MULTIPLE TREND BREAKS AND THE UNIT-ROOT HYPOTHESIS

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Abstract—Ever since Nelson and Plosser (1982) found evidence in favor of the unit-root hypothesis for 13 long-term annual macro series, observed unit-root behavior has been equated with persistence in the economy. Perron (1989) questioned this interpretation, arguing instead that the "observed" behavior may indicate failure to account for structural change. Zivot and Andrews (1992) restored confidence in the unit-root hypothesis by incorporating an endogenous break point into the specification. By allowing for the possibility of two endogenous break points, we find more evidence against the unit-root hypothesis than Zivot and Andrews, but less than Perron.

I. Introduction

FOR over a decade the unit-root hypothesis has served as the basis for testing the degree of persistence in the economy. Nelson and Plosser (1982) found evidence in favor of this hypothesis for 13 out of 14 long-term annual macro series. Perron (1989) suggested that the observed "unit-root" behavior may have been the result of failure to account for a structural change in the data and demonstrated this by including a dummy variable in the specification to allow for this structural change. In doing so, he reversed the Nelson and Plosser conclusions in 10 of the 13 series.

Subsequent literature, including Christiano (1992), Banerjee et al. (1992), Zivot and Andrews (1992), and Perron (1994), has incorporated an endogenous break point into the model specification. In particular, using asymptotic critical values, Zivot and Andrews fail to reject the unit-root hypothesis at the 5% level for four of the 10 Nelson–Plosser series—real per-capita GNP, the GNP deflator, money stock, and real wages—for which Perron rejects the hypothesis. With finite-sample critical values, they fail to reject the unit-root null at the 5% level for three more series—employment, nominal wages, and common stock prices.

To date the endogenous break literature has focused on testing the unit-root null against a one-break alternative, but it is far from obvious that one break is a good characteristic of long-term macro series. For example, Jones (1995) plots annual per-capita gross domestic product (GDP) for the United States from 1880 to 1987. Without any statistical examination it is plausible that there are two breaks, the first for the Great Depression and the second for World War II.

This paper extends the endogenous break methodology to allow for a two-break alternative. We then reexamine the unit-root hypothesis for the Nelson–Plosser data by considering the possibility that two break points occurred over the relevant time period. Even within the class of endogenous break models, results regarding tests of the unit-root hypothesis are sensitive to the number of breaks in the alternative specification. We find more evidence against the unit-root hypothesis than Zivot and Andrews (1992), but less than Perron (1989). Specifically, we can reject the unit-root hypothesis at the 5% level for seven of the 13 series and at the 10% level for two additional series. In particular, we reject the unit-root null at the 5% level for three of the seven Nelson–Plosser series for which Perron rejects, but Zivot and Andrews fail to reject, in favor of an alternative model with two breaks. These results illustrate the need for tests that are robust to misspecification with respect to the number of structural breaks.

Inference about the break points themselves is less sensitive than inference regarding persistence to the assumptions about the number of breaks. Perron imposed 1929 (the Great Crash), which was confirmed by Zivot and Andrews' endogenous one-break model for eight of the 13 series. Our evidence confirms these previous findings that the Great Crash is the major cause of the breaks. Most of the other breaks coincide with World War I or World War II.

The paper is organized as follows. Tests of the unit-root hypothesis against an alternative of trend stationarity with two endogenously determined breaks are developed in section II. These tests are applied to the Nelson–Plosser data in section III. Conclusions are presented in section IV.

II. Unit-Root Tests with a Two-Break Alternative Hypothesis

A. Theory

This section considers the behavior of sequences of Dickey–Fuller (1979) t-tests for a unit root. It is similar in spirit to the sequential tests for changes in coefficients of Banerjee et al. (1992), in the case where there is only one structural break. The statistic considered here is computed using the full sample, allowing for two shifts in the deterministic trend at distinct unknown dates.[1] The model considered is

$$\Delta y_t = \mu + \beta t + \theta DU1_t + \gamma DT1_t + \omega DU2_t + \psi DT2_t + \alpha y_{t-1} + \sum_{i=1}^{k} c_i \Delta y_{t-i} + \epsilon_t \tag{1}$$

for $t = 1, \ldots, T$, where $c(L)$ is a lag polynomial of known order k and $1 - c(L)L$ has all its roots outside the unit circle,

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[1] That is, it is assumed that the two dates are separated by a sample of data of positive measure. Intuitively such an assumption is necessary for identification of the breaks. For a more detailed discussion, see Bai and Perron (1995).

