International Journal of Systems Science, 2001, volume 32, number 9, pages 1185–1192

Detection of change points in time series analysis with fuzzy statistics

KULDEEP KUMAR† and BERLIN WU‡

Changes in time series often occur gradually so that there is a certain amount of fuzziness in the change point. In this paper we have presented an integrated identification procedure for change-point detection based on fuzzy logic. The membership function of each datum corresponding to the cluster centres is calculated and is used for performance index grouping. We have also suggested a test for the change in level and the change in slope for testing a hypothesis about change points. We have made simulation studies to demonstrate the whole procedure. Finally an empirical study about change-point identification in the exchange rate data of six Asian nations has been demonstrated using the algorithm of the paper

1. Introduction

The weakness of many models for time series analysis clearly resides in the impossibility of having no significant structure changes during the whole dynamic process. Two fundamental questions that often arisen are as follows.

(1) Does there exist a *good* model that leads this dynamic process?

(2) Does a single model fit the dynamic process?

(Need we use more than one model to fit the time series?)

As a result, model identification of a time series from a model-based system becomes an important procedure before *a priori* model selection (Wu 1995). If an *a priori* model family is correctly chosen, procedures of model construction such as parameter estimation, diagnosis and forecasting will make sense. On the other hand, if the underlying time series demonstrates certain structure changes, it is natural to detect those change points or change periods before modelling the whole process. Otherwise, using the traditional techniques for model construction, we may hardly obtain an appropriate *good* model to explain the nonlinear time series.

The problem of change point detection in time series has been examined by many researchers. For instance, Tsay (1990) proposed some procedures for detecting outliers, level shifts and variance changes in univariate time series. The procedures that he suggested are particularly useful and relatively easy to implement, while Balke (1993) pointed out that Tsay's procedures do not always perform satisfactorily when level shifts are present. Inclan and Tiao (1994) proposed an iterative procedure to detect variance changes based on a centred version of the cumulative sums of squares presented by Brown *et al.* (1975).

Methods for change points detection include the modified Page (1955) (MPAGE) method and the cumulative sum (CUSUM) method proposed by Hinkley (1971). Hsu (1979, 1982) investigated the detection of a variance shift at an unknown point in a sequence of independent observations, focusing on the detection of points of change one at a time because of the heavy computational burden. Worsley (1986) used maximum liklihood (ML) methods to test a change in mean for a sequence of independent exponential family random variables. Sastri *et al.* (1989) presented a study of performance comparison for six time-series change detection procedures. Recently, Rukhin (1997) studied the classical change-point estimation problem in the

Received 2 February 2000. Accepted 1 December 2000.

† School of Information Technology, Bond University, Gold Coast, Queensland 4229, Australia. Tel: +61-7-55953305; Fax: +61-7-55953320; e-mail: kkumar@bond.edu.au

‡ Department of Mathematical Sciences, National Chengchi University, Taiwan.

International Journal of Systems Science ISSN 0020–7721 print/ISSN 1464–5319 online © 2001 Taylor & Francis Ltd
http://www.tandf.co.uk/journals
DOI: 10.1080/00207720110034698

Bayesian setting, that is point estimation of the change-point parameter is considered after the data have been observed and the change point is known to occur.

However, those detecting techniques are based on the assumption that the underlying time series conducts a significant change point characteristic (Wu and Chen 1999). Using the concept of fuzzy set theory, Wu and Chen (1999) proposed a procedure about change period detection for nonlinear time series. Nevertheless, we must indicate that, in dealing with time series with switching regimes, we should consider not only the change-point detection but also the properties of change periods. Because many patterns of change structure in time series occur over a time interval, those phenomena should not be treated as a mere sudden turning at a certain time. For instance

(i) the exchange rate may go up or down gradually after a new financial policy performance or
(ii) The supply of M_1 or M_2 may change their trend at different period of time according to the national economic conditions.

In fact, the semantics of the term 'change point' is not very clear and well understood (interested readers may refer to any popular dictionary such as *Webster's New Dictionary*).

Fuzzy set theory has received much attention recently. The proposed method for change-point detection uses the concept of fuzzy entropy. The term 'entropy' comes from thermodynamics. Entropy can be considered as a measure of how close a system is to equilibrium. It can also be considered as a measure of disorder in the system. For instance, the classical clustering method separates the data into k categories, while in many cases there are elements, which cannot be contained, in a specific category. They belong to two or more categories simultaneously. A more detailed concept of fuzzy entropy has been given in the seminal paper by Wu and Chen (1999).

