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STATISTICAL CONSIDERATIONS ON THE RANDOMNESS OF ANNUAL MAXIMUM DAILY RAINFALL[1]

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ABSTRACT: Annual maximum daily rainfall data from nine stations throughout the southern slopes of the Eastern Italian Alps with record length of 67-68 years have been analyzed with the aim of verifying if their internal structure justifies the assumption of independence and identical distribution, or the "White noise hypothesis." The approach is to consider the hypothesis H_0 of white noise as the intersection of several sub-hypotheses, each concerning one of the characteristics of a white noise process. To this end the nine series were subjected to various statistical tests regarding randomness, independence, change-points, and predictability. The results are examined first individually and then globally. They indicate that in eight of the nine considered time series the "white noise hypothesis" was rejected.
(KEY TERMS: annual maximum daily rainfall; white noise; change-point; prediction.)

INTRODUCTION

The basic objectives of applying statistics to hydrology are the derivation of information from observed hydrologic phenomena of the past, and the prediction of what is expected in the future (Yevjevich, 1972).

In this context, the data used most in hydrologic design problems are those related to variations in the pattern of rainfall and those related to the maximum intensity (depth/time) in a given time interval. These data, in the form of long records, are analyzed to define the frequency difference between rainfall measured over different time periods and to detect the rainfall occurrence pattern for one day or for other definite time intervals.

In the specific fields of the derivation of river discharges and of mitigation of risk connected with flood events, one of the hydrologic variables used most is annual maximum daily rainfall. In general, these hydrologic data often appear randomly scattered around a mean value (Figure 1). For this reason they can be thought of as a sequence generated by a "white noise" (WN) process, that is a realization of independent and identically distributed (i.i.d.) random variables.

In Italy annual maximum daily rainfall represents the maximum cumulated rainfall, falling between 09.00 h of the previous day and the same hour of the considered day, for each calendar year. The precipitation is sampled by tipping-bucket raingauges with capture funnels of 0,1 m^2.

In this work, nine time series of annual maximum daily rainfall published in the Hydrologic Bulletins of the Italian Hydrologic Office are analyzed.

The series belong to stations located on the southern side of the Alps (more precisely in Eastern Italy), in areas characterized by different precipitation regimes and different average annual rainfall. The selected time series are those of Monte Maria-Marienberg (1335 m a.s.l., 673 mm/y.), Vipiteno-Sterzing (948 m a.s.l., 773 mm/y.), Bressanone-Brixen (560 m a.s.l. 668 mm/y.), S. Martino in Badia-St.Martin in Thurn (1393 m a.s.l., 748 mm/y.) and Dobbiaco-Toblach (1250 m a.s.l., 880 mm/y.) located in the South Tyrol (which is a bilingual region). Musi (633 m a.s.l., 3320 mm/y.), Venzone (230 m a.s.l., 2160 mm/y.), Udine (146 m a.s.l., 1370 mm/y.) and Schio (234 m a.s.l., 1520 mm/y.) are located, respectively, in Friuli-Venezia Giulia and in Veneto, at the foot of the prealpine relief. All the series run, without missing data, from 1924 to 1990 or 1991.