$DU1_t$ and $DU2_t$ are indicator dummy variables for a mean shift occurring at times $TB1$ and $TB2$, respectively, and $DT1_t$ and $DT2_t$ are the corresponding trend shift variables. That is, $DU1_t = \mathbf{1}(t > TB1)$, $DU2_t = \mathbf{1}(t > TB2)$, $DT1_t = (t - TB1)\mathbf{1}(t > TB1)$, and $DT2_t = (t - TB2)\mathbf{1}(t > TB2)$. In principle, this model also could include additional stationary regressors.

As in Banerjee et al. (1992), it is convenient to define transformed regressors $Z_t = [Z_t^1, 1, (y_t - \bar{\mu}_0 t), t+1, DU1_{t+1}, DU2_{t+1}, DT1_{t+1}, DT2_{t+1}]'$, where $Z_t^1 = (\Delta y_t - \bar{\mu}_0, \ldots, \Delta y_{t-k+1} - \bar{\mu}_0)$, and $\bar{\mu}_0 = E(\Delta y_t)$, and a transformed parameter vector Ξ, so that equation (1) can be rewritten $y_t = \Xi' Z_{t-1} + \epsilon_t$. This transformation is adopted from and discussed by Sims et al. (1990). Let $\Rightarrow$ denote weak convergence on $D[0, 1]$. The errors are assumed to satisfy the following assumption:

ASSUMPTION A: ϵ_t is a martingale difference sequence and satisfies $E(\epsilon_t^2 | \epsilon_{t-1}, \ldots) = \sigma^2$, $E(|\epsilon_t|^i | \epsilon_{t-1}, \ldots) = \kappa_i$ ($i = 3, 4$), and $\sup_t E(|\epsilon_t|^{4+\xi} | \epsilon_{t-1}, \ldots) = \bar{\kappa} < \infty$ for some $\xi > 0$.

Under Assumption A, $T^{-1/2} \sum_{t=1}^{[T\lambda]} \epsilon_t \Rightarrow \sigma W(\lambda)$, uniformly, for $\lambda \in [0, 1]$, where W is a standard one-dimensional Brownian motion. Also, as noted in Banerjee et al. (1992),

$$T^{-1} \sum_{t=1}^{T} Z_{t-1}^1 Z_{t-1}^{1'} \xrightarrow{p} \Omega_k, \qquad T^{-1/2} \sum_{t=1}^{T} Z_{t-1}^1 \epsilon_t \Rightarrow \sigma B(1),$$

$$T^{-3/2} \sum_{t=1}^{T} Z_{t-1}^1 y_t \Rightarrow 0$$

where Ω_k is a nonrandom positive semidefinite matrix, and $B(1)$ is a k-dimensional Brownian motion with covariance matrix Ω_k, independent of W.

The estimators and test statistics are computed using the full T observations for distinct pairs of values of (k_1, k_2) for $k_1 = k_0, k_0 + 1, \ldots, T - k_0$, and $k_2 = k_0, k_0 + 1, \ldots, T - k_0$, where $k_0 = [T\delta_0]$, $k_1 \neq k_2$, $k_1 \neq k_2 \pm 1$, and δ_0 represents some startup fraction of the sample. (Throughout this paper, we use $\delta_0 = 0.01$.) Define δ_1 and δ_2 as the fractions of the sample at which the first and second breaks, respectively, occur, that is, $\delta_1 = TB1/T$ and $\delta_2 = TB2/T$.

Because elements of Ξ converge at different rates, define the scaling matrix

$$\Upsilon_T = \mathrm{diag}(T^{1/2} I_k, T^{1/2}, T, T^{3/2}, T^{1/2}, T^{1/2}, T^{3/2}, T^{3/2})$$

partitioned conformably with Z_t and Ξ. For $\delta_0 \leq \delta_1, \delta_2 \leq 1 - \delta_0$, the sequential ordinary least-squares (OLS) estimator of the coefficient vector is $\hat{\Xi}(\delta_1, \delta_2) = (\Sigma_{t=1}^T Z_{t-1}([T\delta_1], [T\delta_2]) Z_{t-1}([T\delta_1], [T\delta_2]))^{-1} (\Sigma_{t=1}^T Z_{t-1}([T\delta_1], [T\delta_2]) y_t)$, and define

$$\Gamma_T(\delta_1, \delta_2) = \Upsilon_T^{-1} \sum_{t=1}^{T} Z_{t-1}([T\delta_1], [T\delta_2]) Z_{t-1}([T\delta_1], [T\delta_2])' \Upsilon_T^{-1}$$

and

$$\Psi_T(\delta_1, \delta_2) = \Upsilon_T^{-1}(\delta_1, \delta_2) \sum_{t=1}^{T} Z_{t-1}([T\delta_1], [T\delta_2]) \epsilon_t .$$

The following theorem provides asymptotic representations for the standardized coefficients.

