COMMUNICATIONS IN STATISTICS
Theory and Methods
Vol. 33, No. 8, pp. 1863–1876, 2004

A Bayesian Analysis of a Structural Change in the Parameters of a Time Series

Boualem Kezim[1,*] and Zahia Abdelli[2]

[1]Girard School of Business and International Commerce, Merrimack College, N. Andover, Massachusetts, USA
[2]Operations Research Department, Universite des Sciences et de la Technologie Houari Boumediene, Algiers, Algeria

ABSTRACT

The purpose of this paper is to make a Bayesian analysis of a first-order autoregressive process subject to one change in both the variance of the error terms and the autocorrelation coefficients at an unknown time point. The main emphasis is to derive the posterior distributions of the change point, the autocorrelation parameter and the variance ratio. A numerical illustration is provided using the Gibbs sampler.

Key Words: Bayesian analysis; Change-point; Change in variance; Autoregressive process; Posterior distribution; Gibbs sampler.

*Correspondence: Boualem Kezim, Girard School of Business and International Commerce, Merrimack College, 315 Turnpike St., N. Andover, MA 01845, USA; Fax: (978) 837-5013; E-mail: kezimb@merrimack.edu.

DOI: 10.1081/STA-120037446 0361-0926 (Print); 1532-415X (Online)

1. INTRODUCTION

The change point problem has been considered by many authors from various viewpoints. From a Bayesian viewpoint, primary interest in the study of structural change has focused on models assuming independence between observations. In practice this assumption may be violated, as in the case of an autoregressive time series. In addition, most studies have been concerned with the problem of a change in the mean of a sequence of independent random variables. There are many contributions to this area since introduced by Page (1954). From a Bayesian viewpoint, Chernoff and Zacks (1964) and Kander and Zacks (1966) derived Bayesian tests for detecting a change in the mean of a sequence of independent normal random variables.

Changes in regression have been considered by Holbert and Broemeling (1977), Chin Choy and Broemeling (1980), and Salazar et al. (1981). There are many other papers which appeared in the literature from a non-Bayesian viewpoint. Broemeling and Tsurumi (1987) gave a review of the literature.

While many Bayesian studies assume homoscedasticity in the error terms, only a few studies have investigated the problem of a change in the variance at an unknown time point (e.g., Smith, 1975). He gave posterior probabilities for the time point when a variance change occurs. Using sampling theory, Hsu (1977) derived two tests for a variance shift in a sequence of independent random variables. For a sequence of independent and normally distributed random variables, Menzefricke (1981) obtained the posterior distributions of both change point and variance.

Abraham and Wei (1984) derived the posterior distributions of the parameters of a time series assuming a known inflation or deflation of the errors variance. Assuming the change point known, Broemeling and Tsurumi (1987) considered a linear model with correlated error terms and where the variance and the correlation coefficient change. Ng Vee Ming (1990) analyses a linear model in which both the mean and the precision change once at an unknown time point. The posterior distributions of the change point and the ratio of precisions are derived. Recently, a few studies have appeared in the literature concerning the problem of multiple change points in the variance of a sequence of independent and autocorrelated observations. Tsay (1988) proposed a procedure to detect outliers, level shifts, and variance changes in an autoregressive moving average model. Inclan and Tiao (1994) considered the problem of multiple change points in the variance of a sequence of independent observations. They proposed a procedure to detect variance changes based on an iterated cumulative sums of squares algorithm. Recently, Wang and Zivot (2000) considered a Bayesian time series model of multiple structural changes in level, trend, and variance. They, however, assumed constant autoregressive parameters. Several financial series that exhibit sudden changes of variance have been studied by many authors (See, Hsu, 1977; Wang and Zivot, 2000).

While Salazar (1982), and Wang and Zivot (2000) assume a constant autocorrelation coefficient, the purpose of this paper is to make a Bayesian analysis of a first-order autoregressive process (AR(1)) subject to one change in both the variance of the error terms and the autocorrelation coefficient at an unknown time point. We would like to point out that, in some situations, the variance of the error terms and the autocorrelation can change in such a way that the variance of the observations do not change. The main emphasis is to derive the posterior distributions of the change point, the autocorrelation parameter and the variance ratio. To overcome the numerical difficulties encountered when Bayesian methods are used, we will apply the Gibbs sampler algorithm to a simulated series.