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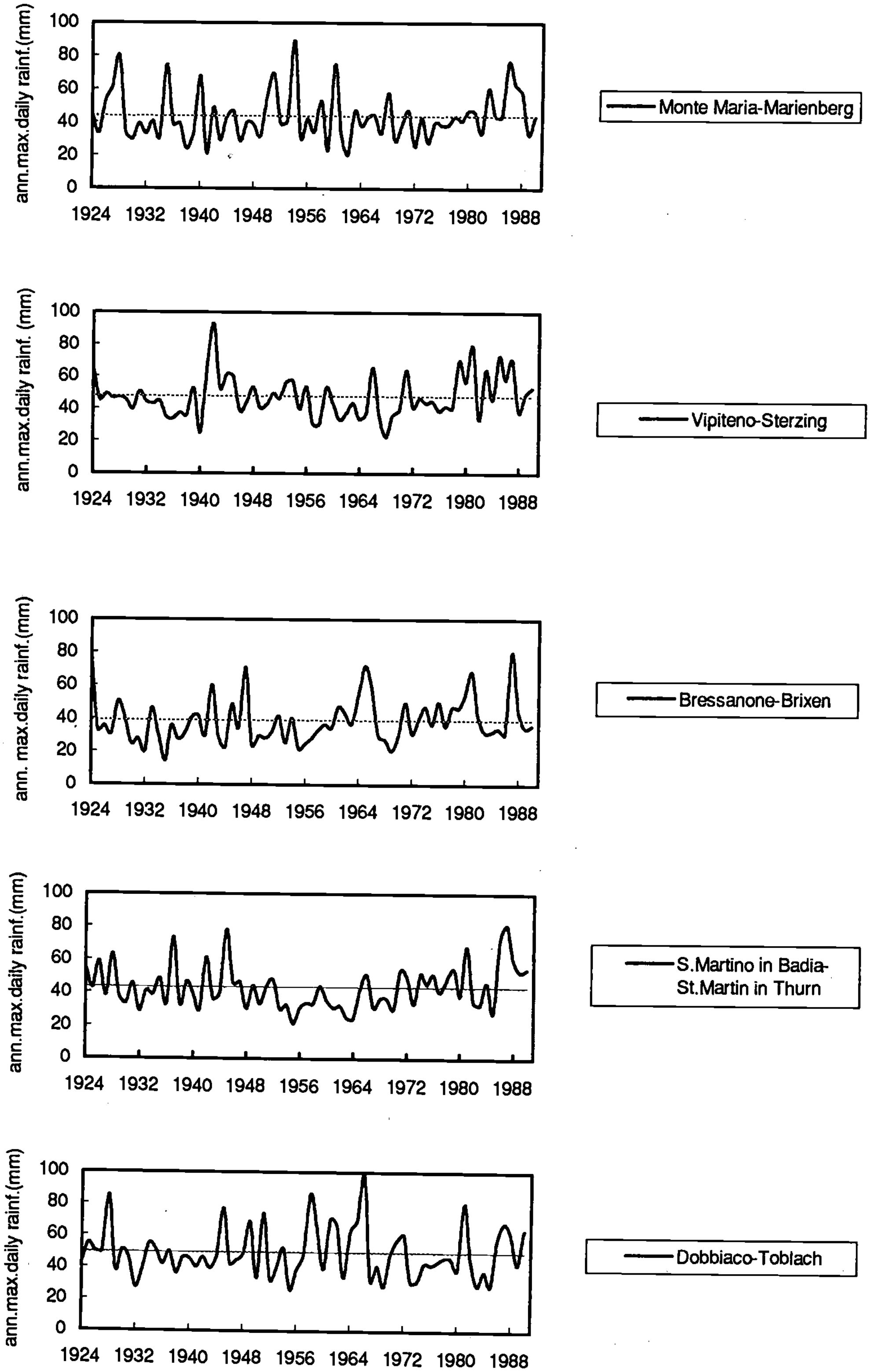

Figure 1. Time Series of the Annual Maximum Daily Rainfall
(dashed line represents the average).