THEOREM 1: Suppose that y_t is generated according to equation (1) with $\beta = \theta = \gamma = \omega = \psi = 0$ and $\alpha = 0$ and that Assumption A holds. Let $\Delta \equiv \max(\delta_1, \delta_2)$. Then $\Upsilon_T(\hat{\Xi}(\cdot) - \Xi) = \Gamma_T(\cdot)^{-1}\Psi_T(\cdot) \Rightarrow \Gamma(\cdot)^{-1}\Psi(\cdot)$, where $\Psi(\delta_1, \delta_2) = \sigma[B(1)', W(1), \int_0^1 J(s)dW(s), W(1) - \int_0^1 W(s)ds, W(1) - W(\delta_1), W(1) - W(\delta_2), (1-\delta_1)W(1) - \int_{\delta_1}^1 W(s)ds, (1-\delta_2)W(1) - \int_{\delta_2}^1 W(s)ds]'$ and $\Gamma_{11} = \Omega_k$, $\Gamma_{1j} = 0$ ($j = 2, \ldots, 8$), $\Gamma_{22} = 1$, $\Gamma_{32} = \int_0^1 J(s)ds$, $\Gamma_{33} = \int_0^1 J^2(s)ds$, $\Gamma_{42} = ½$, $\Gamma_{43} = \int_0^1 sJ(s)ds$, $\Gamma_{44} = ⅓$, $\Gamma_{i,2:4} = [1 - \delta_k, \int_{\delta_k}^1 J(s)ds, ½(1 - \delta_k^2)]$ and $\Gamma_{ii} = \Gamma_{i2}$ (for $i = 5, 6$ and $k = i - 4$), $\Gamma_{i,2:4} = [½(1-\delta_k)^2, \int_{\delta_k}^1 (s - \delta_k)J(s)ds, ⅙(\delta_k^3 - 3\delta_k + 2)]$, $\Gamma_{i,i-2} = \Gamma_{i2}$, $\Gamma_{ii} = ⅓(1 - \delta_k)^3$, and $\Gamma_{i,13-i} = ½(1 - \Delta^2) - \delta_k(1 - \Delta)$ (for $i = 7, 8$, and $k = i - 6$), $\Gamma_{65} = 1 - \Delta$, and $\Gamma_{87} = ⅓ - ½(\delta_1 + \delta_2)(1 - \Delta^2) + \delta_1\delta_2(1 - \Delta) - ⅓\Delta^3$, where $\Gamma(\delta_1, \delta_2)$ is symmetric, $W(\delta)$ is a standard Brownian motion process, $J(s) = \sigma b W(s)$, and $b = [1 - c(1)]^{-1}$.

Proof: The proof obtains by direct calculation, following arguments in Banerjee et al. (1992).

There are several things to note:

1. We consider the unit-root hypothesis that $\alpha = 0$.[2] The test statistic of interest is the t-statistic associated with this hypothesis. Its asymptotic distribution can be stated[3] as $\hat{t}$

[2] Due to the empirical question investigated here, an assumption we have made is that there is no break under the null hypothesis of a unit root. In other contexts it might be desirable to test for a break without making assumptions about whether or not there is a unit root, as proposed by Perron (1991) and Vogelsang (1994).

[3] We thank an anonymous referee for suggesting this expression for the t-statistic, which follows the representations for models with one break derived in Zivot and Andrews (1992) and Perron (1994). The referee also pointed out that a key benefit of this representation is that no trimming is required and the break points need not necessarily be distinct in order to derive the asymptotic distribution for this particular statistic (the t-statistic on α). That this is also true for the two-break case is a plausible conjecture. The proof approach we take instead provides the joint limiting representations of all the OLS estimators, conditional on a fixed choice of the trimming parameter δ_0, as these may be useful in other contexts (constructing, for instance, sequential F-statistics as in Banerjee et al. (1992)).

