Detection of change points in time series analysis with fuzzy statistics

KULDEEP KUMAR† and BERLIN WU‡

Changes in time series often occur gradually so that there is a certain amount of fuzziness in the change point. In this paper we have presented an integrated identification procedure for change-point detection based on fuzzy logic. The membership function of each datum corresponding to the cluster centres is calculated and is used for performance index grouping. We have also suggested a test for the change in level and the change in slope for testing a hypothesis about change points. We have made simulation studies to demonstrate the whole procedure. Finally an empirical study about change-point identification in the exchange rate data of six Asian nations has been demonstrated using the algorithm of the paper.

1. Introduction

The weakness of many models for time series analysis clearly resides in the impossibility of having no significant structure changes during the whole dynamic process. Two fundamental questions that often arisen are as follows.

(1) Does there exist a good model that leads this dynamic process?

(2) Does a single model fit the dynamic process?

(Need we use more than one model to fit the time series?)

As a result, model identification of a time series from a model-based system becomes an important procedure before a priori model selection (Wu 1995). If an a priori model family is correctly chosen, procedures of model construction such as parameter estimation, diagnosis and forecasting will make sense. On the other hand, if the underlying time series demonstrates certain structure changes, it is natural to detect those change points or change periods before modelling the whole process. Otherwise, using the traditional techniques for model construction, we may hardly obtain an appropriate good model to explain the nonlinear time series.

The problem of change point detection in time series has been examined by many researchers. For instance, Tsay (1990) proposed some procedures for detecting outliers, level shifts and variance changes in univariate time series. The procedures that he suggested are particularly useful and relatively easy to implement, while Balke (1993) pointed out that Tsay’s procedures do not always perform satisfactorily when level shifts are present. Inclan and Tiao (1994) proposed an iterative procedure to detect variance changes based on a centred version of the cumulative sums of squares presented by Brown et al. (1975).

Methods for change points detection include the modified Page (1955) (MPAGE) method and the cumulative sum (CUSUM) method proposed by Hinkley (1971). Hsu (1979, 1982) investigated the detection of a variance shift at an unknown point in a sequence of independent observations, focusing on the detection of points of change one at a time because of the heavy computational burden. Worsley (1986) used maximum likelihood (ML) methods to test a change in mean for a sequence of independent exponential family random variables. Sastri et al. (1989) presented a study of performance comparison for six time-series change detection procedures. Recently, Rukhin (1997) studied the classical change-point estimation problem in the

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Bayesian setting, that is point estimation of the change-point parameter is considered after the data have been observed and the change point is known to occur.

However, those detecting techniques are based on the assumption that the underlying time series conducts a significant change point characteristic (Wu and Chen 1999). Using the concept of fuzzy set theory, Wu and Chen (1999) proposed a procedure about change period detection for nonlinear time series. Nevertheless, we must indicate that, in dealing with time series with switching regimes, we should consider not only the change-point detection but also the properties of change periods. Because many patterns of change structure in time series occur over a time interval, those phenomena should not be treated as a mere sudden turning at a certain time. For instance

(i) the exchange rate may go up or down gradually after a new financial policy performance or
(ii) The supply of M1 or M2 may change their trend at different period of time according to the national economic conditions.

In fact, the semantics of the term ‘change point’ is not very clear and well understood (interested readers may refer to any popular dictionary such as Webster’s New Dictionary).

Fuzzy set theory has received much attention recently. The proposed method for change-point detection uses the concept of fuzzy entropy. The term ‘entropy’ comes from thermodynamics. Entropy can be considered as a measure of how close a system is to equilibrium. It can also be considered as a measure of disorder in the system. For instance, the classical clustering method separates the data into k categories, while in many cases there are elements, which cannot be contained, in a specific category. They belong to two or more categories simultaneously. A more detailed concept of fuzzy entropy has been given in the seminal paper by Wu and Chen (1999).