In this research we present an integrated identification procedure for change-point and change-periods detection. The membership function of each datum corresponding to the cluster centres is calculated and as performance index grouping. Two tests on the change in level and change in slope with a simple regression model are suggested for testing hypotheses about the change points. The simulation and empirical results showed that our detection procedure is an efficient and realistic procedure in detecting the structure change of a time series. In particular, when the change is gradual, our model construction procedure demonstrated a better performance in prediction.

2. Detecting process for change points and periods

2.1. *Estimation*

In order to classify the vague points, we use the following definitions.

Definition 1—α-level of fuzzy change points: Let $\{y_t,\ t = 1, 2, \ldots, N\}$ be a time series. Suppose that we decompose $\{y_t\}$ into k clusters according to a set of fuzzy cluster centres, where μ_{it} is the degree of membership that y_t holds for cluster centre C_i. For each y_t, $t = 1, 2, \ldots, N$, if

$$\max\{\mu_{it}\} - \min\{\mu_{it}\} < 1 - k\alpha, \quad \alpha \in \left(0, \frac{1}{k}\right) \tag{1}$$

Then we call y_t an α level of the fuzzy change point. Moreover, if $\max\{\mu_{it}\} - \min\{\mu_{it}\} = 1$, we call y_t an absolutely fuzzy point. If $\max\{\mu_{it}\} - \min\{\mu_{it}\} = 0$, we call y_t a crisp point. □

Definition 2—Fuzzy entropy: Let μ_{it} be the membership of y_t to the cluster $C_i, i = 1, 2, \ldots, k$. The fuzzy entropy of y_t is defined as

$$\delta(y_t) = -\frac{1}{k}\sum_{i=1}^{k} \mu_{it} \ln(1-\mu_{it}). \tag{2}$$

□

The proposed fuzzy detecting procedures of fuzzy change periods with α level include the following:

(i) fuzzy clustering;
(ii) deciding the α level of a fuzzy point;
(iii) detecting the α level of fuzzy change periods.

A detailed algorithm is illustrated below.

Algorithm for change period detection:

Step 1. Input the time series $\{y_t\}$. Find $C_i (i = 1, 2, 3)$ the set of cluster centres for $\{y_t\}$. Classify $\{y_t\}$ into three categories.

Step 2. Let μ_{it} be the degree of membership of each observation y_t to each cluster C_i. Compute the membership of μ_{it} by

$$\mu_{it} = 1 - \frac{(y_t - C_{it})^2}{\sum_{i=1}^{3}(y_t - C_{it})^2}, \quad i = 1, \ldots, 3, \quad t = 1, \ldots, N. \tag{3}$$

Step 3. Calculate the fuzzy entropy of y_t by using its memberships:

$$\delta(y_t) = -\tfrac{1}{3}\sum_{i=1}^{3} \mu_{it}(1 - \mu_{it}). \tag{4}$$

Step 4. Compute the average of the cumulative fuzzy entropy for each t by

$$MS(\delta(y_t)) = \frac{1}{t}\sum_{k=1}^{t} \delta(y_t), \quad t = 1, 2, \ldots, N \tag{5}$$

and find $\mathrm{Med}[MS(\delta(y_t))]$, the median of $MS(\delta(y_t))$.

Step 5. Choose a proper λ threshold level and classify the fuzzy time series as

$$C_t^* = \begin{cases} 0, & \text{if Md}\,[MS(\delta(y_t))] \in [0, \mathrm{Med}\,[MS\,(\delta(y_t)) - \lambda], \\ 1, & \text{if Md}\,[MS(\delta(y_t))] \in [\mathrm{Med}\,[MS(\delta(y_t))] - \lambda, \\ & \qquad \mathrm{Med}\,[MS(\delta(y_t))] + \lambda], \\ 2, & \text{if Md}\,[MS(\delta(y_t))] \in [\mathrm{Med}[MS(\delta(y_t))] + \lambda, 1]. \end{cases} \tag{6}$$

Step 6. Decide the α significant level of change periods, that is, for each categories 0, 1 and 2, if the elements of category 1 contain sample points (successive data) greater than $[\alpha N]$, then we reject the hypothesis that there is no structural change.

2.2. *Testing*

The Chow (1960) test for the constancy of the regression coefficient vector over the sample is one of the most widely used diagnostic tests in applied statistics. It is known that the statistics associated with various forms of the Chow test are F distributed under the null hypothesis of parameter stability.