$\hat{}(\delta_1, \delta_2) \Rightarrow \int_0^1 W^*(s)dW(s)/[\int_0^1 W^*(s)^2 ds]^{1/2}$, where W^* is the continuous-time residual from a projection of a Brownian motion onto the functions $\{1, s, \mathbf{1}(s > \delta_1), \mathbf{1}(s > \delta_2), (s - \delta_1)\mathbf{1}(s > \delta_1), (s - \delta_2)\mathbf{1}(s > \delta_2)\}$.

2. If $DU2$ and $DT2$ are omitted from equation (1), this is Zivot and Andrews' (1992) model C. If in addition $DT1$ is omitted, this is their model A while if $DU1$ is omitted, this is their model B. The nomenclature for their models is similar to that used by Perron (1989) with δ_1 fixed.[4] Extending this convention, therefore, we refer to equation (1) as model CC, with model AA corresponding to equation (1) with $DT1$ and $DT2$ omitted and model CA corresponding to equation (1) with $DT2$ omitted. The limiting distributions for models AA and CA, therefore, are of the form expressed in Theorem 1, omitting the relevant rows and columns, with matrix and vector expressions reflecting the appropriate reduction in dimension.
3. As stated above, we rule out the possibility that the two breaks occurred on consecutive dates. That is, we do not consider a positive shock followed by a negative shock (or vice versa) as being two separate episodes.
4. As in Banerjee et al. (1992), this result applies for $0 < \delta_0 \le \delta_1, \delta_2 \le (1 - \delta_0) < 1$. Thus the test for the change in the coefficients is constrained not to be at the ends of the sample. In practice, this requires choosing a "trimming" value, $k_0 = [T\delta_0]$. In the empirical section we follow Zivot and Andrews (1992) and estimate equation (1) for values of δ_1 and δ_2 between $2/T$ and $(T - 1)/T$. Note that in estimation of model CA, we do not impose $\delta_1 < \delta_2$; we allow δ_1 and δ_2 to vary over the entire range, with δ_1 denoting a joint mean and trend break and δ_2 denoting a mean break.
5. This result provides joint uniform convergence of all the estimators and test statistics. Thus the asymptotic representation of continuous functions of these processes is obtained via the continuous mapping theorem. In this paper we will use $\min_{k_0 \le k_1, k_2 \le T - k_0} \hat{t}(k_1/T, k_2/T; \delta_0)$, that is, the minimum of the sequence of t-statistics computed over the two-dimensional grid of possible combinations of k_1 and k_2. By assumption, $k_1 \neq k_2$ (if $\delta_1 = \delta_2$, this is a one-break model and Γ is noninvertible) and by note 3 above, $k_1 \neq k_2 \pm 1$.
6. The results of the theorem hold for a fixed (known) value of k and not necessarily for an estimated k, as we use in the empirical section. We have chosen to follow Campbell and Perron (1991), Zivot and Andrews (1992), and Perron (1994) in designating a maximum lag length and using an augmented Dickey–Fuller procedure to determine the lag length for each series. However, as noted by Zivot and Andrews (1992) in the one-break context, the extension of these results to the case of $k \xrightarrow{p} \infty$ is nontrivial and beyond the scope of this paper.

B. *Critical Values*

We compute critical values three ways:

1. Critical values were computed for models AA, CA, and CC, using 125 observations and 500 replications. We use the method of endogenously determining the appropriate lag length in computing critical values, with a maximum of eight lags. The data were generated under the null hypothesis, that is, $\Delta y_t = \epsilon_t$.
2. We also computed critical values using the exact number of observations in each series. The results are qualitatively the same using these critical values.
3. We follow Zivot and Andrews' procedure of bootstrapping finite-sample critical values by estimating ARMA models for each series (under the null hypothesis of no break) and assuming that the estimated ARMA model was the true data-generating process. This captures possible finite-sample dependence. Results using these critical values are again qualitatively the same, except as noted below. A complete set of results using each of the critical values is available on request from the authors.