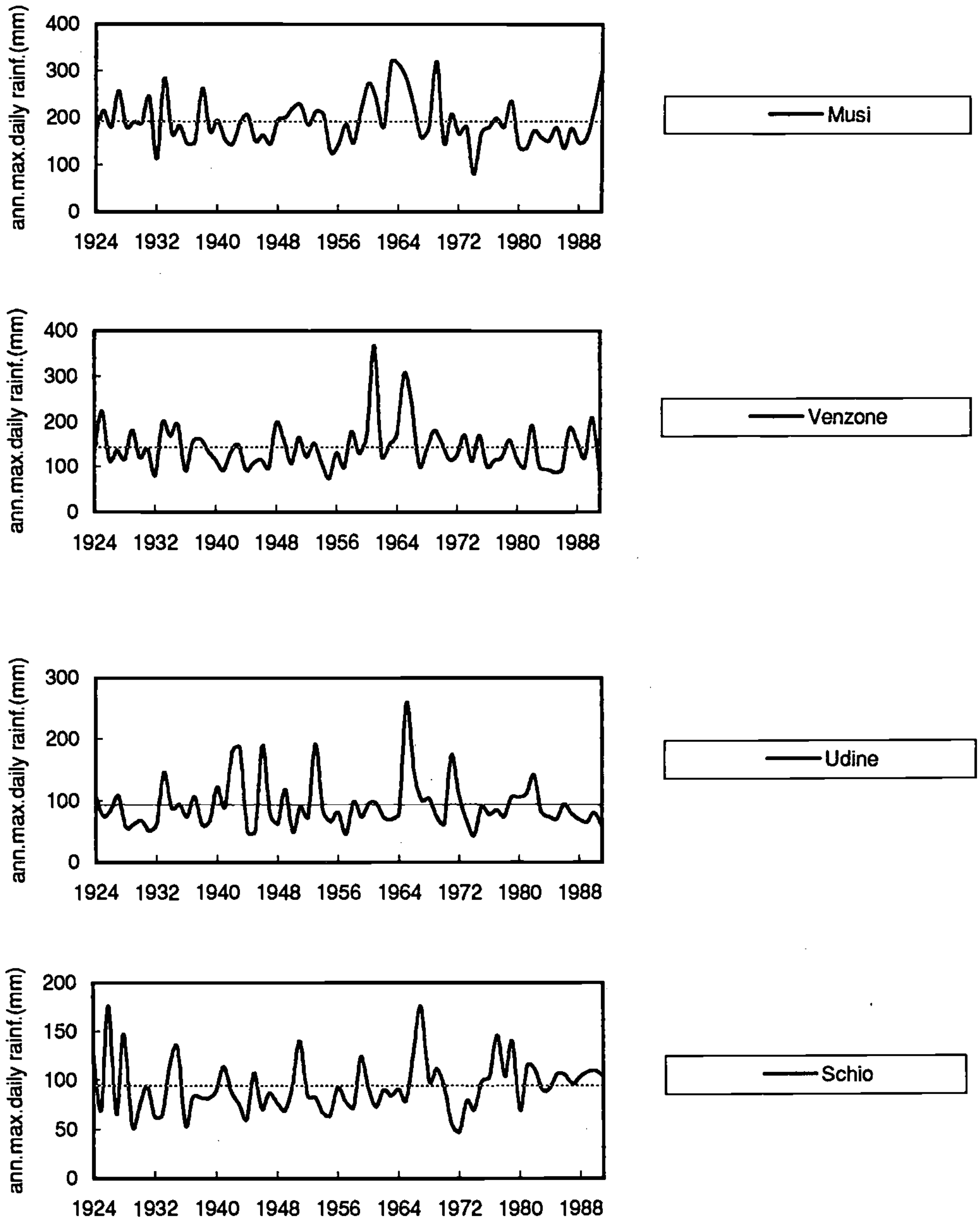

Figure 1. Time Series of the Annual Maximum Daily Rainfall (dashed line represents the average) (cont'd.)

The aim of the present paper is to verify if these time series can really be thought of as realizations of a white noise process. The topic of characterizing the internal structure of time series has already been considered in the literature, above all, in studies on climatic variations (W.M.O. , 1992; Cavadis, 1992; Mitosek 1992). However, with the extension of the time series, hydrologists should be aware that the standard assumption of the use of most common probability distribution functions might not be verified. For this reason a method of verifying the "white noise hypothesis" is proposed. It is based on a series of tests: they are first individually evaluated and then related to each other.

TESTING APPROACH

We can think of a white noise as:

$$x_t = \mu + \varepsilon_t \qquad (1)$$

where μ is a constant (which represents the average) and ε_t is a random error such that $E(\varepsilon_t) = 0 \ \forall t, \mathrm{Var}(\varepsilon_t)$

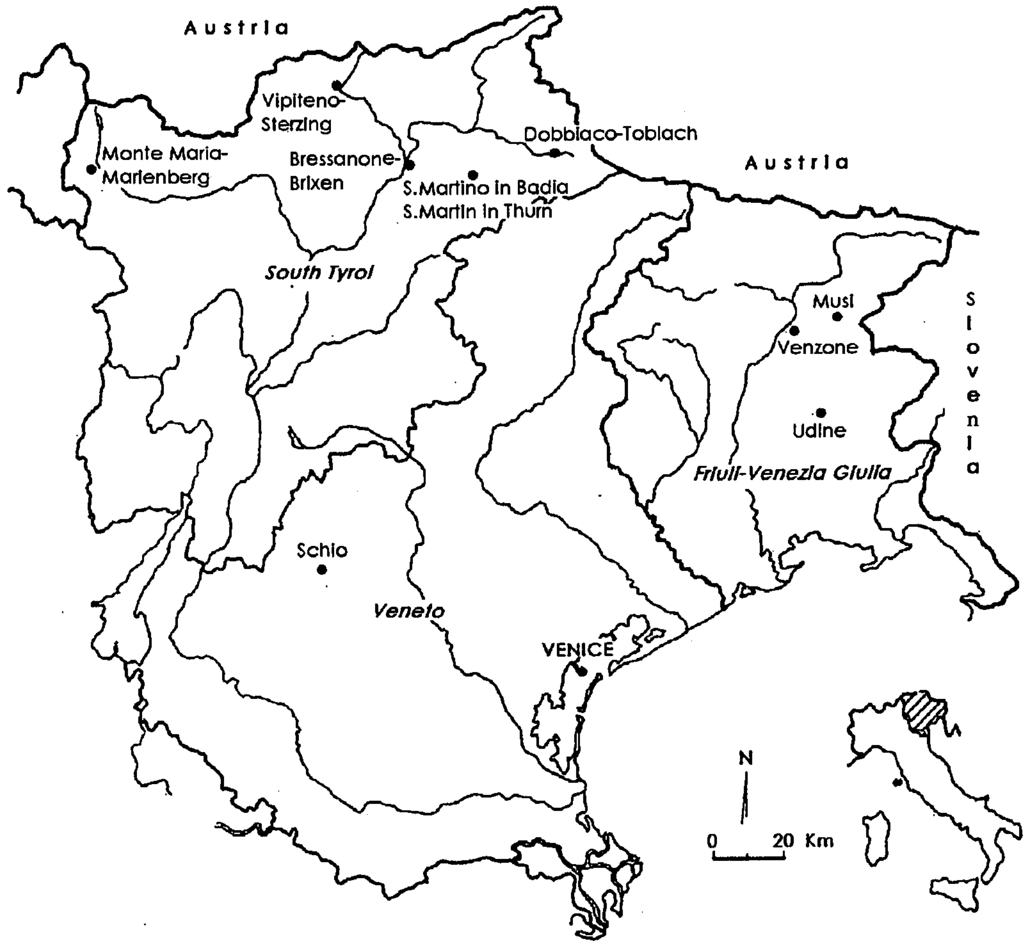

Figure 2. Map of the Eastern Italy, Showing the Location of the Nine Rainfall Gauging Stations.