Consider a sample (X, Y) with $N = N_1 + N_2$ observations and suppose that we have two regression models

$$Y_t = \begin{cases} \beta_{10} + \beta_{11}X_t + \varepsilon_t, & 1 \leqslant t \leqslant N_1, \\ \beta_{20} + \beta_{21}X_t + \varepsilon_t, & N_1 < t \leqslant N_2. \end{cases} \tag{7}$$

The most common form of the Chow test for parameter stability assuming that $\beta_{10} = \beta_{20}$, tests the hypothesis $\mathrm{H}_0 : \beta_{11} = \beta_{21}$ versus the hypothesis $\mathrm{H}_1 : \beta_{11} \neq \beta_{21}$. In this research we shall generalize the test procedure to a more complete form. We consider the following hypothesis:

(i) The change in level can be tested by testing the hypothesis: $\mathrm{H}_0 : \beta_{10} = \beta_{20}$ versus the hypothesis $\mathrm{H}_1 : \beta_{10} \neq \beta_{20}$.

(ii) The change in slope can be tested by testing the hypothesis $\mathrm{H}_0 : \beta_{11} = \beta_{21}$ versus the hypothesis $\mathrm{H}_1 : \beta_{11} \neq \beta_{21}$.

There are two general approaches for comparing the slope and intercept of two regression lines. In the first approach we fit the data from two different regimes separately and then make an appropriate two-sample t-test. In the second approach we define a dummy variable Z to be 0 if the data come from regime 1 and to be 1 if the data come from regime 2. Then, for the combined data, the single multiple regression model is defined as

$$Y_t = \beta_0 + \beta_1 X_t + \beta_2 Z + \beta_3 X_t Z + \varepsilon_t, \tag{8}$$

which yields the following two models for the two values of Z. That is, if $Z = 0$, then $Y_t = \beta_0 + \beta_1 X_t + \varepsilon_t$; if $Z = 1$, then $Y_t = (\beta_0 + \beta_2) + (\beta_1 + \beta_3)X_t + \varepsilon_t$. This allows us to write the regression coefficients for the separate models in terms of the coefficients of the model (7). It follows that $\beta_{10} = \beta_0$, $\beta_{20} = \beta_0 + \beta_2$, $\beta_{11} = \beta_1$ and $\beta_{21} = \beta_1 + \beta_3$.

Testing that the slope of two regression lines is same, can be conducted by testing the hypothesis $H_0 : \beta_3 = 0$. If $\beta_3 = 0$, then $\beta_{12} = \beta_1$, which is also the slope of the first line. The statistics for testing $\mathrm{H}_0 : \beta_3 = 0$ is the partial F statistic for the significance of the addition of the variable XZ to a model already containing X and Z. The test statistics are computed as follows:

$$F(XZ|X, Z) = \frac{\text{regression } SS(X, Z, XZ) - \text{regression } SS(X)}{MS \text{ residual } (X, Z, XZ)}. \tag{9}$$

The test statistic to test that the two intercepts are equal is equivalent to testing the hypothesis $\mathrm{H}_0 : \beta_2 = 0$ for the overall model. The test is computed as

$$F(Z|X) = \frac{\text{regression } SS(Z, X) - \text{regression } SS(X)}{MS \text{ residual } (X, Z, XZ)}. \tag{10}$$

The details of these two methods can be found in the book by Kleinbaum *et al*. (1998, chapter 14). They have also shown that these two methods give exactly the same result. For simplicity we have used second approach in this paper.

Testing rules for a change point in the switching time series model:

Step 1. Choose the estimated change points $i = 1, \ldots, k$, according to the procedure in § 2.1

Step 2. For each period $i, i = 1, \ldots, k$, fit a linear regression model with time.

Step 3. Test the hypothesis that there is a difference of slope of the two consecutive lines. If the difference is significant, then it suggests a change point. (The difference in the slopes could be in the sign from positive to negative.) Go to step 5.

Step 4. Test the hypothesis that there is a significant difference in the levels of the two consecutive lines at $t = t_i$. If the difference is significant, then it suggests a change point. (If there is a significant difference in the intercept of the two regression lines then it will also suggest a change point, but this change point is due to the change in level rather than to the change in slope.) A significant difference in slope could well result in a significant difference between the intercepts.

Step 5. The time series has change points at $t = t_i$.

3. Simulation studies

Four time series data of size 450 are generated from four models. The first is an AR(1) model with a switch in level. The second is also an AR(1) model with a switch in the slope. The third model is a switching time series regression model with a switch in slope and the fourth is also a switching time series regression model with a switch in level and also in slope. These models are shown in figures 1, 2, 3 and 4 respectively.