III. Unit Roots in the Nelson–Plosser Data

We apply the tests developed above to the long-term macro series for the United States analyzed first by Nelson and Plosser (1982) and subsequently by Perron (1989) and Zivot and Andrews (1992). As in the previous section, we use 1% trimming ($\delta_0 = 0.01$) and estimate the model over distinct pairs of values (k_1, k_2), with $k_1 \neq k_2$ and $k_1 \neq k_2 \pm 1$. The data consist of annual observations for 14 series, beginning between 1860 and 1909, and ending in 1970. Since Nelson and Plosser found, using augmented Dickey–Fuller tests, the unemployment rate to be stationary, we follow Perron and Zivot and Andrews and consider the remaining 13 series.[5] In order to facilitate comparison with our work, some results of Perron and of Zivot and Andrews are summarized in table 1. The unit-root null is rejected by Perron for 10 of the 13 series at the 5% significance level, while in Zivot and Andrews the null is rejected, using asymptotic critical values, for 6 series at the 5% level and one more at the 10% level.[6]

[4] Perron's null models additionally include a crash dummy (equal to 1 at the date of the break and 0 elsewhere) and/or a mean-shift dummy variable (equal to $DU1_{1929}$ above). For model B, Perron uses a two-step estimation procedure.

[5] Not surprisingly, our tests also reject the unit-root null for unemployment.

[6] The evidence in Zivot and Andrews (1992) is weakened using bootstrapped critical values (as in computation 3 above), where the null is rejected for only three series at the 5% level and two more at the 10% level. Perron (1994), using additional tests and lag length selection criteria, finds some evidence against the unit-root hypothesis in the Nelson–Plosser data.

TABLE 1.—SINGLE-BREAK UNIT-ROOT TESTS
$\Delta y_t = \mu + \beta t + \theta DU_t + \gamma DT_t + \alpha y_{t-1} + \Sigma_{i=1}^{k} c_i \Delta y_{t-i} + \epsilon_t$

Series	T	Model	Perron (1989) t_α	Perron (1989) Reject	Zivot and Andrews (1992) Break	Zivot and Andrews (1992) t_α	Reject Asymptotic	Reject Finite
Real GNP	62	A	−5.03	1%	1929	−5.58	1%	5%
Nominal GNP	62	A	−5.42	1%	1929	−5.82	1%	2.5%
Per-capita real GNP	62	A	−4.09	2.5%	1929	−4.61	10%	
Industrial production	111	A	−5.47	1%	1929	−5.95	1%	1%
Employment	81	A	−4.51	1%	1929	−4.95	5%	
GNP deflator	82	A	−4.04	2.5%	1929	−4.12		
Consumer prices	111	A	−1.28		1873	−2.76		
Nominal wages	71	A	−5.41	1%	1929	−5.30	2.5%	10%
Real wages	71	C	−4.28	5%	1940	−4.74		
Money stock	82	A	−4.29	2.5%	1929	−4.34		
Velocity	102	A	−1.66		1949	−3.39		
Interest rate	71	A	−0.45		1932	−0.98		
Common stock prices	100	C	−4.87	2.5%	1936	−5.61	1%	10%

Notes: The columns labeled "reject" indicate the significance level at which the unit-root hypothesis can be rejected. A blank space indicates that the null cannot be rejected at the 10% level. "Asymptotic" and "finite" indicate that the rejections are based on asymptotic and finite sample critical values, respectively. $DU_t = \mathbf{1}\ (t > TB)$ and $DT_t = (t - TB)\mathbf{1}\ (t > TB)$. Model A does not include DT_t. Perron's regressions include an additional dummy variable, $D(TB)_t = \mathbf{1}\ (t = TB + 1)$.

Since considerable evidence exists that data-dependent methods to select the value of the truncation lag k are superior to choosing a fixed k a priori, we follow Zivot and Andrews (1992) and use the procedure suggested by Perron (1989).[7] Start with an upper bound k_{max} for k. If the last included lag is significant, choose $k = k_{max}$. If not, reduce k by 1 until the last lag becomes significant. If no lags are significant, set $k = 0$. We set $k_{max} = 8$ and use the (approximate) 10% value of the asymptotic normal distribution, 1.60, to assess the significance of the last lag.[8]

An issue which has not received much attention in the literature involves determining which model to select. Perron (1989) estimates model A for all variables except real wages and common stock prices, for which he estimates model C. Zivot and Andrews (1992) follow Perron's choices. We estimate models AA, CA, and CC for all 13 series. In the absence of a statistically accepted procedure for selecting among models, we extend Zivot and Andrews' choice of models and select model CC for real wages and stock prices and model AA for the others when comparing our results to theirs.

The results for model AA, where both breaks in the trend function are restricted to the intercept, are reported in table 2. The unit-root null is rejected in favor of the two-break alternative at the 1% level for nominal GNP and industrial production, at the 2.5% level for real GNP, per-capita real GNP, and employment, and at the 10% level for the money supply. The results also illustrate the importance of the Great Crash. 1929 is one of the break years for all five series for which the unit-root null can be rejected at the 2.5% level.[9]

[7] Ng and Perron (1995) use simulations to show that these sequential tests have an advantage over information-based methods since the former produce tests with more robust size properties without much loss of power.