In this research we present an integrated identification procedure for change-point and change-periods detection. The membership function of each datum corresponding to the cluster centres is calculated and as performance index grouping. Two tests on the change in level and change in slope with a simple regression model are suggested for testing hypotheses about the change points. The simulation and empirical results showed that our detection procedure is an efficient and realistic procedure in detecting the structure change of a time series. In particular, when the change is gradual, our model construction procedure demonstrated a better performance in prediction.

2. Detecting process for change points and periods

2.1. Estimation

In order to classify the vague points, we use the following definitions.

Definition 1—α-level of fuzzy change points: Let \( \{y_t, t = 1, 2, \ldots, N\} \) be a time series. Suppose that we decompose \( \{y_t\} \) into \( k \) clusters according to a set of fuzzy cluster centres, where \( \mu_{it} \) is the degree of membership that \( y_t \) holds for cluster centre \( C_i \). For each \( y_t, t = 1, 2, \ldots, N \), if

\[
\max \{\mu_{it}\} - \min \{\mu_{it}\} < 1 - k\alpha, \quad \alpha \in \left(0, \frac{1}{k}\right) \tag{1}
\]

Then we call \( y_t \) an \( \alpha \) level of the fuzzy change point. Moreover, if \( \max \{\mu_{it}\} - \min \{\mu_{it}\} = 1 \), we call \( y_t \) an absolutely fuzzy point. If \( \max \{\mu_{it}\} - \min \{\mu_{it}\} = 0 \), we call \( y_t \) a crisp point.

Definition 2—Fuzzy entropy: Let \( \mu_{it} \) be the membership of \( y_t \) to the cluster \( C_i, i = 1, 2, \ldots, k \). The fuzzy entropy of \( y_t \) is defined as

\[
\delta(y_t) = -\frac{1}{k} \sum_{i=1}^{k} \mu_{it} \ln (1-\mu_{it}). \tag{2}
\]

The proposed fuzzy detecting procedures of fuzzy change periods with \( \alpha \) level include the following:

(i) fuzzy clustering;
(ii) deciding the \( \alpha \) level of a fuzzy point;
(iii) detecting the \( \alpha \) level of fuzzy change periods.

A detailed algorithm is illustrated below.

Algorithm for change period detection:

Step 1. Input the time series \( \{y_t\} \). Find \( C(i = 1, 2, 3) \) the set of cluster centres for \( \{y_t\} \). Classify \( \{y_t\} \) into three categories.

Step 2. Let \( \mu_{it} \) be the degree of membership of each observation \( y_t \) to each cluster \( C_i \). Compute the membership of \( \mu_{it} \) by

\[
\mu_{it} = 1 - \frac{(y_t - C_i)^2}{\sum_{i=1}^{3} (y_t - C_i)^2}, \quad i = 1, \ldots, 3, \quad t = 1, \ldots, N. \tag{3}
\]

Step 3. Calculate the fuzzy entropy of \( y_t \) by using its memberships:

\[
\delta(y_t) = -\frac{1}{k} \sum_{i=1}^{3} \mu_{it}(1-\mu_{it}). \tag{4}
\]

Step 4. Compute the average of the cumulative fuzzy entropy for each \( t \) by
The Chow (1960) test for the constancy of the regression coefficient vector over the sample is one of the most widely used diagnostic tests in applied statistics. It is known that the statistics associated with various forms of the Chow test are $F$ distributed under the null hypothesis of parameter stability.

Consider a sample $(X, Y)$ with $N = N_1 + N_2$ observations and suppose that we have two regression models

$$Y_t = \begin{cases} 
\beta_{10} + \beta_{11}X_t + \varepsilon_t, & 1 \leq t \leq N_1, \\
\beta_{20} + \beta_{21}X_t + \varepsilon_t, & N_1 < t \leq N_2.
\end{cases}$$

(7)

The most common form of the Chow test for parameter stability assuming that $\beta_{10} = \beta_{20}$, tests the hypothesis $H_0 : \beta_{11} = \beta_{21}$ versus the hypothesis $H_1 : \beta_{11} \neq \beta_{21}$. In this research we shall generalize the test procedure to a more complete form. We consider the following hypothesis:

(i) The change in level can be tested by testing the hypothesis $H_0 : \beta_{10} = \beta_{20}$ versus the hypothesis $H_1 : \beta_{10} \neq \beta_{20}$.