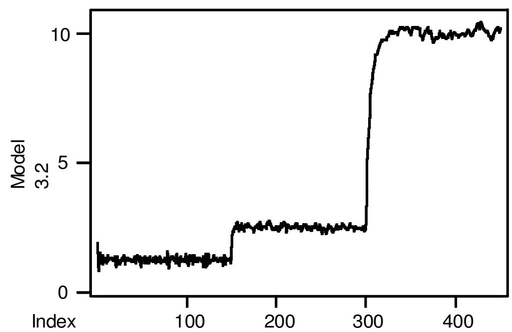

Figure 1. Simulation data for model (11).

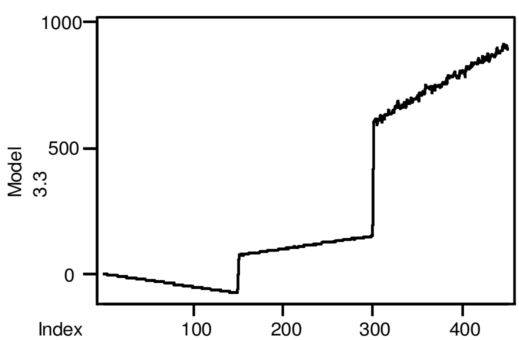

Figure 2. Simulation data for model (12).

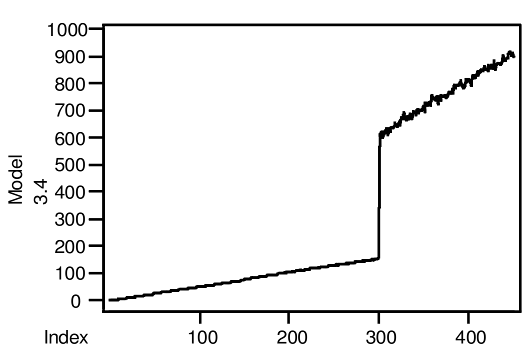

Figure 3. Simulation data for model (13).

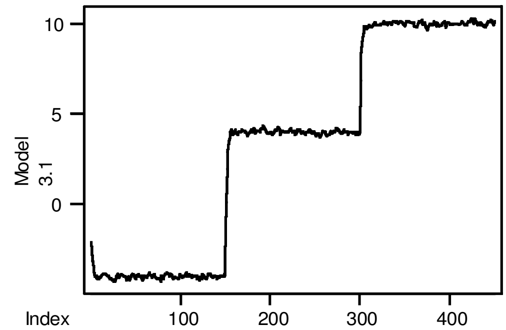

Figure 4. Simulation data for model (14).

$$X_t = \begin{cases} -2 + 0.5X_{t-1} + \varepsilon_t, & \varepsilon_t \sim N(0, 0.1) \quad \text{if } 1 \leqslant t \leqslant 150, \\ 2 + 0.5X_{t-1} + \varepsilon_t, & \varepsilon_t \sim N(0, 0.1) \quad \text{if } 151 \leqslant t \leqslant 300, \\ 5 + 0.5X_{t-1} + \varepsilon_t, & \varepsilon_t \sim N(0, 0.1) \quad \text{if } 301 \leqslant t \leqslant 450, \end{cases} \tag{11}$$

$$X_t = \begin{cases} 2 + 0.6X_{t-1} + \varepsilon_t, & \varepsilon_t \sim N(0, 0.1) \quad \text{if } 1 \leqslant t \leqslant 150, \\ 2 + 0.2X_{t-1} + \varepsilon_t, & \varepsilon_t \sim N(0, 0.1) \quad \text{if } 151 \leqslant t \leqslant 300, \\ 2 + 0.8X_{t-1} + \varepsilon_t, & \varepsilon_t \sim N(0, 0.1) \quad \text{if } 301 \leqslant t \leqslant 450, \end{cases} \tag{12}$$

$$X_t = \begin{cases} 2 + 0.5t + \varepsilon_t, & \varepsilon_t \sim N(0, 0.1) \quad \text{if } 1 \leqslant t \leqslant 150, \\ 2 + 0.5t + \varepsilon_t, & \varepsilon_t \sim N(0, 1) \quad \text{if } 151 \leqslant t \leqslant 300, \\ 2 + 2t + \varepsilon_t, & \varepsilon_t \sim N(0, 10) \quad \text{if } 301 \leqslant t \leqslant 450, \end{cases} \tag{13}$$

$$X_t = \begin{cases} 1 + 0.5t + \varepsilon_t, & \varepsilon_t \sim N(0, 0.1) \quad \text{if } 1 \leqslant t \leqslant 150, \\ 5 + 0.5t + \varepsilon_t, & \varepsilon_t \sim N(0, 0.1) \quad \text{if } 151 \leqslant t \leqslant 300, \\ 10 + 2t + \varepsilon_t, & \varepsilon_t \sim N(0, 10) \quad \text{if } 301 \leqslant t \leqslant 450. \end{cases} \tag{14}$$