[8] Following Perron (1989) and Zivot and Andrews (1992), we do not increase the upper bound when the procedure selected $k = k_{max}$.

[9] Results using bootstrapped critical values (as in computation 3 above) are qualitatively similar, although the significance levels are weaker. The unit-root null is rejected at the 5% level for real GNP, nominal GNP, per-capita real GNP, industrial production, and employment, and at the 10% level for the money stock.

TABLE 2.—MODEL AA
$\Delta y_t = \mu + \beta t + \theta DU1_t + \omega DU2_t + \alpha y_{t-1} + \Sigma_{i=1}^{k} c_i \Delta y_{t-i} + \epsilon_t$

Series	TB1 TB2	α	θ	ω	k
Real GNP	1929	−0.479	−0.148	0.122	1
	1940	(−6.65)[b]	(−4.98)	(4.12)	
Nominal GNP	1920	−0.531	−0.180	−0.325	8
	1929	(−7.42)[a]	(−3.83)	(−6.17)	
Per-capita real GNP	1929	−0.529	−0.140	0.150	2
	1939	(−6.67)[b]	(−4.76)	(4.66)	
Industrial production	1918	−0.914	−0.166	−0.363	8
	1929	(−7.78)[a]	(−4.51)	(−6.36)	
Employment	1929	−0.596	−0.106	−0.054	8
	1956	(−6.83)[b]	(−5.32)	(−3.72)	
GNP deflator	1929	−0.204	−0.085	0.046	1
	1945	(−4.64)	(−3.44)	(2.35)	
Consumer prices	1915	−0.103	0.062	0.070	5
	1945	(−4.41)	(3.22)	(3.46)	
Nominal wages	1930	−0.416	−0.163	0.059	7
	1949	(−5.59)	(−3.82)	(1.94)	
Real wages	1940	−0.479	0.094	0.060	1
	1954	(−6.06)[d]	(4.79)	(3.57)	
Money stock	1930	−0.362	−0.157	−0.078	8
	1958	(−6.03)[d]	(−4.56)	(−3.37)	
Velocity	1884	−0.283	−0.094	0.113	1
	1949	(−4.77)	(−3.05)	(3.76)	
Interest rate	1932	−0.169	−0.615	1.034	0
	1967	(−3.46)	(−4.30)	(6.18)	
Common stock prices	1930	−0.372	−0.119	0.362	1
	1953	(−5.52)	(−2.10)	(4.60)	

Notes: The critical values are −6.94 (1%), −6.53 (2.5%), −6.24 (5%), and −5.96 (10%). t-statistics are in parentheses.
[a] Significant at the 1% level.
[b] Significant at the 2.5% level.
[c] Significant at the 5% level.
[d] Significant at the 10% level.

Model CC allows for two breaks in both the intercept and the slope of the trend function. The results for this model, reported in table 3, show that allowing slope breaks provides little additional evidence against the unit-root hypothesis.

TABLE 3.—MODEL CC

$$\Delta y_t = \mu + \beta t + \theta DU1_t + \gamma DT1_t + \omega DU2_t + \psi DT2_t + \alpha y_{t-1} + \sum_{i=1}^{k} c_i \Delta y_{t-i} + \epsilon_t$$