(ii) The change in slope can be tested by testing the hypothesis $H_0 : \beta_{11} = \beta_{21}$ versus the hypothesis $H_1 : \beta_{11} \neq \beta_{21}$.

There are two general approaches for comparing the slope and intercept of two regression lines. In the first approach we fit the data from two different regimes separately and then make an appropriate two-sample $t$-test. In the second approach we define a dummy variable $Z$ to be 0 if the data come from regime 1 and to be 1 if the data come from regime 2. Then, for the combined data, the single multiple regression model is defined as

$$Y_t = \beta_0 + \beta_1X_t + \beta_2Z + \beta_3X_tZ + \varepsilon_t,$$

(8)

which yields the following two models for the two values of $Z$. That is, if $Z = 0$, then $Y_t = \beta_0 + \beta_1X_t + \varepsilon_t$; if $Z = 1$, then $Y_t = (\beta_0 + \beta_2) + (\beta_1 + \beta_3)X_t + \varepsilon_t$. This allows us to write the regression coefficients for the separate models in terms of the coefficients of the model (7). It follows that $\beta_{10} = \beta_0$, $\beta_{20} = \beta_0 + \beta_2$, $\beta_{11} = \beta_1$ and $\beta_{21} = \beta_1 + \beta_3$.

Testing that the slope of two regression lines is same, can be conducted by testing the hypothesis $H_0 : \beta_3 = 0$. If $\beta_3 = 0$, then $\beta_{21} = \beta_{11}$, which is also the slope of the first line. The statistics for testing $H_0 : \beta_3 = 0$ is the partial $F$ statistic for the significance of the addition of the variable $XZ$ to a model already containing $X$ and $Z$. The test statistics are computed as follows:

$$F(XZ|X, Z) = \frac{\text{regression } SS(X, Z, XZ) - \text{regression } SS(X)}{MS \text{ residual } (X, Z, XZ)}.$$  

(9)

The test statistic to test that the two intercepts are equal is equivalent to testing the hypothesis $H_0 : \beta_2 = 0$ for the overall model. The test is computed as

$$F(Z|X) = \frac{\text{regression } SS(Z, X) - \text{regression } SS(X)}{MS \text{ residual } (X, Z, XZ)}.$$  

(10)

The details of these two methods can be found in the book by Kleinbaum et al. (1998, chapter 14). They have also shown that these two methods give exactly the same result. For simplicity we have used second approach in this paper.

### Testing rules for a change point in the switching time series model:

**Step 1.** Choose the estimated change points $i = 1, \ldots, k$, according to the procedure in §2.1

**Step 2.** For each period $i, i = 1, \ldots, k$, fit a linear regression model with time.

**Step 3.** Test the hypothesis that there is a difference of slope of the two consecutive lines. If the difference is significant, then it suggests a change point. (The difference in the slopes could be in the sign from positive to negative.) Go to step 5.

**Step 4.** Test the hypothesis that there is a significant difference in the levels of the two consecutive lines at $t = t_i$. If the difference is significant, then it suggests a change point. (If there is a significant difference in the intercept of the two regression lines then it will also suggest a change point, but this change point is due to the
change in level rather than to the change in slope.) A significant difference in slope could well result in a significant difference between the intercepts.

Step 5. The time series has change points at $t = t_i$.

3. Simulation studies

Four time series data of size 450 are generated from four models. The first is an AR(1) model with a switch in level. The second is also an AR(1) model with a switch in the slope. The third model is a switching time series regression model with a switch in slope and the fourth is also a switching time series regression model with a switch in level and also in slope. These models are shown in figures 1, 2, 3 and 4 respectively.