Table 1 shows the fitted regression lines for models (11)–(14) and table 2 illustrates the result of change point estimation as well as testing.

It can be observed that the p value corresponding to the F test is very low, so, under any reasonable α level of significance, say, $0.005 \leqslant \alpha \leqslant 0.1$, we reject the null hypothesis, that is there exist change points. From table 2 we can see that our identification procedure demonstrated a very efficient result in detecting the change points for model (11)–(14).

Table 1. Fitted regressioin lines for models (11)–(14)

Model	Regression 1[a]	Regression 2[a]	Regression 3[a]
(11)	$X_t = -3.93 - 0.0007t$	$X_t = 3.58 + 0.0017t$	$X_t = 9.18 + 0.0021t$
	[0.034] [0.000 39]	[0.17] [0.000 72]	[0.22] [0.000 57]
(12)	$X_t = 1.25 - 0.00003t$	$X_t = 2.54 - 0.0002t$	$X_t = 6.54 + 0.0087t$
	[0.023] [0.000 26]	[0.042] [0.000 18]	[0.054] [0.0014]
(13)	$X_t = 1.97 - 0.500t$	$X_t = 1.83 + 0.501t$	$X_t = -0.52 + 2.01t$
	[0.017] [0.000 19]	[0.44] [0.0019]	[6.8] [0.018]
(14)	$X_t = 0.975 + 0.500t$	$X_t = 4.83 + 0.501t$	$X_t = 7.84 + 2.01t$
	[0.008] [0.000 19]	[0.97] [0.0019]	[6.8] [0.018]

[a] The values inside square brackets are the standard errors of the coefficients for the regression line.

Table 2. Change-point detection for models (11)–(14)

	First change point			Second change point		
Model	Real	Estimate	F test (p value)	Real	Estimate	F test (p value)
(11)	151	150	0.0037	301	295	0.0002
(12)	151	155	0.0029	301	305	0.0001
(13)	151	145	0+[a]	301	300	0+[a]
(14)	151	146	0+[a]	301	300	0+[a]

[a] 0+ stands for values that are very small and close to 0.

4. Empirical examples

The data analysed here come from the foreign exchange rate database from the Central Bank of Taiwan for the period from 2 July 1997 to 31 October 1998. The local currencies that we selected are new Taiwan (NT) dollar (Taiwan), yen (Japan), won (South Korea), indo (Indonesia), baht (Thailand) and ringgit (Malaysia). Each local currency is given a total number of observations of 454. The purpose of this study is to examine the change point of the eastern Asia financial crisis. Figure 5 provides flow charts of the empirical analysis process.

On 2 July, 1997, the Thailand Government announced adoption of the floating exchange rate system against the US dollar, the local currencies of Thailand and other Asian countries, except for Hong Kong and PR China, continually depreciate against US dollar. From figure 5(*e*), we can see the level of depreciation of the Thai baht is about 20% in the first 2 months, and the ringgit follows the baht at about 10%. The yen, won and NT dollar depreciated equally at under 10%. After November 1997, the Thai baht depreciated sharply by above 50%, and the won devalued abruptly by above 40%. In the beginning, the won was influenced and resembled the NT dollars. After 4 months, the won followed the same group with the baht and ringgit having similar severe devaluations.

We can see that the Asian financial crisis began with the devaluation of the Thai baht in July, 1997 and the worst depreciation was in January 1998 and continually fell after January. Then the crisis spilled over to neighbouring countries such as Malaysia and Indonesia. The main reasons are the similarity of the financial environments of these three countries. It is significant that South Korea, Taiwan, Singapore and Hong Kong had similar economic advantages before 1996. At the beginning of the economic crisis, the exchange rates of the yen, NT dollar and won depreciated slightly, but the won devaluation turned out to be deeper after November 1997. Collectively, the crisis-hit nations had the worst effects in January 1998 and then the impact of the devaluation was mitigated.