$= \sigma^2 \; \forall t$ and ε_t independent of ε_s for every $s \neq t$. By such a definition it immediately follows that, with respect to a time series $\{x_t\}_{t=1}^{N}$, the hypotheses system to verify is the following:

$$\begin{cases} H_0: \{x_t\} \text{ is generated by a WN process} \\ H_1: \{x_t\} \text{ is generated by another type of process} \end{cases} \quad (2)$$

When the data generator process (DPG) is a WN, the resulting series must not show any trend component, periodic component, change-point, or serial correlation. Moreover, the series have to be completely unpredictable. After these propositions, the system (Equation 2) can be replaced by a new system, built considering the intersection of several sub-hypotheses, each referred to a specific characteristic of a WN process.

$$\begin{cases} H_0: \bigcap_{i=1}^{k} H_{0,i} \\ H_1: \bigcup_{i=1}^{k} H_{1,i} \end{cases} \quad (3)$$

H_0 is accepted only if all the sub-hypotheses are accepted, at a fixed significance level, and rejected if at the least one of the sub-hypotheses is rejected. In this study, we look at this diagnostic problem considering the following four sub-hypotheses:

$H_{0,1}$: randomness (no trend or systematic oscillations).
$H_{0,2}$: no serial correlation.
$H_{0,3}$: no change-point.
$H_{0,4}$: unpredictability.

However, this set of sub-hypotheses can be further extended, when other characteristics of a WN are individuated.

The decision of which test to use, on the basis of the data characteristics and also of the test characteristics (known versus unknown distribution, known dependencies in the data, number of data . . .), led us to a nonparametric approach as most recommended in the specific literature. While some tests are very common, others are not so well known. Amongst the latter we have considered the contributions coming from the literature of chaos.

A problem that arises from the hypothesis system (Equation 3) is the significance level of the global test. When the sub-hypotheses are tested independently (i.e., on different data sets) then the global level α_g is

$$\alpha_g = \prod_{i=1}^{k} \alpha_i$$

It easy to see that the resulting test is conservative with respect to H_0. In fact, to have a global significance level of 5 percent, with α_i constant, it is necessary to choose $\alpha_i = 0.05^{1/k}$. Another approach for the determination of the global level, in the independent case, is that suggested by Liverey and Chen (1983).

When the tests, as in this case are not independent, the only thing that can be said about the significance level of the global tests is that:

$$1-\sum_{i=1}^{k} \left[1-\alpha_i\right] \le \alpha_g \le \min\left\{\alpha_i\right\} \tag{4}$$

which does not permit us to determine the exact level of the test. Nevertheless, in our case, due to Equation (4) the first type error in the global test does not exceed 5 percent.

RANDOMNESS

The first sub-hypothesis formulated ($H_{0,1}$) concerns the randomness of the series. When, as said before, the DGP is a WN, the series should not exhibit any temporal trend or any structural or systematic fluctuations. The system hypotheses to be verified is:

$$\begin{cases} H_{0,1}: \text{no trend or systematic oscillations} \\ H_{1,1}: \overline{H_{0,1}} \end{cases} \tag{5}$$

The system (Equation 5) was verified using the above-below runs test, which is a particular type of Mann-Whitney test, widely used in the literature (see Sneyers, 1995). We assume we have a numerical sequence. Let's transform this sequence into a binary sequence according with:

$$y_i = \begin{cases} 1 & \text{if } x_i - \bar{x} \ge 0 \\ 0 & \text{if } x_i - \bar{x} < 0 \end{cases}$$

A run is defined as a succession of one or more identical symbols which are followed by a different symbol or no symbol at all. In the above-below runs test when the jth element is larger than the mean of the time series, a run above starts; when it is smaller, a run below starts. We can observe when a sequence is above its mean, and for how long, and when it is below its mean, and for how long.

Clues to the lack of randomness are provided by any tendency of the symbols to exhibit a definite pattern in the sequence. Too few runs, or an excessive number of long runs, is usually indicative of some sort of trend, while an excessive number of short runs can indicate some regular oscillation (for example 001100110011 . . .).

Under the null hypothesis $H_{0,1}$ the expected number of runs, R, is neither too large nor too small. So $H_{0,1}$ is rejected for too large or too small values of R. If we were interested only in a trend alternative, we would reject H_0 only for small values of R.

For small values of N, let us say $N \le 25$, the distribution of R is tabulated (Siegel and Castellan, 1988). For large values of N we can use the statistic

$$Z = \frac{R + h - 2mn/N - 1}{\sqrt{[2mn(2mn-N)]/\left[N^2(N-1\right]}}$$

where m is the number of symbols "1" and $n = N - m$ is the number of "0;" $h = \pm 0.5$ according to whether r is smaller or larger than $(2mn + 1)$. Z has a standard normal distribution, so that, for large N, the test is easily performed. The empirical results of the application of the test on the nine series are summarized in Table 1. In all cases the null-hypothesis is accepted at a significance level of 5 percent.