$$X_t = \begin{cases} 
-2 + 0.5X_{t-1} + \epsilon_t, & \epsilon_t \sim N(0, 0.1) \text{ if } 1 \leq t \leq 150, \\
2 + 0.5X_{t-1} + \epsilon_t, & \epsilon_t \sim N(0, 0.1) \text{ if } 151 \leq t \leq 300, \\
5 + 0.5X_{t-1} + \epsilon_t, & \epsilon_t \sim N(0, 0.1) \text{ if } 301 \leq t \leq 450,
\end{cases}$$  \hspace{1cm} (11)

$$X_t = \begin{cases} 
2 + 0.6X_{t-1} + \epsilon_t, & \epsilon_t \sim N(0, 0.1) \text{ if } 1 \leq t \leq 150, \\
2 + 0.2X_{t-1} + \epsilon_t, & \epsilon_t \sim N(0, 0.1) \text{ if } 151 \leq t \leq 300, \\
2 + 0.8X_{t-1} + \epsilon_t, & \epsilon_t \sim N(0, 0.1) \text{ if } 301 \leq t \leq 450,
\end{cases}$$  \hspace{1cm} (12)

$$X_t = \begin{cases} 
2 + 0.5t + \epsilon_t, & \epsilon_t \sim N(0, 0.1) \text{ if } 1 \leq t \leq 150, \\
2 + 0.5t + \epsilon_t, & \epsilon_t \sim N(0, 1) \text{ if } 151 \leq t \leq 300, \\
2 + 2t + \epsilon_t, & \epsilon_t \sim N(0, 10) \text{ if } 301 \leq t \leq 450,
\end{cases}$$  \hspace{1cm} (13)

$$X_t = \begin{cases} 
1 + 0.5t + \epsilon_t, & \epsilon_t \sim N(0, 0.1) \text{ if } 1 \leq t \leq 150, \\
5 + 0.5t + \epsilon_t, & \epsilon_t \sim N(0, 0.1) \text{ if } 151 \leq t \leq 300, \\
10 + 2t + \epsilon_t, & \epsilon_t \sim N(0, 10) \text{ if } 301 \leq t \leq 450.
\end{cases}$$  \hspace{1cm} (14)

Table 1 shows the fitted regression lines for models (11)–(14) and table 2 illustrates the result of change point estimation as well as testing.

It can be observed that the $p$ value corresponding to the $F$ test is very low, so, under any reasonable $\alpha$ level of significance, say, $0.005 \leq \alpha \leq 0.1$, we reject the null hypothesis, that is there exist change points. From table 2 we can see that our identification procedure demonstrated a very efficient result in detecting the change points for model (11)–(14).
4. Empirical examples

The data analysed here come from the foreign exchange rate database from the Central Bank of Taiwan for the period from 2 July 1997 to 31 October 1998. The local currencies that we selected are new Taiwan (NT) dollar (Taiwan), yen (Japan), won (South Korea), indr (Indonesia), baht (Thailand) and ringgit (Malaysia). Each local currency is given a total number of observations of 454. The purpose of this study is to examine the change point of the eastern Asia financial crisis. Figure 5 provides flow charts of the empirical analysis process.

On 2 July, 1997, the Thailand Government announced adoption of the floating exchange rate system against the US dollar, the local currencies of Thailand and other Asian countries, except for Hong Kong and PR China, continually depreciate against US dollar. From figure 5(e), we can see the level of depreciation of the Thai baht is about 20% in the first 2 months, and the ringgit follows the baht at about 10%. The yen, won and NT dollar depreciated equally at under 10%. After November 1997, the Thai baht depreciated sharply by above 50%, and the won devalued abruptly by above 40%. In the beginning, the won was influenced and resembled the NT dollars. After 4 months, the won followed the same group with the baht and ringgit having similar severe devaluations.

We can see that the Asian financial crisis began with the devaluation of the Thai baht in July, 1997 and the worst depreciation was in January 1998 and continually fell after January. Then the crisis spilled over to neighboring countries such as Malaysia and Indonesia. The main reasons are the similarity of the financial environments of these three countries. It is significant that South Korea, Taiwan, Singapore and Hong Kong had similar economic advantages before 1996. At the beginning of the economic crisis, the exchange rates of the yen, NT dollar and won depreciated slightly, but the won devaluation turned out to be deeper after November 1997. Collectively, the crisis-hit nations had the worst effects in January 1998 and then the impact of the devaluation was mitigated.