In these figures, the yen–US dollar exchange rate fluctuated sharply in the period of estimation and forecasting; the NT dollar against the US dollar devalued sharply and fluctuated in the middle of the estimation period; the baht–US dollar changes rose to a peak in January 1998 during this crisis. The NT dollar and Thai baht both changed steadily against the US dollar at the

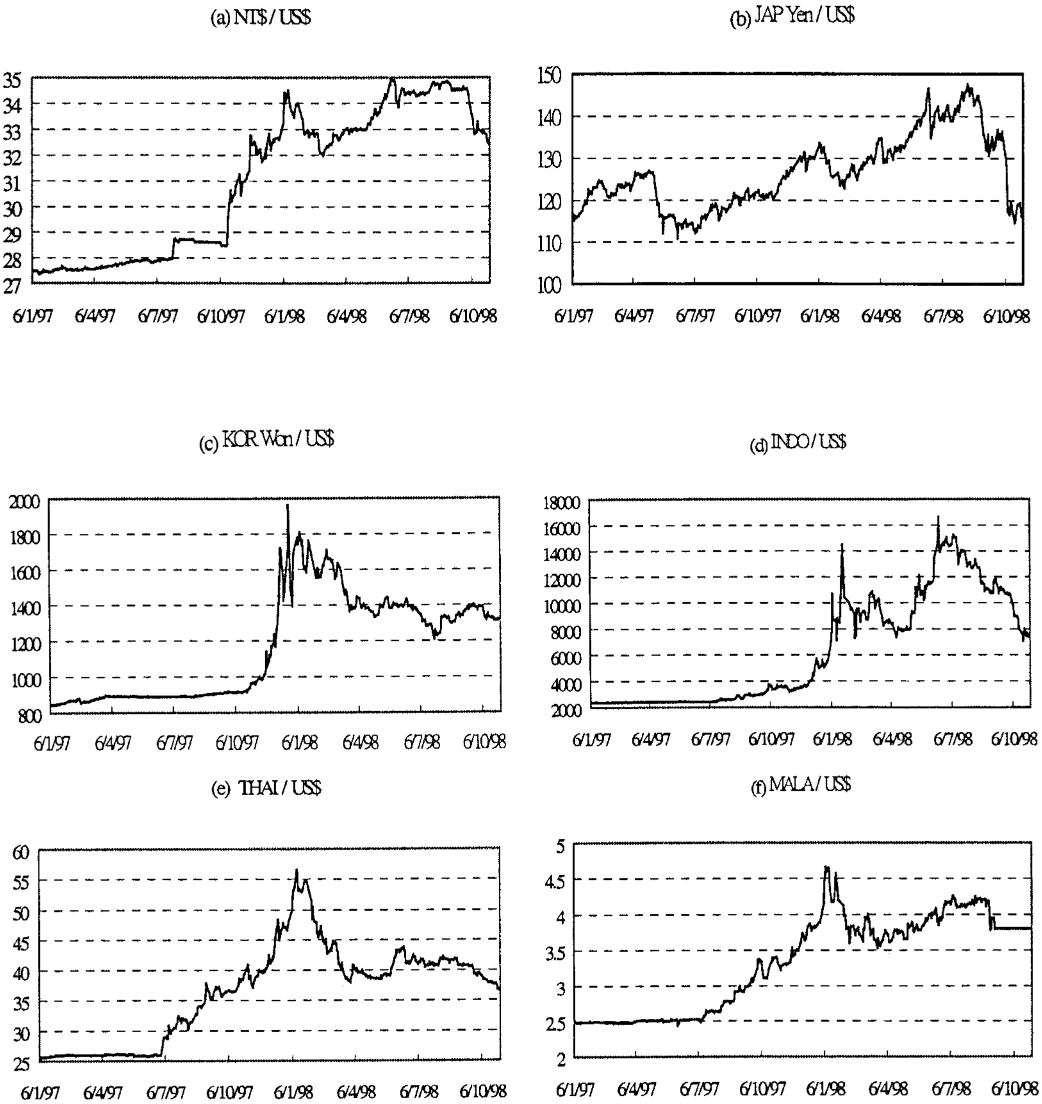

Figure 5. The exchange rates of six Asian countries against the US dollar.

end periods. Almost without exception, the depreciation of the exchange rate of crisis-hit nations was significantly worse than before.

Table 3 demonstrates the change points and change periods for the exchange rate of six Asian countries.

