}^{\tilde{z}_D^{PJC}} \int_{0}^{\tilde{z}_K^{PJC}} f^{\psi,T}(z_D, z_K \,|\, H_{d-1}) dz_D dz_K, \tag{4}$$

where $\left(\tilde{z}_D^{PJC};\ \tilde{z}_K^{PJC}\right)$ is the pair of critical values from distribution given by Eq. (1) under condition (2).

Data generating process was formulated as $y_t = \alpha_0 + \alpha_1 y_{t-1} + \alpha_2 t + \xi_t$ and $\xi_t = \eta_t + \theta\eta_{t-1}$, where $\xi_t$ and $\eta_t$ are i.i.d., come from standardized, normal distribution, N(0;1), and $y_0 = 0$. In the data generating process the following values of coefficients were assumed: (a) $\alpha_0 = \alpha_2 = 0$ –stationarity of $y_t$ with zero expectation, (b) $\alpha_0 = 1 \wedge \alpha_2 = 0$ –stationarity of $y_t$ with non-zero expectation, (c) $\alpha_0 = 1 \wedge \alpha_2 = 0,\ 1$ –trendstationarity of $y_t$. Sample size was assumed as a hundred and the number of replications was fixed at a hundred thousand. As $\left(\tilde{z}_D^{PJC};\ \tilde{z}_K^{PJC}\right)$ the asymptotic counterparts were accepted for $PJC$ equal to 0.95. The results for each alternative hypothesis are given in Table 3. The unit values of the power are the result of rounding and finite number of replications.

The power of the joint ADF–KPSS test decreases as the coefficient $\alpha_1$ increases towards 1, and rises as the coefficient $\theta$ drops towards $-1$. This is the result of approaching to unit root in characteristic equation associated with autoregressive representation of $y_t$, and with moving average representation of $y_t$, respectively. The results can be considered as an evidence of the consistency of the test.

The experiments were repeated for the variable generated by stochastic process with structural breaks:

$$y_t = \alpha_0 + \alpha_1 y_{t-1} + \alpha_2 t + \alpha_3 BM_t + \alpha_4 BT_t + \xi_t, \tag{5a}$$

$$\xi_t = \eta_t + \theta\eta_{t-1} \tag{5b}$$

where

$$BM_t = 1 \quad \text{if } t > T_B, \quad BT_t = t - T_B \quad \text{if } t > T_B,$$

$$BM_t = 0 \quad \text{if } t \le T_B, \quad BT_t = 0 \quad \text{if } t \le T_B,$$

and $T_B$ denotes the moment preceding structural break. Coefficients $\alpha_3$, $\alpha_4$ connected with dummies representing structural breaks equal to 0.5 and 0.05, respectively, the number of replications was fixed at ten thousand and other parameters remain unchanged. The results are presented in Fig. 1, where the dotted line denotes the power when the break in mean of the process occurs, the solid line stands for the

Table 3
Power of the joint ADF–KPSS test

| $(\alpha_1;\ \theta)$ | Alternative hypotheses | | |
|---|---|---|---|
| | Stationarity of $y_t$, $E(y_t)=0$ | Stationarity of $y_t$, $E(y_t)\neq 0$ | Trendstationarity of $y_t$ |
| (0; 0) | 0.997 | 0.996 | 0.997 |
| (0; − 0.5) | 0.999 | 0.999 | 0.999 |
| (0; − 0.8) | 1.000 | 1.000 | 1.000 |
| (0.2; 0) | 0.996 | 0.994 | 0.996 |
| (0.2; − 0.5) | 0.999 | 0.998 | 1.000 |
| (0.2; − 0.8) | 1.000 | 1.000 | 1.000 |
| (0.5; 0) | 0.991 | 0.989 | 0.990 |
| (0.5; − 0.5) | 0.996 | 0.993 | 0.994 |
| (0.5; − 0.8) | 1.000 | 1.000 | 0.999 |
| (0.8; 0) | 0.943 | 0.919 | 0.894 |
| (0.8; − 0.5) | 0.965 | 0.953 | 0.924 |
| (0.8; − 0.8) | 0.996 | 0.992 | 0.963 |