2. THE FIRST-ORDER AUTOREGRESSIVE PROCESS

Assuming a change in both the autocorrelation coefficient of an AR(1) process and the variance of the error terms at an unknown time point m, the model is given by:

$$y_i - \mu = \rho_1(y_{i-1} - \mu) + e_i; \quad i = 1, \dots, m$$

$$y_i - \mu = \rho_2(y_{i-1} - \mu) + e_i; \quad i = m+1, \dots, n$$

where ρ_1 and ρ_2 are unknown autocorrelation coefficients, y_i is the ith observation of the dependent variable, the error terms e_i are independent random variables and follow a $N(0, \sigma_1^2)$ for $i = 1, 2, \dots, m$ and a $N(0, \sigma_2^2)$ for $i = m+1, \dots, n$. m is the unknown change point and y_0 is the initial quantity. Without loss of generality, we assume $\mu = 0$.

The prior distributions of the unknown parameters are assigned as follows: m has a uniform distribution over $[1, n-1]$, the precision $\eta_i = \sigma_i^{-2}$ has a gamma distribution with parameters $a_i/2$ and $b_i/2$; a_i, $b_i > 0$; $i = 1, 2$, and the conditional distribution of ρ_i given η_i is normal with mean μ_i and precision η_i; $i = 1, 2$. Furthermore, we assume that (ρ_1, η_1) and (ρ_1, η_2) are independent random variables.

The likelihood function is given by

$$L(\theta/y) \propto \eta_1^{\frac{m}{2}} \exp\left\{-\frac{\eta_1}{2}\sum_{i=1}^{m}(y_i - \rho_1 y_{i-1})^2\right\}$$
$$\times\, \eta_2^{\frac{n-m}{2}} \exp\left\{-\frac{\eta_2}{2}\sum_{i=m+1}^{n}(y_i - \rho_2 y_{i-1})^2\right\}$$

where $\theta = (m, \rho_1, \rho_2, \eta_1, \eta_2)$ while the prior distribution of the parameters θ is given by

$$P(\theta) \propto \eta_1^{\frac{a_1-1}{2}} \eta_2^{\frac{a_2-1}{2}} \exp\left\{-\frac{\eta_1}{2}(\rho_1 - \mu_1)^2\right\} \exp\left\{-\frac{\eta_2}{2}(\rho_2 - \mu_2)^2\right\}.$$

By Baye's theorem, the joint posterior distribution of θ is

$$L(\theta/y) \propto \eta_1^{\frac{m+a_1-1}{2}} \exp\left\{-\frac{\eta_1}{2}\sum_{i=1}^{m}(y_i - \rho_1 y_{i-1})^2 + b_1\right\}$$
$$\times\, \eta_2^{\frac{n-m+a_2-1}{2}} \exp\left\{-\frac{\eta_2}{2}\sum_{i=m+1}^{n}(y_i - \rho_2 y_{i-1})^2 + b_2\right\}.$$

Performing some calculations, the following expressions can be rewritten as follows:

$$\sum_{i=1}^{m}(y_i - \rho_1 y_{i-1})^2 = A_1(\rho_1 - \hat{\rho}_1)^2 + S(\hat{\rho}_1) \quad \text{and}$$
$$\sum_{i=m+1}^{n}(y_i - \rho_1 y_{i-1})^2 = A_2(\rho_2 - \hat{\rho}_2)^2 + S(\hat{\rho}_2) \quad \text{where,}$$

$$A_1 = \sum_{i=1}^{m} y_{i-1}^2,\ \hat{\rho}_1 = \frac{1}{A_1}\sum_{i=1}^{m} y_i y_{i-1},\ S(\hat{\rho}_1) = \sum_{i=1}^{m}(y_i - \hat{\rho}_1 y_{i-1})^2$$
$$A_2 = \sum_{i=m+1}^{n} y_{i-1}^2,\ \hat{\rho}_2 = \frac{1}{A_2}\sum_{i=m+1}^{n} y_i y_{i-1},\ \text{and}\ S(\hat{\rho}_2) = \sum_{i=m+1}^{n}(y_i - \hat{\rho}_2 y_{i-1})^2.$$