Series	TB1 TB2	α	θ	γ	ω	ψ	k
Real GNP	1931 1945	−0.619 (−6.80)[d]	−0.256 (−4.98)	0.031 (5.91)	−0.119 (−3.43)	−0.023 (−5.11)	1
Nominal GNP	1920 1929	−0.530 (−7.38)[a]	−0.218 (−2.84)	0.029 (0.62)	−0.360 (−6.04)	−0.011 (−1.27)	8
Per-capita real GNP	1931 1945	−0.624 (−6.80)[d]	−0.248 (−4.84)	0.035 (6.12)	−0.130 (−3.67)	−0.028 (−5.52)	2
Industrial production	1918 1929	−0.958 (−7.51)[a]	−0.169 (−2.90)	−0.003 (−0.32)	−0.352 (−5.47)	0.001 (0.15)	8
Employment	1929 1956	−0.618 (−6.40)	−0.113 (−5.28)	0.000 (0.14)	−0.045 (−2.37)	−0.002 (−1.06)	8
GNP deflator	1915 1940	−0.289 (−6.63)[d]	0.158 (5.36)	−0.009 (−5.27)	0.121 (5.22)	0.012 (5.87)	1
Consumer prices	1915 1945	−0.233 (−5.76)	0.128 (5.24)	0.001 (1.34)	0.060 (2.82)	0.005 (3.67)	5
Nominal wages	1915 1940	−0.276 (−6.36)	0.191 (5.23)	−0.005 (−1.35)	0.143 (4.91)	0.012 (5.36)	7
Real wages	1922 1940	−0.899 (−6.63)[d]	0.063 (2.94)	−0.003 (−1.33)	0.131 (5.60)	0.012 (4.82)	1
Money stock	1930 1958	−0.398 (−6.09)	−0.175 (−4.84)	−0.001 (−0.38)	−0.049 (−1.60)	−0.005 (−1.47)	8
Velocity	1897 1929	−0.478 (−6.37)	−0.025 (−0.81)	0.012 (4.79)	−0.147 (−4.49)	0.006 (3.73)	1
Interest rate	1932 1965	−0.185 (−3.70)	−0.607 (−4.34)	0.012 (2.01)	−0.077 (−0.33)	0.258 (3.57)	0
Common stock prices	1926 1939	−0.567 (−6.11)	0.267 (2.72)	−0.029 (−2.62)	−0.148 (−1.55)	0.066 (4.56)	1

Notes: The critical values are −7.34 (1%), −7.02 (2.5%), −6.82 (5%), and −6.49 (10%). *t*-statistics are in parentheses.
[a] Significant at the 1% level.
[b] Significant at the 2.5% level.
[c] Significant at the 5% level.
[d] Significant at the 10% level.

Except for the GNP deflator, the unit-root null cannot be rejected at a higher significance level with model CC than with model AA and, for four series, the unit-root null cannot be rejected at the same level. This may be because the power of these tests declines when unnecessary breaks are included.

These results, like Perron's, illustrate the rule in Campbell and Perron (1991) that nonrejection of the unit-root hypothesis may be due to misspecification of the deterministic trend. The most direct comparison of our results is with Zivot and Andrews (1992), who can only reject the unit-root hypothesis for three of the 13 series at the 5% level and two more at the 10% level using bootstrapped (as in computation 3 above) critical values. By incorporating two, rather than one, trend breaks, we reject the unit-root null with bootstrapped critical values for five series at the 5% level and two more at the 10% level.

Our results can be divided into three categories. For the three series—real GNP, nominal GNP, and industrial production—for which the unit-root hypothesis is rejected (at the 5% level) by both Perron (1989) and Zivot and Andrews (1992), the null is also rejected by our two-break model. There are seven series for which Perron rejects, but Zivot and Andrews fail to reject, the unit-root null. Among these seven series, we reject the unit-root null for per-capita real GNP and employment and fail to reject (at the 5% level) the null for the GNP deflator, nominal wages, real wages, the money stock, and common stock prices. Finally there are three series for which both Perron and Zivot and Andrews fail to reject the unit-root null. Extending Zivot and Andrews' models to two breaks does not change this conclusion.

Of course there is little justification for the assumption that both breaks were of the same form. As an alternative, model CA allows for one break in both the intercept and the slope (*TB*1), and the second in just the intercept (*TB*2), of the trend function. The results, reported in table 4, are similar to those of model AA for most of the series. The unit-root hypothesis can be rejected at the 1% level for nominal GNP and industrial production, at the 2.5% level for employment,

TABLE 4.—MODEL CA

$$\Delta y_t = \mu + \beta t + \theta DU1_t + \gamma DT1_t + \omega DU2_t + \alpha y_{t-1} + \sum_{i=1}^{k} c_i \Delta y_{t-i} + \epsilon_t$$