INDEPENDENCE

The sub-hypothesis $H_{0,2}$ concerns the absence of any type of serial correlation in the data. To verify independence in a preliminary way, tests based on classic linear autocorrelation, partial and not, were used. By way of analogy with previous works (Srikanthan and Stewart, 1991; Sneyers, 1995) we also used

TABLE 1. Results of the Run Test at 5 Percent Level.

Station	Z	p-Value	H0,1
Monte Maria-Marienberg	0.009	0.496	Accepted
Vipiteno-Sterzing	-0.005	0.502	Accepted
Bressanone-Brixen	-0.010	0.504	Accepted
S.Martino in Badia-St.Martin in Thurn	0.006	0.497	Accepted
Dobbiaco-Toblach	0.008	0.503	Accepted
Musi	0.035	0.498	Accepted
Venzone	-0.012	0.495	Accepted
Udine	0.000	0.500	Accepted
Schio	-0.028	0.511	Accepted

the Spearman and Kendall nonparametric tests. They are based, respectively, on the correlation coefficients:

$$C_s = 1 - \frac{6\sum d_i^2}{N^3 - N}$$

and

$$C_K = \frac{2\sum\sum_{i \neq j} S_{ij}}{N(N-1)}$$

where $d_i = r(A_i) - r(B_i)$; $S_{ij} = +1$ if $r(A_i) \geq r(B_j)$, $S_{ij} = -1$ if $r(A_i) < r(B_j)$ and $r(A_i)$, $r(B_i)$ $(i = 1, \ldots N)$ are the ranks in two sets (A and B) of observations. In this case, relatively to j^{th} the lag two samples are $\{x_t\}_{t=1}^{N-j}$, $\{x_t\}_{t=j}^{N}$. The use of these indicators did not show significant autocorrelation. However, one should take note that these tests refer to linear relationships. Since it is known that the absence of linear correlation does not insure independence, we have considered a test able to capture also non linear correlations.

The test used was the BDS (Brock *et al.*, 1986) which belongs to the chaotic systems literature. It tests the null hypothesis of independence for a time series $\{x_t\}$ by means of the system:

$$\begin{cases} H_{0,2} : \text{the series } \{x_t\} \text{ is independent} \\ H_{1,2} : \overline{H_{0,2}} \end{cases}$$

The hypothesis of independence is tested using the concept of spatial correlation. To this end the series $\{x_t\}$ must be embedded in m-dimensional space by means of the construction of vectors of the type (Sauer *et al.*, 1991).

$$X_t = (x_t, x_{t-1}, \ldots, x_{t-m+1}) \qquad t = 1,2,\ldots, N - m + 1.$$

The dependence of the series is examined using the so-called "correlation integral," a quantity which examines the distances between points in $\Re^m$. For each embedding dimension m and for each choice of ε, the correlation integral is defined as

$$C(\varepsilon, m, N) = \frac{1}{N_m(N_m - 1)} \sum_{t \neq s} I\left[\|X_t - X_s\| < \varepsilon\right]$$

where $N_m = N - m + 1$ and I represents the indicator function. Brock and Baek (1991) considered the statistic

$$BDS = \sqrt{N}\left[C(\varepsilon, M, N) - C(\varepsilon, 1, N)^m\right] / \sqrt{V}$$

where V, the variance of $C(\varepsilon, m, N)$, under the null-hypothesis is

$$V = 4\Big\{K(\varepsilon)^m + 2\Big[\sum k(\varepsilon)^{m-1} c(\varepsilon)^{2i}$$
$$+ (m-1)^2 c(\varepsilon)^{2m} - m^2 k(\varepsilon) c(\varepsilon)^{2m-2}\Big]\Big\}$$

with $c(\varepsilon) = E(I[\|X_t - X_s\| < \varepsilon])$ and $k(\varepsilon) = E(I[\|X_t - X_s\| < \varepsilon]\, I[\|X_s - X_r\| < \varepsilon])$. These authors showed that under the null hypothesis H_0 the BDS statistic is asymptotically standard normally distributed. The interest for the BDS resides in the fact that in certain circumstances it is able to point out dependencies which can completely escape linear methods.

The results furnished by this test are illustrated in Table 2, which shows that the null hypothesis of independence at a significance level of 5 percent is rejected everywhere, with the exception of Udine.

As far as ε is concerned there are no automatic criteria of choice; it depends on the quantity and quality of data. Hsieh and Le Baron (1988) suggested choosing ε depending on the standard deviation of the data. The choice of m depends on the length of the time series. The authors give also finite sample properties of the test using Monte Carlo simulations. Further considerations have been provided by Brock *et al.* (1991).

For computations carried out in this work, we chose ε equal to 0.3 and 0.5 times the standard deviation and we calculated the BDS statistics by m = 1, 2, 3, 4, 5. Finally, we would like to point out that the BDS test is unadvisable with sample sizes less than those employed in this work.

TABLE 2. Results of the BDS Test at 5 Percent Level.