Therefore, the joint posterior distribution of θ can rewritten as follows:

$$L(\theta/y) \propto \eta_1^{\frac{m+a_1-1}{2}} \exp\left\{-\frac{\eta_1}{2}\left[(\rho_1 - \hat{\rho}_1)^2 A_1 + S(\hat{\rho}_1) + b_1 + (\rho_1 - \mu_1)^2\right]\right\}$$
$$\times\, \eta_2^{\frac{n-m+a_2-1}{2}} \exp\left\{-\frac{\eta_2}{2}\left[(\rho_2 - \hat{\rho}_2)^2 A_2 + S(\hat{\rho}_2) + b_2 + (\rho_2 - \mu_2)^2\right]\right\}.$$

Furthermore, we have

$$A_i(\rho_i - \hat{\rho}_i)^2 + S(\hat{\rho}_i) + b_i + (\rho_i - \mu_i)^2 = A_i[(\rho_i - \tilde{\rho}_i)^2 + T(\tilde{\rho}_i)],$$

$$\tilde{\rho}_i = \frac{A_i\hat{\rho}_i + \mu_i}{A_i}, \quad \text{and} \quad T(\tilde{\rho}_i) = \frac{A_i\hat{\rho}_i + S(\hat{\rho}_i) + b_i + \mu_i^2}{A_i}$$

$$\text{with } T(\tilde{\rho}_i) > 0, \ \text{for } i = 1, 2.$$

Finally, the joint posterior distribution of θ can be written as follows:

$$P(\theta/y) \propto \eta_1^{\frac{m+a_1-1}{2}} \exp\Big\{-\frac{\eta_1}{2}\left[(\rho_1 - \tilde{\rho}_1)^2 + T(\tilde{\rho}_1)\right]A_1\Big\} \times \eta_2^{\frac{n-m+a_2-1}{2}} \exp\Big\{-\frac{\eta_2}{2}\left[(\rho_2 - \tilde{\rho}_2)^2 + T(\tilde{\rho}_2)\right]A_2\Big\}.$$

Integrating $P(\theta/y)$ on (η_1, η_2) and (ρ_1, ρ_2), and using normal and Fisher integrals, respectively, leads to the posterior distribution of the change point m, giving

$$P(m/y) \propto \Gamma\Big(\frac{m+a_1}{2}\Big)\Gamma\Big(\frac{n-m+a_2}{2}\Big) \times (A_1)^{-\frac{m+a_1+1}{2}}(A_2)^{-\frac{n-m+a_2+1}{2}}T(\tilde{\rho}_1)^{\frac{m+a_1}{2}}T(\tilde{\rho}_2)^{\frac{n-m+a_2}{2}}.$$

3. POSTERIOR DISTRIBUTION OF THE VARIANCE RATIO

The posterior distribution of the variance ratio $\tau = \sigma_1^2/\sigma_2^2$ is given by

$$P(\tau/y) = \sum_{m=1}^{n-1} P(\tau/m, y)P(m/y)$$

where

$$P(\tau/m, y) \propto K(m, \tilde{\rho}_1, \tilde{\rho}_2)[\tau K(m, \tilde{\rho}_1, \tilde{\rho}_2)]^{-\frac{n-m+a_2-2}{2}} \times [1 + \tau K(m, \tilde{\rho}_1, \tilde{\rho}_2)]^{-\frac{n+a_1+a_2}{2}}$$

with $K(m, \tilde{\rho}_1, \tilde{\rho}_2) = ((T(\tilde{\rho}_2)A_2)/(T(\tilde{\rho}_1)A_1))$.