Series	TB1 TB2	α	θ	γ	ω	k
Real GNP	1929 1940	−0.550 (−6.90)[c]	−0.150 (−4.74)	0.002 (0.94)	0.121 (4.03)	2
Nominal GNP	1929 1940	−0.529 (−7.43)[a]	−0.356 (−6.06)	−0.010 (−1.16)	−0.238 (−3.47)	8
Per-capita real GNP	1929 1939	−0.545 (−6.65)[c]	−0.131 (−4.16)	0.002 (0.82)	0.147 (4.53)	2
Industrial production	1929 1918	−0.946 (−7.76)[a]	−0.361 (−6.31)	−0.001 (−0.99)	−0.183 (−4.51)	8
Employment	1941 1929	−0.603 (−7.14)[b]	0.052 (3.11)	−0.004 (−4.08)	−0.118 (−5.67)	8
GNP deflator	1930 1915	−0.418 (−6.61)[d]	−0.128 (−4.47)	0.008 (4.81)	0.147 (4.76)	8
Consumer prices	1930 1915	−0.217 (−6.91)[c]	−0.051 (−3.33)	0.007 (6.32)	0.138 (6.00)	8
Nominal wages	1940 1915	−0.269 (−6.20)	0.132 (4.69)	0.012 (5.16)	0.203 (5.70)	1
Real wages	1931 1940	−0.676 (−6.48)[d]	−0.064 (−3.59)	0.004 (3.20)	0.089 (4.56)	3
Money stock	1930 1942	−0.397 (−6.96)[c]	−0.176 (−5.52)	−0.008 (−4.81)	0.151 (4.67)	6
Velocity	1929 1884	−0.406 (−5.87)	−0.106 (−3.68)	0.007 (4.65)	−0.136 (−4.06)	1
Interest rate	1964 1932	−0.204 (−4.04)	−0.242 (−1.15)	0.269 (4.78)	−0.639 (−4.50)	0
Common stock prices	1936 1953	−0.484 (−6.05)	−0.249 (−3.29)	0.018 (3.18)	0.224 (2.28)	3

Notes: The critical values are −7.24 (1%), −7.02 (2.5%), −6.65 (5%), and −6.33 (10%). t-statistics are in parentheses.
[a] Significant at the 1% level.
[b] Significant at the 2.5% level.
[c] Significant at the 5% level.
[d] Significant at the 10% level.

at the 5% level for real GNP, per capita real GNP, consumer prices, and money supply, and at the 10% level for the GNP deflator and real wages.[10]

The differences between the models fall into two categories. For consumer prices and the GNP deflator, where the unit-root null could not be rejected at the 10% level with model AA, the null can be rejected at the 5 and 10% levels, respectively. For these series, the inability to reject the unit root hypothesis with model AA can be attributed to the failure to properly model breaks in the trend function. In addition, the null for the money stock can be rejected at the 5% level (with model CA), compared to the 10% level with model AA. The second category consists of two series, real GNP and per-capita real GNP, where model CA provides less evidence against the unit-root hypothesis than model AA.

As mentioned above, there is no clearly accepted way to distinguish between the models. Suppose that instead of extending Zivot and Andrews' (1992) choice of models, we choose the model that is most negative to the unit-root hypothesis. In this case we reject the null hypothesis for seven series at the 5% level and two more at the 10% level.

IV. Conclusions

This paper has attempted to resume debate regarding the relationship between the unit-root hypothesis and structural breaks. We have shown that the econometric theory of endogenous one-break models extends to the case of two breaks. Limiting distributions allow standard inference and we have computed critical values with which to consider our empirical results. In particular, we have used a well-known example to illustrate that inference related to unit roots is sensitive to the number of assumed structural breaks. We have shown that the results obtained using one endogenous

[10] As with model AA, results using bootstrapped critical values (as in computation 3 above) for model CA are qualitatively similar, although the significance level is weaker (rejection at the 10% level) for real GNP, per-capita real GNP, and industrial production; the null hypothesis is no longer rejected for real wages. Rejection significance levels for consumer prices, the money supply, nominal GNP, employment, and the GNP deflator are the same as the levels obtained using critical values 1 above.

break are often reversed when a model with two breaks is estimated. The Nelson–Plosser data seem to exhibit two breaks; these are most often centered around the Great Depression, World War I, and World War II.

An obvious criticism of this approach is that we have little reason to expect that there have been exactly two breaks in the economy over the last century. In addition, our results do not address the possibility that even higher order models are more appropriate. This begs the question of where to go next—to a model with three breaks? In response to such criticism, we reiterate that the focus of this paper is not to assert a preference for models with a specific number of breaks. Indeed similar dissatisfaction also applies to previous work that has assumed only one break. We provide these results to suggest and emphasize that, rather than narrowly considering alternatives with a specific number of breaks, subsequent literature should focus on model selection, in determining both the number of breaks and also the type of break. The need for such results is substantial. Vogelsang (1994) shows that power is nonmonotonic when a one-break model is estimated on data that contain two breaks. This is similar to the original point made by Perron (1989); models that do not account sufficiently for structural change are misspecified and inferences may then suggest excessive persistence.

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