Station	$H_{0,2}$
Monte Maria-Marienberg	Rejected
Vipiteno-Sterzing	Rejected
Bressanone-Brixen	Rejected
S.Martino in Badia-St.Martin in Thurn	Rejected
Dobbiaco-Toblach	Rejected
Musi	Rejected
Venzone	Rejected
Udine	Accepted
Schio	Rejected

CHANGE-POINTS

To verify the hypothesis $H_{0,3}$ on the methods suggested by the literature (Leith, 1973; Katz, 1977; Klugman and Klugman, 1981) we used the nonparametric test proposed by Pettitt (1979).

The test supposes that initially the DGP has a certain location parameter (median) which undergoes one or more changes over time. It is therefore clear that the existence of only one change-point invalidates the H_0 hypothesis. The hypothesis system to verify is

$$\begin{cases} H_{0,3}: \text{no change-points} \\ H_{1,3}: \overline{H_{0,3}} \end{cases}$$

The test statistic is:

$$K_t = \max_{t=1,\ldots,N-1} |2W_t - t(N-1)| \tag{6}$$

where $W_t = \sum_{i=1}^{t} r_i (t = 1,\ldots,N-1)$, with $r_i = rank(x_i)$. K_t divides the series into two parts: the first, $\{x_s\}_{s=1}^{t}$, is represented by the first m observations and the second, $\{x_s\}_{s=t+1}^{N}$, by the remaining $N - t$. The value of t^* which maximizes Equation (6) is the estimated change-point. To check the significance of this change it is possible, for large N, to use the statistic

$$Z = \frac{K_{t*} + h - [t^*(N-1)/2]}{\sqrt{t^*(N-t^*)(N+1)/12}}$$

where $h = \pm\ 0.5$ according to whether K_{t*} is greater or smaller than $t^*(N - 1)/2$. Z is asymptotically standard normally distributed.

The results obtained from Pettitt's test are summarized in Table 3 and, for some series, in the Figure 3. They show the presence of change-points in several of the series considered. More precisely, in six of the nine series the null hypothesis has to be rejected at a level of 5 percent. In the table, all the detected change-points are reported but it is good to remember that, for the aim of the present work, thus far it has been sufficient to establish the first change-point.

TABLE 3. Results of Pettitt's Change-Point Test at 5 Percent Level.

Station	Z	Change Points	Years	$H_{0,3}$
Monte Maria-Marienberg	-2.14	1	1977	Rejected
Vipiteno-Sterzing	-2.74	3	1940, 1956, 1978	Rejected
Bressanone-Brixen	-2.83	1	1960	Rejected
S.Martino in Badia-St.Martin in Thurn	-2.77	2	1952, 1970	Rejected
Dobbiaco-Toblach	1.41	0	–	Accepted
Musi	2.65	2	1958, 1969	Rejected
Venzone	1.59	0	–	Accepted
Udine	1.45	0	–	Accepted
Schio	-2.97	2	1965, 1969	Rejected