In these figures, the yen–US dollar exchange rate fluctuated sharply in the period of estimation and forecasting; the NT dollar against the US dollar devalued sharply and fluctuated in the middle of the estimation period; the baht–US dollar changes rose to a peak in January 1998 during this crisis. The NT dollar and Thai baht both changed steadily against the US dollar at the

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### Table 1. Fitted regression lines for models (11)–(14)

<table>
<thead>
<tr>
<th>Model</th>
<th>Regression 1&lt;sup&gt;a&lt;/sup&gt;</th>
<th>Regression 2&lt;sup&gt;a&lt;/sup&gt;</th>
<th>Regression 3&lt;sup&gt;a&lt;/sup&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>(11)</td>
<td>( X_t = -3.93 - 0.0007t )</td>
<td>( X_t = 3.58 + 0.0017t )</td>
<td>( X_t = 9.18 + 0.0021t )</td>
</tr>
<tr>
<td></td>
<td>[0.034] [0.00039]</td>
<td>[0.17] [0.00072]</td>
<td>[0.22] [0.00057]</td>
</tr>
<tr>
<td>(12)</td>
<td>( X_t = 1.25 - 0.00003t )</td>
<td>( X_t = 2.54 - 0.0002t )</td>
<td>( X_t = 6.54 + 0.0087t )</td>
</tr>
<tr>
<td></td>
<td>[0.023] [0.00026]</td>
<td>[0.042] [0.00018]</td>
<td>[0.054] [0.0014]</td>
</tr>
<tr>
<td>(13)</td>
<td>( X_t = 1.97 - 0.50t )</td>
<td>( X_t = 1.83 + 0.501t )</td>
<td>( X_t = -0.52 + 2.01t )</td>
</tr>
<tr>
<td></td>
<td>[0.017] [0.00019]</td>
<td>[0.44] [0.00019]</td>
<td>[6.8] [0.018]</td>
</tr>
<tr>
<td>(14)</td>
<td>( X_t = 0.975 + 0.50t )</td>
<td>( X_t = 4.83 + 0.501t )</td>
<td>( X_t = 7.84 + 2.01t )</td>
</tr>
<tr>
<td></td>
<td>[0.008] [0.00019]</td>
<td>[0.97] [0.00019]</td>
<td>[6.8] [0.018]</td>
</tr>
</tbody>
</table>

<sup>a</sup>The values inside square brackets are the standard errors of the coefficients for the regression line.

### Table 2. Change-point detection for models (11)–(14)

<table>
<thead>
<tr>
<th>Model</th>
<th>First change point</th>
<th>Second change point</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Real</td>
<td>Estimate</td>
</tr>
<tr>
<td>(11)</td>
<td>151</td>
<td>150</td>
</tr>
<tr>
<td>(12)</td>
<td>151</td>
<td>155</td>
</tr>
<tr>
<td>(13)</td>
<td>151</td>
<td>145</td>
</tr>
<tr>
<td>(14)</td>
<td>151</td>
<td>146</td>
</tr>
</tbody>
</table>

<sup>a</sup>0+ stands for values that are very small and close to 0.
end periods. Almost without exception, the depreciation of the exchange rate of crisis-hit nations was significantly worse than before.

Table 3 demonstrates the change points and change periods for the exchange rate of six Asian countries.

In table 3, the detected times of change points and change periods are close to the usual observation in the first and second or third periods of the regime. Furthermore, the trends of the two regimes are stable compared with the change in the true observations which fluctuate sharply. The leading trend of these time series are consistent with the trend of the estimated observations and it is not like the previous trend which approaches some value. However, the estimates of the six time series change little after the last period.

5. Conclusion
In the economic and financial analysis discussed above the time series data have the switching regime property. If we use the univariate autoregressive moving average family, or any single model, to analyse these data, it will not solve the orientation problem. The contribution of this paper is that it provides a new method to detect change points. In this paper, we introduced three detection procedures that can effectively detect multiple change periods for a nonlinear time series. The proposed algorithm also combines with the concept of a fuzzy set. We have demonstrated how to find a α level change period to help to model a time series model with multiple change periods.
Simulation results show that our proposed techniques of change period detection are simple and efficient. Our algorithm is highly recommended practically for detecting the $\alpha$ level change period and is supported by the empirical results. A major advantage of this framework is that our detection procedures do not require any initial knowledge about the structure in the data and can take full advantage of the model-free approach. 