In table 3, the detected times of change points and change periods are close to the usual observation in the first and second or third periods of the regime. Furthermore, the trends of the two regimes are stable compared with the change in the true observations which fluctuate sharply. The leading trend of these time series are consistent with the trend of the estimated observations and it is not like the previous trend which approaches some value. However, the estimates of the six time series change little after the last period.

5. Conclusion

In the economic and financial analysis discussed above the time series data have the switching regime property. If we use the univariate autoregressive moving average family, or any single model, to analyse these data, it will not solve the orientation problem. The contribution of this paper is that it provides a new method to detect change points. In this paper, we introduced three detection procedures that can effectively detect multiple change periods for a nonlinear time series. The proposed algorithm also combines with the concept of a fuzzy set. We have demonstrated how to find a α level change period to help to model a time series model with multiple change periods.

Table 3. Change points and change periods for the exchange rates of six Asian countries

Country	Currency	Cluster centre	Change point ($\alpha = 0.05$)	Change period[a]
Taiwan	NTS	High, 34.9	29.5 (17 October 1997)	28.52 ↑ 30.45 (16 October 1997–20 October 1997)
		Low, 27.3		
Japan	Yes	High, 147.4	(1) 131.7 (30 March 1998)	(1) 129.9 ↑ 133.3 (24 March 1998–1 April 1998)
		Low, 110.9	(2) 128.2 (7 October 1998)	(2) 130.1 ↓ 122.5 (6 October 1998–8 October 1998)
South Korea	Won	High, 1962	(1) 999 (11 October 1997)	(1) 957.6 ↑ 1187 (28 October 1997–1 December 1997)
		Low, 844.6	(2)1 448 (20 March 1998)	(2) 1460 ↓ 1366 (16 March 1998–23 March 1997)
Indonesia	Indo	High, 16 593	(1) 5 600 (24 December 1997)	(1) 5 000 ↑ 6750 (15 December 1997–31 December 1999)
		Low, 2361	(2) 9050 (6 May 1998)	(2) 8 150 ↑ 9450 (29 April 1998–12 May 1998)
			(3) 11 100 (25 August 1998)	(3) 11 700 ↓ 10 800 (18 August 1998–4 September 1998)
Thailand	Baht	High, 56.5	(1) 27.5 (2 July 1997)	(1) 25.9 ↑ 27.5 (30 June 1994–2 July 1997)
		Low, 25.6	(2) 48.6 (5 February 1998)	(2) 51.4 ↓ 47.9 (2 February 1998–9 February 1998)
			(3) 40.2 (12 August 1998)	(3) 41.2 ↓ 38.7 (13 March 1998–23 March 1998)
Malaysia	Ringgir	High, 4.66	(1) 2.56 (14 July 1997)	(1) 2.50 ↑ 2.65 (9 July 1997–18 July 1997)
		Low, 2.44	(2) 3.98 (3 February 1998)	(2) 4.20 ↓ 3.60 (21 January 1997–10 February 1998)
			(3) 3.89 (1 September 1998)	(3) 4.19 ↓ 3.79 (27 August 1998–2 September 1998)

[a] ↑, The time series is steadily increasing during the change period; ↓, the time series is steadily decreasing during the change period.

Simulation results show that our proposed techniques of change period detection are simple and efficient. Our algorithm is highly recommended practically for detecting the α level change period and is supported by the empirical results. A major advantage of this framework is that our detection procedures do not require any initial knowledge about the structure in the data and can take full advantage of the model-free approach.

Wu and Chen (2000) have compared this technique of change point detection with other change point detection techniques. It was observed that techniques based on fuzzy entropy work better than other techniques such as MPAGE and CUSUM mentioned in § 1.

However, certain challenging problems still remain open, such as:

(1) The semantics of the term 'stationarity' need to be redefined carefully. It seems that the term 'change points', which may stand for change in mean, change in variance, change in parameter or change in model needs to be clarified before performing a detection process.

(2) In the case of random-walk processes, an appropriate detection procedure needs to be developed for prior recognition and model identification.

(3) What knowledge base is required to obtain specific behaviour of time series under certain multivariate endogenous variables?

(4) The convergence of the algorithm for classification and the proposed statistics have not been well proved, although the algorithms and the proposed statistics are known as fuzzy criteria. This needs further investigation.

Although there remain many problems to be overcome, we think fuzzy statistical methods will be a worthwhile approach and will stimulate more future empirical work in time series analysis.

Acknowledgements

We are grateful to the editor and two referees for the comments on the earlier version of the paper.

References

Andel, J., 1993, A time series model with suddenly changing parameters. *Journal of Time Series Analysis*, **14**, 111–123.