4. POSTERIOR DISTRIBUTION OF THE SHIFT IN THE AUTOCORRELATION COEFFICIENTS: $\delta = \rho_2 - \rho_1$

The joint posterior distribution of the parameters $\xi = (m, \rho_1, \delta, \sigma_1, \sigma_2)$ is given by

$$P(\xi/y) \propto \frac{1}{\sigma_1^{m+a_1+2}} \exp\left\{ -\frac{1}{2\sigma_1^2} A_1 \cdot T(\tilde{\rho}_1)\left[1 + \frac{(\rho_1 - \tilde{\rho}_1)^2}{T(\tilde{\rho}_1)}\right]\right\}$$
$$\times \frac{1}{\sigma_2^{n-m+a_2+2}} \exp\left\{ -\frac{1}{2\sigma_2^2} A_2 \cdot T(\tilde{\rho}_2)\left[1 + \frac{(\rho_2 - \tilde{\rho}_2)^2}{T(\tilde{\rho}_2)}\right]\right\}.$$

Integrating $P(\xi/y)$ with respect to (σ_1, σ_2), the joint posterior distribution of (m, ρ_1, δ) is obtained as

$$P(m, \rho_1, \delta/y) \propto \Gamma\left(\frac{m + a_1 + 1}{2}\right)\Gamma\left(\frac{n - m + a_2 + 1}{2}\right)[T(\tilde{\rho}_1)A_1]^{-\frac{m+a_1+1}{2}}$$
$$\times [T(\tilde{\rho}_2)A_2]^{-\frac{n-m+a_2+1}{2}}\left[1 + \frac{(\rho_1 - \tilde{\rho}_1)^2}{T(\tilde{\rho}_1)}\right]^{-\frac{m+a_1+1}{2}}$$
$$\times \left[1 + \frac{(\rho_1 + \delta - \tilde{\rho}_2)^2}{T(\tilde{\rho}_2)}\right]^{-\frac{n-m+a_2+1}{2}}.$$

Thus, the posterior distribution of the shift in the autocorrelation coefficients $\delta = \rho_2 - \rho_1$ is given by

$$P(\delta/y) \propto \sum_{m=1}^{n-1} \int_{\rho_1} P(m, \rho_1, \delta/y) d\rho_1.$$

Note. (1) The integral $\int_{\rho_1} [1 + (\rho_1 - \tilde{\rho}_1)^2/T(\tilde{\rho}_1)]^{-((m+a_1+1)/2)} \times [1 + (\rho_1 + \delta - \tilde{\rho}_2)/T(\tilde{\rho}_2)]^{-((n-m+a_2+1)/2)} d\rho_1$ can be obtained using numerical integration or the Gibbs sampler.

(2) The joint posterior distribution of the autocorrelation coefficients (ρ_1, ρ_2) is given by $P(\rho_1, \rho_2/y) = \sum_{m=1}^{n-1} P(\rho_1, \rho_2/m, y)P(m/y)$ where $P(\rho_1, \rho_2/m, y)$ is proportional to a double-t product distribution. Thus, $P(\rho_1, \rho_2/y)$ is a mixture of double-t product distributions, with $m + a_1$ degrees of freedom, location $\tilde{\rho}_1$, and precision $T(\tilde{\rho}_1)/(m + a_1)$ for the first factor and $n - m + a_2$ degrees of freedom, location $\tilde{\rho}_2$ and precision $T(\tilde{\rho}_2)/(n - m + a_2)$ for the second factor.

5. A NUMERICAL STUDY

For illustrative purposes, in this section our goal is to present a simulated example. In order to overcome the numerical difficulties of approximating the marginal posterior distributions of the model parameters, we apply the Gibbs sampler algorithm (See for more details Carlin et al., 1992; Gelfand et al., 1990) to our model. The model with generated data is given by

$$y_t - \mu = 0.3(y_{t-1} - \mu) + e_t; \quad t = 1, \ldots, 105$$
$$y_t - \mu = 0.5(y_{t-1} - \mu) + e_t; \quad t = 106, \ldots, 200$$

with $e_t \sim N(0, 1)$ for $t = 1, \ldots 105$; $e_t \sim N(0,3)$ for $i = 106, \ldots 200$; $y_0 = 0.1$. Without loss of generality, we assume $\mu = 0$.

We chose $m^* = 105$ as the true value of the change point m and $\rho_1{}^* = 0.3$ and $\rho_2{}^* = 0.5$ as the true values of the autocorrelation coefficients ρ_1 and ρ_2. The following plot (Fig. 1) represents the simulated 200 observations according to the above autoregressive process of order one. Notice that even though the series has a structural change, it is not completely clear where the change occurred.