Wu and Chen (2000) have compared this technique of change point detection with other change point detection techniques. It was observed that techniques based on fuzzy entropy work better than other techniques such as MPAGE and CUSUM mentioned in §1.

However, certain challenging problems still remain open, such as:

1. The semantics of the term 'stationarity' need to be redefined carefully. It seems that the term 'change points', which may stand for change in mean, change in variance, change in parameter or change in model needs to be clarified before performing a detection process.

2. In the case of random-walk processes, an appropriate detection procedure needs to be developed for prior recognition and model identification.

3. What knowledge base is required to obtain specific behaviour of time series under certain multivariate endogenous variables?

4. The convergence of the algorithm for classification and the proposed statistics have not been well proved, although the algorithms and the proposed statistics are known as fuzzy criteria. This needs further investigation.

Although there remain many problems to be overcome, we think fuzzy statistical methods will be a worthwhile approach and will stimulate more future empirical work in time series analysis.

Acknowledgements

We are grateful to the editor and two referees for the comments on the earlier version of the paper.

References


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Table 3. Change points and change periods for the exchange rates of six Asian countries

<table>
<thead>
<tr>
<th>Country</th>
<th>Currency</th>
<th>Cluster centre</th>
<th>Change point ($\alpha = 0.05$)</th>
<th>Change period$^a$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Taiwan</td>
<td>NTS</td>
<td>High, 34.9</td>
<td>29.5 (17 October 1997)</td>
<td>28.52 ↑ 30.45 (16 October 1997–20 October 1997)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Low, 27.3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Japan</td>
<td>Yes</td>
<td>High, 147.4</td>
<td>(1) 131.7 (30 March 1998)</td>
<td>(1) 129.9 ↑ 133.3 (24 March 1998–1 April 1998)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Low, 110.9</td>
<td>(2) 128.2 (7 October 1998)</td>
<td>(2) 130.1 ↑ 122.5 (6 October 1998–8 October 1998)</td>
</tr>
<tr>
<td>South Korea</td>
<td>Won</td>
<td>High, 1962</td>
<td>(1) 999 (11 October 1997)</td>
<td>(1) 957.6 ↑ 1187 (28 October 1997–1 December 1997)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Low, 844.6</td>
<td>(2) 1448 (20 March 1998)</td>
<td>(2) 1460 ↑ 1366 (16 March 1998–23 March 1997)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Low, 2361</td>
<td>(2) 9050 (6 May 1998)</td>
<td>(2) 8120 ↑ 9450 (29 April 1998–12 May 1998)</td>
</tr>
<tr>
<td>Thailand</td>
<td>Baht</td>
<td>High, 56.5</td>
<td>(1) 27.5 (2 July 1997)</td>
<td>(1) 25.9 ↑ 27.5 (30 June 1994–2 July 1997)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Low, 25.6</td>
<td>(2) 48.6 (5 February 1998)</td>
<td>(2) 51.4 ↑ 47.9 (2 February 1998–9 February 1998)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(3) 40.2 (12 August 1998)</td>
<td>(3) 41.2 ↑ 38.7 (13 March 1998–23 March 1998)</td>
</tr>
<tr>
<td>Malaysia</td>
<td>Ringgit</td>
<td>High, 4.66</td>
<td>(1) 2.56 (14 July 1997)</td>
<td>(1) 2.50 ↑ 2.65 (9 July 1997–18 July 1997)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Low, 2.44</td>
<td>(2) 3.98 (3 February 1998)</td>
<td>(2) 4.20 ↑ 3.60 (21 January 1997–10 February 1998)</td>
</tr>
</tbody>
</table>

$^a$ ↑, The time series is steadily increasing during the change period; ↓, the time series is steadily decreasing during the change period.