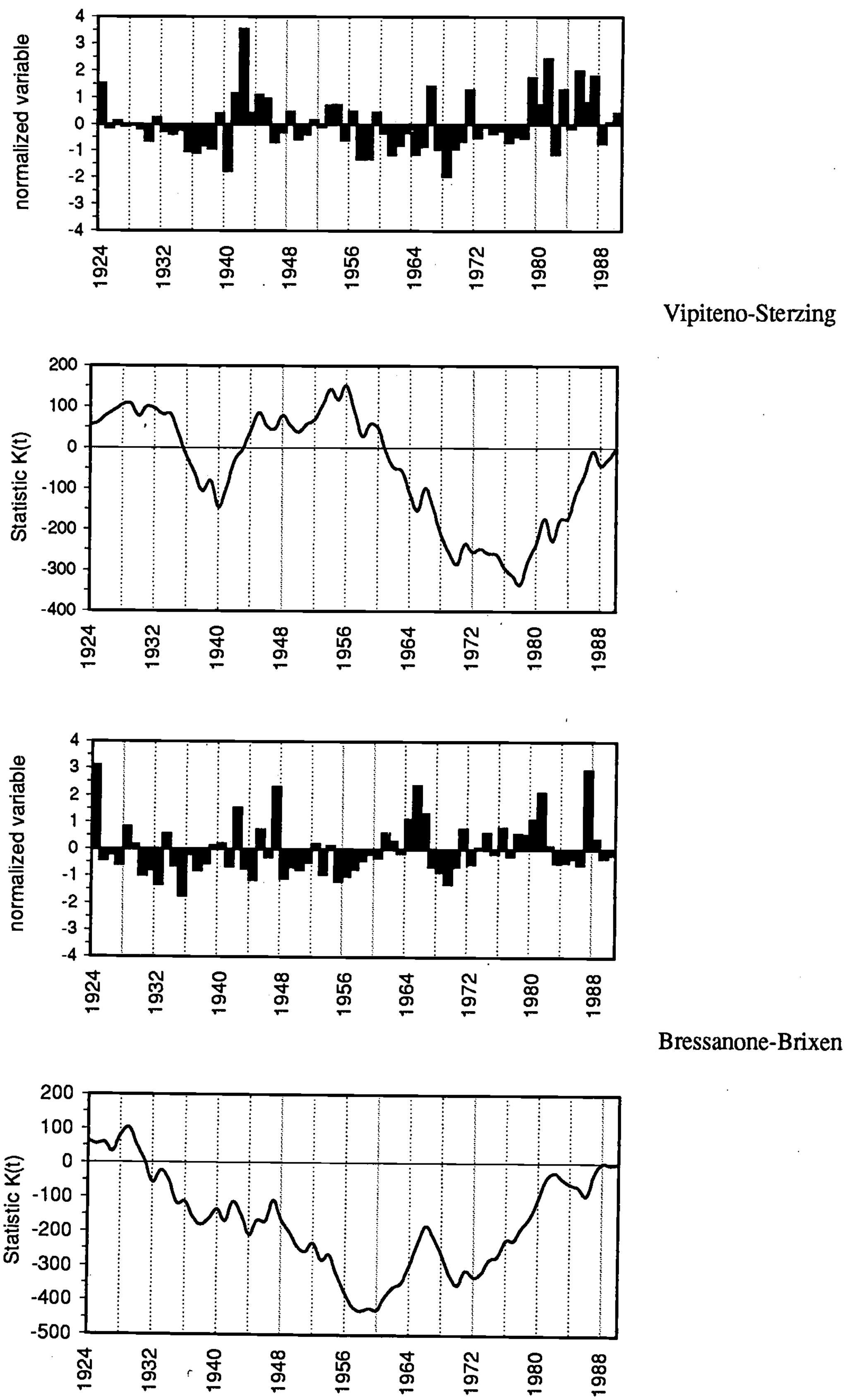

Figure 3. K(t) Statistic for the Time Series of Vipiteno-Sterzing, Bressanone-Brixen, and Udine.

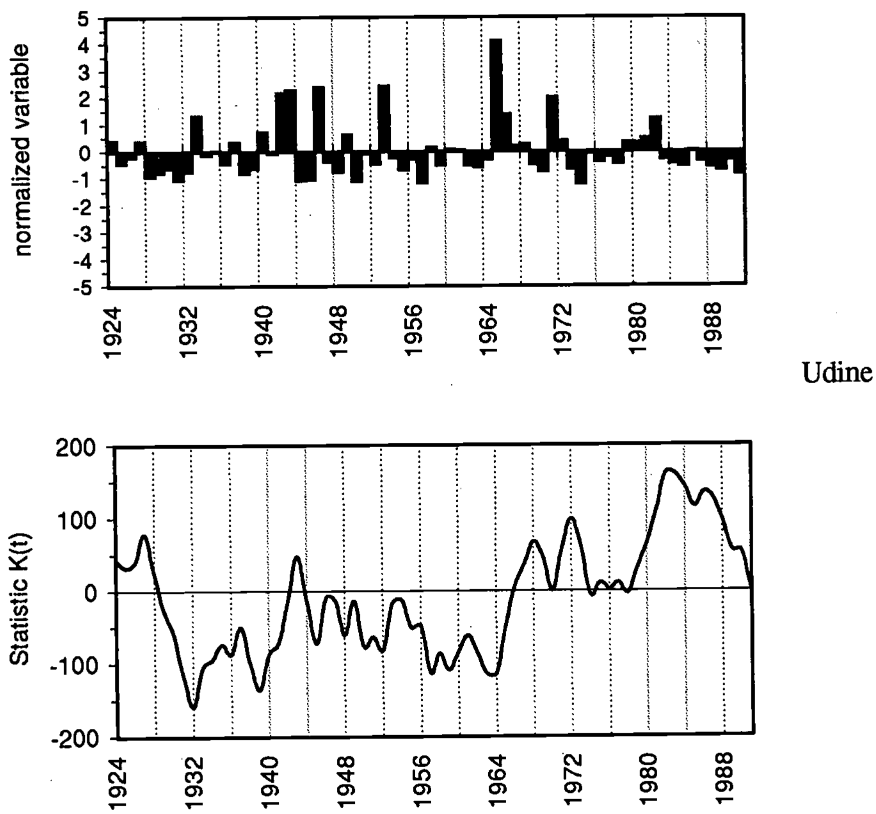

Figure 3. K(t) Statistic for the Time Series of Vipiteno-Sterzing, Bressanone-Brixen, and Udine (cont'd.)

PREDICTION

To verify the hypothesis $H_{0,4}$, the best predictor under the null hypothesis was compared with an alternative predictor. Of course if $H_{0,4}$ is true no one alternative predictor can be significantly better than P_0.

Under the null hypothesis H_0 the best predictor is $P_0 = E\left[x_t | \{x_s\}_{s=1}^{t-1} \{\varepsilon_s\}_{s=1}^{t-1}\right] = \bar{x}$, which is the sample average. The chosen alternative predictor P_0 comes from the chaotic system theory (Farmer and Sidorowich, 1987). It is a linear predictor with time varying parameters or, in other words, a local linear predictor, namely $x_t = \beta_0^{(t-1)} + \beta_1^{(t-1)} x_{t-1} + \ldots + \beta_d^{(t-1)} x_{t-d}$ (Lisi, 1995).