Now, using the Gibbs sampler with 2,200 replicates, the posterior distributions of the change point, the precision and the autocorrelation coefficient before and after the change have been simulated. Table 1 displays the estimates of the model parameters from the Gibbs sampling algorithm. It provides us with the posterior mean and median estimates and their respective standard deviations as well as the 2.5 and the 97.5 percentiles.

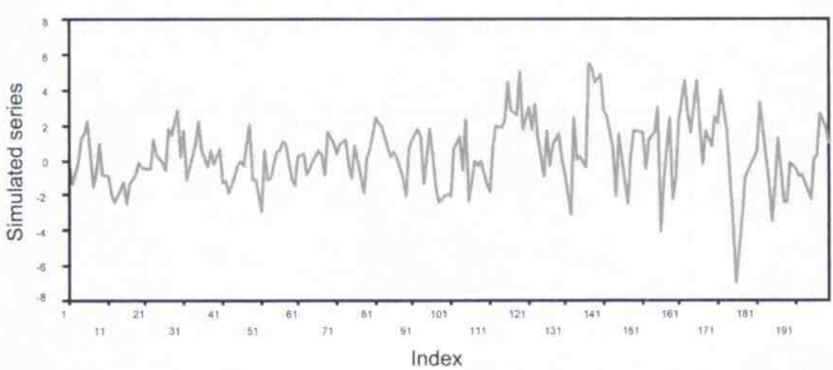

Figure 1. A simulated series. (*View this art in color at www.dekker.com.*)

Table 1. Posterior estimates.

Parameters	Mean	SD	2.5%	Median	97.5%	Sample
k	104.1	1.352	101.0	104.0	105.0	2300
eta[1]	0.8605	0.121	0.6494	0.8531	1.112	2200
eta[2]	0.1124	0.0162	0.0828	0.1116	0.1457	2200
rho[1]	0.4138	0.0919	0.2358	0.4125	0.5964	2100
rho[2]	0.5609	0.0858	0.3852	0.5612	0.7311	2100

Notation: eta[1] = η_1; eta[2] = η_2; rho[1] = ρ_1; rho[2] = ρ_2; $k = m$ is the change-point.

First, we can readily see from Fig. 2 that the posterior mode is equal to 105 which is the true value of the change point m. The posterior mean and median is both equal to 104. Here, we find three largest spikes of m equal to 103, 104 and 105 with the 105th point taking a dominant 43%. It seems that the marginal posterior distribution of m gives a clear indication about the change point when the change in the precisions is large.

We obtained also the unconditional posterior density functions of the precisions η_1, η_2 and the autocorrelation coefficients ρ_1, ρ_2. The resulting curves are displayed in Figs. 3 and 4, respectively.

From Fig. 3, one can notice that the posterior distributions of the precision η_1 and η_2 are visibly well separate here indicating that it is extremely unlikely that the values of η_1 and η_2 can be equal. These distributions do not overlap which shows that there is evidence of a change in the precisions. Table 1 summarizes the posterior estimates for the simulated series. The posterior mean (0.1124) and median (0.1116) of η_2 estimate fairly well the true value of η_2 which equals 0.1111 while the posterior mean (0.8605) and median (0.8531) of η_1 exhibit some bias since the true value equals 1. We have a similar scenario for

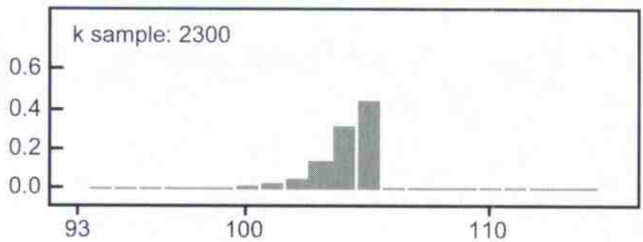

Figure 2. Posterior distribution of the change-point. (*View this art in color at www.dekker.com.*)