Balke, N. S., 1993, Detecting level shifts in time series. *Journal of Business and Economic Statistics*, **11**, 81–92.

Barry, D., and Hartigan, J. A., 1993, A Bayesian analysis for change point problems. *Journal of the American Statistical Association*, **88**, 309–319.

Bleary, M., 1990, Some comparisons of the relative power of simple tests for structure change in regression models. *Journal of Forecasting*, **9**, 437–444.

Brown, R., Durbin, J., and Evans, J., 1975, Techniques for testing the constancy of regression relationships over time. *Journal of the Royal Statistical Society, Ser.* B, **37**, 149–163.

Chow, G. C., 1960, Testing for equality between sets of coefficients in two linear regressions. *Econometrica*, **28**, 291–605.

De Gooijer, J. G., and Kumar, K., 1992, Some recent developments in nonlinear time series modeling, testing, and forecasting. *International Journal of Forecasting*, **8**, 135–156.

Hinkley, D. V., 1971, Inference about the change point from cumulative sum test. *Biometrica*, **26**, 279–284.

Holland, J. H., 1975, *Adaptation in Natural and Artificial Systems* (Ann Arbor, Mighican: University of Michigan Press).

Hsu, D. A., 1979, Detecting shifts of parameter in gamma sequences, with applications to stock price and air traffic flow analysis. *Journal of the American Statistical Association*, **74**, 31–40; 1982, A Bayesian robust detection of shift in the risk structure of stock market returns. *Journal of the American Statistical Association*, **77**, 29–39.

INCLAN, C., and TIAO, G. C., 1994, Use of cumulative sums of squares for retrospective detection of changes of variance. *Journal of the American Statistical Association*, **89**, 913–924.

KAO, C., and ROSS, S. L., 1995, A CUSUM test in the linear regression model with serially correlated disturbances. *Econometric Reviews*, **14**, 331–346.

KLEINBAUM, D. G., KUPPER, L. L., MULLER, K. E., and NIZAM, A., 1998, *Applied Regression Analysis and Other Multivariate Methods* (California: Duxbury Press).

KOZA, J. R., 1994, Genetic Programming II: Automatic Discovery of Reusable Programs (Cambridge, Massachusetts: MIT Press).

LORASCHI, A., TETTAMANI, A., TOMASSINI, M., and VERDA, P., 1995, Distributed genetic algorithms with a application to portfolio selection problem. *Artifical Neural Networks and Genetic Algorithms*, edited by N. C. Pearson, N. C. Steele and R. F. Albrecht (Berlin: Springer-Verlag), pp. 384–387.

MITCHELL, M., 1996, *An Introduction to Genetic Algorithms* (Cambridge, Massachusetts: MIT Press).

NYBLOM, J., 1989, Testing for the constance of parameters over time. *Journal of the American Statistical Association*, **844**, 223–230.

PAGE, E. S., 1955, A test for change in a parameter occurring at an unknown point. *Biometrika*, **42**, 523–527.

PLOBERGER , W., and KRAMER, W., 1992, The CUSUM-test with OLS residuals. *Econometrica*, **60**, 271–285.

RUKHIN, A., 1997, Change-point estimation under asymmetric loss. *Statistics and Decisions*, **15**, 141–163.

SASTRI, T., FLORES, B., and VALDES, J., 1989, Detecting points of change in time series, *Computers Open Research*, **16**, 271–293.

TSAY, R. S., 1990, Testing and modeling threshold autoregressive processes. *Journal of the American Statistical Association*, **84**, 231–240; 1991 Detecting and modeling non-linearity in univariate time series analysis. *Statistica Sinica*, **1**, 431–451.

WEISS, A. A., 1986, ARCH and bilinear time series models: comparison and combination. *Journal of Business and Economic Statistics*, **4**, 59–70.

WORSLEY, K. J., 1986, Confidence regions and tests for a change-point in a sequence of exponential family random variables. *Biometrika*, **73**, 91–104.

WU, B., 1994, Identification environment and robust forecasting for nonlinear time series. *Computational Economics*, **7**, 37–53; 1995, Model-free forecasting for nonlinear time series: with application in exchange rates. *Computational Statistics and Data Analysis*, **19**, 433–459.

WU, B., and CHEN, M., 1999, Use of fuzzy statistical methods in change periods detection. *Applied Mathematics and Computation*, **99**, 241–254; 2000, On a comparative study of change point detection techniques. *Applied Mathematics and Computation* (submitted).