For each series, 15 one-step predictions were carried-out with both P_0 and P_1. To compare these predictions we have considered the index $E_j = \sum_i e_{i,j}^2 = \sum_i \left(\hat{x}_{j,i} - x_i\right)^2$, $(j = 0,1)$ where $\hat{x}_{ij}$ is the prediction of x_i with the j[th] predictor. Then the results were synthesized in the statistic $T = E_1/E_0$. It is easy to observe that when P_1 leads to better results than P_0, T is less than 1. The lesser is T, the better is P_1 with respect to P_0.

The results show that in several cases the statistic T is less than 1 (Table 4). However this does not mean that P_1 is also significantly better than P_0. To be sure the values of T are significative and that they are not due to chance, the following statistic, recently developed by Mizrach (1991) was used:

$$M = \sqrt{N_p}\,\frac{\dfrac{1}{N_p}\displaystyle\sum_{j=1}^{N_p} u_j v_j}{\left[\displaystyle\sum_{t=-h}^{h}\left(1 - [|t|/(h+1)]\right) S_{uvuv}(t)\right]^{1/2}}$$

where

$$S_{uvuv}(t) = \begin{cases} \dfrac{1}{N_p}\displaystyle\sum_{j=t+1}^{N_p} u_j v_j u_{j-t} v_{j-t} & \text{if } t \geq 0 \\ \dfrac{1}{N_p}\displaystyle\sum_{j=-t+1}^{N_p} u_{j+t} v_{j+t} u_j v_j & \text{if } t < 0 \end{cases}$$

and $u_j = e_{0j} - e_{1j}$; $v_j = e_{0j} + e_{1j}$; h is the order of dependence (we suppose that e_i is a mixing process), N_p is the number of predictions (in this case $N_p = 15$). Asymptotically, M has a normal standard distribution; moreover, to be applied, M does not require hypotheses of normality or independence of the errors. Since it is known (Mizrach, 1991) that this statistic is particularly conservative with respect to the null hypothesis, the test was performed at a level $\alpha = 0{,}1$.

This test, as shown in Table 4, rejects the null hypothesis in only three cases. The statistic M was calculated only for those stations where T is less than 1, since values of T greater than 1 indicate that P_0 is better than P_1. Bressanone and Monte Maria have similar values of T but very different values of M, due to differences in the variance of the two prediction series.

RESULTS AND CONCLUSIONS

It was stated that the global hypothesis H_0 is accepted only if all the sub-hypotheses are accepted. Looking at Table 5 this condition occurs only for the Udine series. It follows that only this one can be considered as purely random. The DGP of the remaining eight series, on the contrary, is not a WN process.

An exhaustive answer to the reason for these results requires further and specific study. However, it is possible to suppose as first hypothesis, that the general rejection of the could perhaps be attributed to changes occurring in the processes which govern the rainfall formation, particularly in the mountainside of the studied area.

The main scope of this study nevertheless was not the analysis of the features or the relationships between the weather systems and the orographic configuration of Eastern Italy; it was only to verify the effective randomness of time-series before using it.

Such a verification is also important from the point of view of its application, particularly in current hydrology practice in which statistical methods are well established among the various approaches towards the specification of the quantile, as already suggested by Krasovskaia and Gottschalk (1993). Differences in the population properties of the data, added to the normal sources of uncertainty of the same, can increase uncertainty, making the procurement of the necessary information in the field of design storm and flood less reliable.

TABLE 4. Results of Mizrach's Test at 10 Percent Level.

Station	T	M	p-Value	$H_{0,4}$
Monte Maria-Marienberg	0.882	-0.981	0.163	Accepted
Vipiteno-Sterzing	1.043	–	–	Accepted
Bressanone-Brixen	0.865	-1.530	0.063	Rejected
S.Martino in Badia-St.Martin in Thurn	0.949	-1.158	0.123	Accepted
Dobbiaco-Toblach	1.080	–	–	Accepted
Musi	0.478	-1.846	0.032	Rejected
Venzone	0.964	-0.281	0.389	Accepted
Udine	1.154	–	–	Accepted
Schio	0.689	-1.705	0.044	Rejected

TABLE 5. Results for the Global Hypothesis H_0 (R = Rejected, A = Accepted).

Station	$H_{0,1}$	$H_{0,2}$	$H_{0,3}$	$H_{0,4}$	H_0
Monte Maria-Marienberg	A	R	R	A	R
Vipiteno-Sterzing	A	R	R	A	R
Bressanone-Brixen	A	R	R	R	R
S.Martino in Badia-St. Martin in Thurn	A	R	R	A	R
Dobbiaco-Toblach	A	R	R	A	R
Musi	A	R	R	R	R
Venzone	A	R	R	A	R
Udine	A	A	A	A	A
Schio	A	R	R	R	R

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