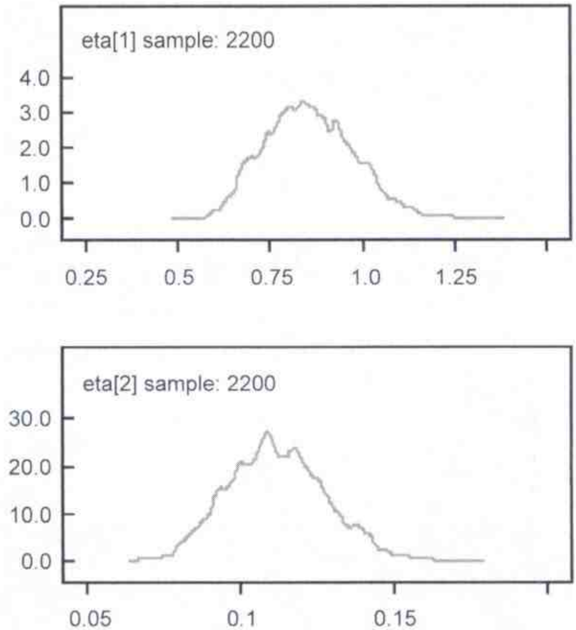

Figure 3. Posterior density functions of the precisions η_1, η_2. (*View this art in color at www.dekker.com.*)

ρ_1 and ρ_2 (see Table 1). All the 95% highest posterior density (h.p.d) intervals of the parameters contain the true value.

Using the same generated series, various simulations were run for various values of the precisions and autocorrelation coefficients. We assumed a change in the middle of the series, namely at the true change point $m^* = 105$, with low, moderate and sharp shift in the autocorrelation coefficients ρ_1 and ρ_2 and similarly for the precisions parameters η_1 and η_2. Using a single iteration of the Gibbs sampler, the posterior estimates for the appropriate series are summarized in Tables 2–5. As the change in variance increases, the probability of detecting the true value of the change point increases for fixed values of ρ_1 and ρ_2 for this particular series. On the other hand, for a large shift in the autocorrelation coefficients with a fixed variability, the posterior estimates of the change-point are not at its true value. The mass functions of m gives wider ranges but cover the true value of the change-point m. This is an indication that the detection of the change point, based on the posterior mode or median, is sensitive to changes in the autocorrelation coefficients as well as in the variability of the variance ratio. Estimation results when $m^* = 105$ are given in Tables 2–5.

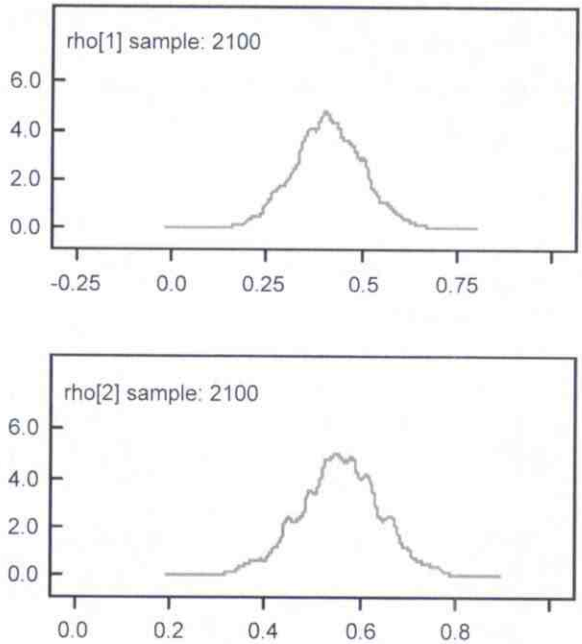

Figure 4. Posterior density functions of the autocorrelation coefficients ρ_1, ρ_2. (*View this art in color at www.dekker.com.*)

Figure 5 shows the posterior mass function of the change point m when the series is subject to a sharp shift in the autocorrelation coefficients with $\rho_1 = 0.3$ and $\rho_2 = 0.7$. As expected, the mass function of m exhibits a wider range which however, covers the true value of m. Neither the mode a posteriori nor the median estimates accurately the true value of the change point m as was the case of a moderate shift in the autocorrelation coefficients with $\rho_1 = 0.3$ and $\rho_2 = 0.5$.

Table 2. Bayes estimates when $m^* = 105$; $\sigma_1 = 1$; $\sigma_2 = 3$; $\rho_1 = 0.3$; $\rho_2 = 0.5$.