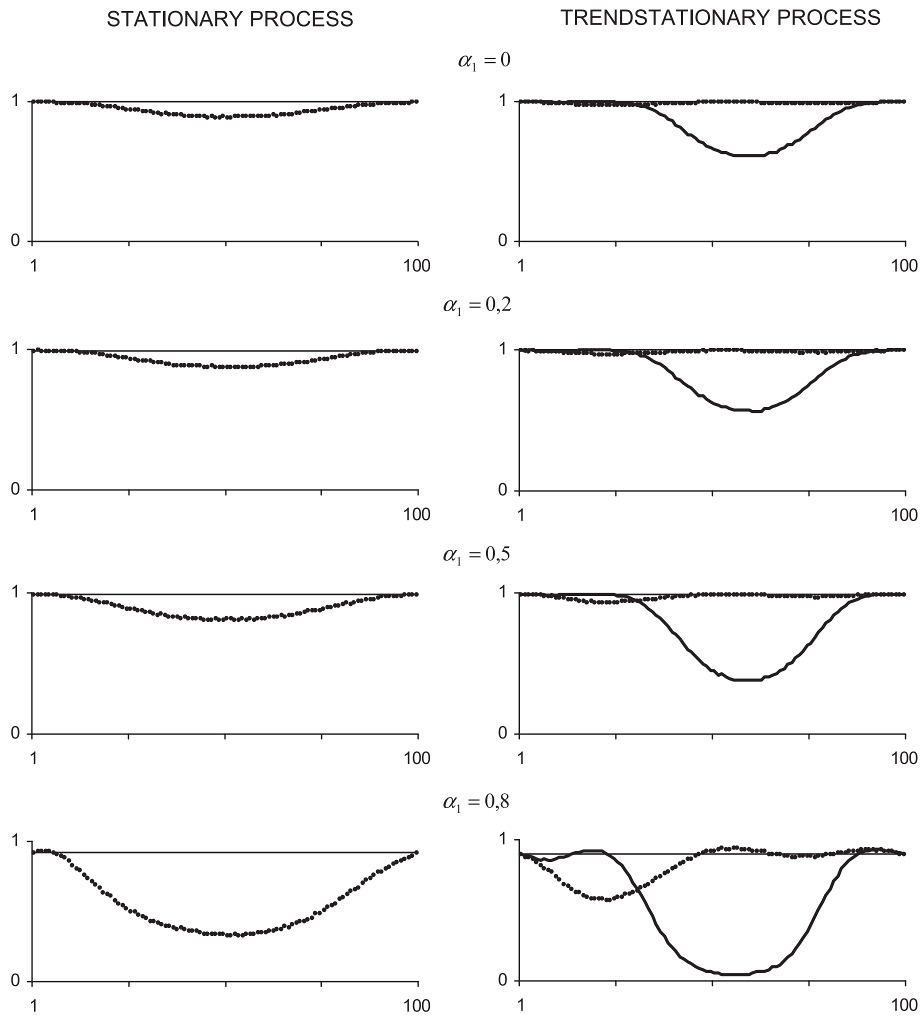


Fig. 1. Power of the joint ADF-KPSS test in the presence of structural break.

power when breaks in mean and trend occur simultaneously and the straight horizontal line indicates the power connected with no-brake data generating process. The experiments were conducted for different alternative hypothesis and moments of structural break. Each point of the dotted and the solid lines represents a distinct experiment and indicates the power of the test when the single break in the sample occurred and the x-axis represents the percentage position of moment of structural break in the sample.

The comparison leads to the following conclusions: (i) there is a significant loss of power in the case of time series generated by stochastic process with structural break, particularly when the break in trend occurs, (ii) a relatively sharp fall in the power is observed when the structural break occurs in the middle of the sample, (iii) in the case of break in the mean the loss of power is smaller if the process is trendstationary. Therefore, the identification of structural breaks and inclusion of appropriate dummies seems to be crucial (see Perron, 1989), and it is inevitable to calculate new critical values for every number, character and moment of structural breaks in stochastic process generating the data.

## 4. Conclusions

The joint ADF–KPSS test ensures a complex approach to the problem of the integration order. It is a powerful tool in inference on short samples if stability of the structural coefficients is ensured by detecting potential structural breaks by appropriate dummies in the models of statistics.

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