Parameter	True values	Mean (SD)	Median	Sample
m	105	104.1(1.35)	104	2300
η_1	1	0.861(0.121)	0.853	2200
η_2	0.111	0.112(0.016)	0.112	2200
ρ_1	0.3	0.414(0.092)	0.413	2100
ρ_2	0.5	0.561(0.858)	0.561	2100

Table 3. Bayes estimates when $m^* = 105$; $\sigma_1 = 1$; $\sigma_2 = 2$; $\rho_1 = 0.3$; $\rho_2 = 0.5$.

Parameter	True values	Mean (SD)	Median	Sample
m	105	107.7(5.31)	107	3000
η_1	1	0.825(0.118)	0.819	2000
η_2	0.25	0.251(0.038)	0.249	2000
ρ_1	0.3	0.396(0.099)	0.396	1000
ρ_2	0.5	0.564(0.874)	0.563	1000

Table 4. Bayes estimates when $m^* = 105$; $\sigma_1 = 1$; $\sigma_2 = 2$; $\rho_1 = 0.3$; $\rho_2 = 0.7$.

Parameter	True values	Mean (SD)	Median	Sample
m	105	108.6(4.34)	109	3300
η_1	1	0.829(0.117)	0.823	3200
η_2	0.25	0.250(0.038)	0.248	3200
ρ_1	0.3	0.409(0.094)	0.409	3100
ρ_2	0.7	0.759(0.069)	0.759	3100

Table 5. Bayes estimates when $m^* = 105$; $\sigma_1 = 1$; $\sigma_2 = 1.5$; $\rho_1 = 0.3$; $\rho_2 = 0.7$.

Parameter	True values	Mean (SD)	Median	Sample
m	105	110(14.28)	111	1415
η_1	1	0.842(0.120)	0.841	1215
η_2	0.444	0.457(0.159)	0.444	1215
ρ_1	0.3	0.425(0.0105)	0.419	1195
ρ_2	0.7	0.752(0.089)	0.754	1195

6. CONCLUDING REMARKS

In this paper, we have derived the posterior distributions of the change point, the variance ratio, and the magnitude of the shift in the autocorrelation parameters of a first-order autoregressive process subject to one change in both the error variance and the autocorrelation coefficient at an unknown time point. We also performed some numerical studies via the Gibbs sampler. We showed how inferences can be made readily by using the Gibbs sampler. The change-point was easily detected when the difference between the variances was moderate to large. As the

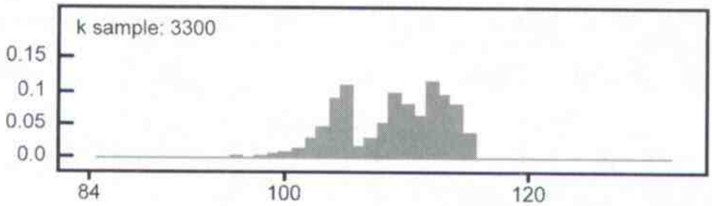

Figure 5. Posterior distribution of the change-point *m*. (*View this art in color at www.dekker.com.*)

magnitude of the difference between the variances decreased for fixed autocorrelations, the mass function of the change-point became wider around the true value thus making detection of the change-point increasingly difficult. The mass function of the change-point also exhibits a wider range with an increased difference in the autocorrelation parameters for fixed variances. This demonstrates that the detection of the structural change-point is sensitive to changes in both the variance and the autocorrelation of the time series. We would like to add that change point problems are encountered in a wide variety of disciplines in the sense that a physical entity might experience structural change as it evolves in time. In economics, an economic policy that was once ineffective may become effective due to a new government program (such as price support) or a major disturbance to the economy (such as an oil embargo or a terrorist action) or a new fiscal policy (such as changes in taxes and government expenditures). In finance, Hsu (1982) gives an example on the stock-market returns based on the Dow–Jones industrial index. In biology, the emphasis may be in detecting the time point in life of an organism at which a change in growth pattern occurs (e.g., see Kivchi et al., 1995). Friede et al. (2001) studied change point estimators applied in dose–response trials for the detection of the lowest plateau dose. Many other researchers have investigated this problem in many other fields.

APPENDIX: BUGS CODE

```
Model Change-Point;

Const

    # Number of Observations
    N
```

```
    # known parameters
    eta.alpha
    eta.beta
    rho.mu
    rho.precision
    mean.mu
    mean.precision
var

    y[N], y0, mu[N], eta[2], sigma[2], rho[2], m, mean, J[N], punif[N];

data in "filename.dat"; #data file
inits in "filename.in"; # initial values file
{ for (i in 2:N) {
    mu[i] < -rho[J[i]]*y[i-1] + mean*(1-rho[J[i]]);
    y[i] ~ dnorm(mu[i], eta[J[i]])
    J[i] < -1 + step(i - (m + 0.5))
                }
    for ( j in 1:2) { eta[j] ~ dgamma (eta.alpha, eta.beta);
    # sigma[ j] < -1.0/sqrt(eta[j])
                }
    mu[1] < -rho[1]*y0 + mean*(1 - rho[1])
    # Priors specification
    m ~ dcat(punif []); # uniform prior over change-point observation
    rho[1] ~ dnorm (rho.mu, rho.precision);
    rho[2] ~ dnorm (rho.mu, rho.precision);
    mean ~ dnorm (mean.mu, mean.precision);
}
```

REFERENCES

Abraham, B., Wei, W. W. S. (1984). Inference about the parameters of a time series model with changing variance. *Metrika* 31:183–194.

Broemeling, L. D., Tsurumi, H. (1987). *Econometrics and Structural Change*. New-York: Marcel Dekker Inc.

Carlin, B. P., Gelfand, A. E., Smith, A. F. M. (1992). Hierarchical Bayesian analysis of changepoint problems. *Appl. Statist.* 41(2): 389–405.

Chernoff, H., Zacks, S. (1964). Estimating the current mean of a normal distribution which is subjected to changes in time. *Ann. Math. Statist.* 35:999–1018.

Chin Choy, J. Y., Broemeling, L. D. (1980). Some Bayesian inferences for a changing linear model. *Technometrics* 22:71–78.

Friede, T., Miller, F., Bischop, W., Kieser, M. (2001). A note on change point estimation in dose-response trials. *Comput. Statist. Data Anal.* 37:219–232.

Gelfand, A. E., Hills, S. E., Racine-Poon, A., Smith, A. F. M. (1990). Illustration of Bayesian inference in normal data models using Gibbs sampling. *JASA* 85:972–985.

Hsu, D. A. (1977). Tests for variance shift at an unknown time point. *JRSS Ser C* 26:279–284.

Hsu, D. A. (1982). A Bayesian robust detection of shift in the risk structure of stock returns. *JASA* 77:29–39.

Holbert, D., Broemeling, L. D. (1977). Bayesian inference related to shifting sequences and two-phase regression. *Comm. Statist* A6(3):265–275.

Inclan, C., Tiao, G. (1994). Use of cumulative sums of squares for retrospective detection of changes of variance. *JASA* 89(427):913–923.

Kander, Z., Zacks, S. (1966). Test procedures for possible changes in parameters of statistical distributions occurring at unknown time points. *Ann. Math. Statist.* 37:1196–1210.

Kivchi, A. S. et al. (1995). Change points in the series of T4 counts prior to AIDS. *Biometrics* 51:236–248.

Menzefricke, U. (1981). A Bayesian analysis of a change in the precision of in a sequence of independent normal random variables at an unknown time point. *Appl. Statist.* 30:141–146.

Ng, V. M. (1990). A Bayesian analysis of linear models exhibiting changes in mean and precision at an unknown time point. *Comm. Statist. Theory Meth.* 19(1):111–120.

Page, E. S. (1954). Continuous inspection schemes. *Biometrika* 41: 100–114.

Salazar, D. (1982). Structural changes in time series models. *J. Econometrics* 10:141–164.

Salazar, D., Broemeling, L. D., Chin, A. (1981). Parameter changes in a regression model with autocorrelated errors. *Comm. Statist. Theory Meth.* A10(17):1751–1758.

Smith, A. F. M. (1975). A Bayesian approach to inference about a change-point on a sequence of random variable. *Biometrika* 62:407–416.

Tsay, R. (1988). Outliers, level shifts, and variance changes in time series. *J. Forecasting* 7:1–20.

Wang, J., Zivot, E. (2000). A Bayesian time series model of multiple structural change in level, trend, and variance. *J. Business Economic Statist.* 18(3):374–386.