ELSEVIER

Economics Letters 50 (1996) 175–178

economics letters

Spurious number of breaks

Luis C. Nunes[a,*], Paul Newbold[b], Chung-Ming Kuan[c]

[a]Universidade Nova de Lisboa, Tv. Estevão Pinto, 1000 Lisboa, Portugal
[b]University of Nottingham, Nottingham NG7 2RD, UK
[c]National Taiwan University, Taipei 10020, Taiwan

Received 17 January 1995; accepted 16 June 1995

Abstract

This paper considers the problem of choosing the number of breaks in the mean or trend of a series. We take the Schwarz–Bayesian criterion (SBC) which can be considered either as a proxy or as an alternative for what might be inferred from visual inspection of the graph of a time series. We show that spuriously many breaks will be inferred when the process is integrated of order one without breaks.

Keywords: Structural change; Unit-root; Model selection

JEL classification: C22

1. Introduction

In the past few years the question of the number of breaks in the level or trend of a time series has impinged on the literature of testing for unit autoregressive roots. For example, in the analysis of a collection of U.S. macroeconomic time series, Nelson and Plosser (1982) assumed no breaks. However, both Perron (1989) and Zivot and Andrews (1992) allowed one break. In the former, the break date was treated as exogenous, while in the latter it was endogenous. It emerges that inference about unit autoregressive roots depends critically on the number of break dates permitted. Since the existence or not of structural breaks appears to be critical, there is, in principle, no reason to be content with a maximum of one possible break.

In practice, the decision about the number of breaks appears to have been made through visual inspection of the graph of a series. This approach was adopted by Perron (1989), and retained by Zivot and Andrews (1992), in assessing the possible nature of a break. It is, of course, impossible formally to analyze the properties of visual selection of this type. Here we

* Corresponding author. Tel.: +351-1-693624, ext. 1363; fax: +351-1-388-6073; e-mail: lcn@feunl.fe.unl.pt.

SSDI 0165-1765(95)00746-6

take the Schwarz–Bayesian criterion (SBC) as the order selection criterion. This approach assumes that the generating model is a mean or trend plus a white noise, and computes the SBC on that basis. One can consider this criterion either as a proxy or as an alternative for what might be inferred from visual selection.

Our main result is that adopting this SBC when the generating process is a random walk without breaks leads in the limit to an estimated number of breaks equal to the maximum permitted. We present simulation evidence to support this conclusion for moderate-sized samples. This result is related to others in Reichlin (1989), Hendry and Neale (1991) and Nunes et al. (1994), where it is shown that a spurious structural change will be estimated when the true process is I(1).

2. Spurious number of breaks

Consider the following simple model of structural change: $y_t = x_t' \beta_t + \epsilon_t$, ϵ_t is i.i.d. $N(0, \sigma^2)$ with $x_t \in \mathbb{R}^m$ for $t = 1, \ldots, T$, and

$$\beta_t = \begin{cases} \beta^1, & 0 < t \leq k_1, \\ \beta^r, & k_{r-1} < t \leq k_r, \quad r = 2, \ldots, R, \\ \beta^{R+1}, & k_R < t \leq T, \end{cases}$$

where the R break points are $0 < k_1 < \cdots < k_R < T$, and $k_r = [T\lambda_r]$, $0 < \lambda_1 < \cdots < \lambda_R < 1$. For example, in the case of a change in mean we have $x_t = 1$, and for a change in trend we have $x_t = (1, t)'$. It is easily seen that the maximum likelihood estimator of σ^2 when there are R break points is

$$\hat{\sigma}_T(R)^2 = \min_{0<k_1<\cdots<k_R<T} \frac{1}{T} \sum_{t=1}^{T} y_t^2 - \frac{1}{T} \sum_{r=1}^{R+1} \left(\sum_{t=k_{r-1}+1}^{k_r} y_t x_t' \right) \left(\sum_{t=k_{r-1}+1}^{k_r} x_t x_t' \right)^{-1} \left(\sum_{t=k_{r-1}+1}^{k_r} x_t y_t \right), \tag{1}$$

where $k_0 = 0$ and $k_{R+1} = T$.

Using the SBC, the estimated number of break points $\hat{R}$ is found by the solution to

$$\min_R \mathrm{SBC}(R) = \log(\hat{\sigma}_T(R)^2) + [m + R(m+1)] \frac{\log T}{T}, \tag{2}$$

subject to $R \leq R_U$, with R_U a given (fixed) upper bound for R. Yao (1988) shows that for the change in mean model, $\hat{R}$ is a consistent estimator of R_0, the true number of breaks, provided $R_0 \leq R_U$ and ϵ_t is normally distributed. The results in Table 1 support this conclusion for the case $R_0 = 0$.

Let us now suppose that the true data-generating process (DGP) for y_t is given by a random walk and there is no structural change:

$$y_t = y_{t-1} + e_t, \quad t = 1, \ldots, T,$$

Table 1
Percentage of breaks selected by the SBC

		DGP is a random walk with no breaks				DGP is a white noise with no breaks			
		Change in mean model		Change in trend model		Change in mean model		Change in trend model	
R_U	$\hat{R}$	$T = 50$	$T = 100$	$T = 50$	$T = 100$	$T = 50$	$T = 100$	$T = 50$	$T = 100$
1	0	1	0	1	0	90	97	84	90
	1	99	100	99	100	10	3	16	10
2	0	0	0	0	0	85	95	83	90
	1	3	1	3	0	9	3	11	9
	2	97	99	97	100	6	2	6	1
3	0	0	0	0	0	71	85	82	88
	1	1	0	4	0	8	7	12	10
	2	8	1	10	1	7	2	3	2
	3	91	99	86	99	14	6	3	0
4	0	0	0	0	0	68	80	79	87
	1	1	0	2	0	8	5	10	10
	2	6	0	9	1	6	4	2	2
	3	15	1	16	1	8	5	2	1
	4	78	99	73	98	10	6	7	0

with e_t obeying a functional central limit theorem. From (1) and the continuous mapping theorem, it is easy to see that

$$\frac{\hat{\sigma}_T(R)^2}{T} \Rightarrow \min_{0<\lambda_1<\cdots<\lambda_R<1} \int_0^1 W(\lambda_1, \ldots, \lambda_R, s)^2 \, \mathrm{d}s \,, \tag{3}$$

where $W(\lambda_1, \ldots, \lambda_R, s)$ is the projection residual of a standard Wiener process $W(s)$ on the subspace generated by the functions $\{\mathbf{1}_{s\in(\lambda_{r-1},\lambda_r)}\}_{r=1}^{R+1}$ for the change in mean model, or the functions $\{\mathbf{1}_{s\in(\lambda_{r-1},\lambda_r)}, s\mathbf{1}_{s\in(\lambda_{r-1},\lambda_r)}\}_{r=1}^{R+1}$ for the change in trend model, with $\lambda_0 = 0$ and $\lambda_{R+1} = 1$.

Since the second term in (2) goes to 0 as T increases, (3) implies that for any fixed R, only the first term in (2) matters asymptotically. Since $\hat{\sigma}_T(R)^2$ is monotonically decreasing in R for any given data set, SBC(R) is minimized at $R = R_U$ for T large enough. Therefore, if we allow for R_U break points, for a large enough sample size T, the SBC will suggest that a random walk is a series generated by a stationary process with R_U break points. This situation is considered in Table 1.

In small samples the estimator of the number of break points will have some distribution on the set $\{0, 1, \ldots, R_U\}$. In Table 1 we present some simulation results based on 5000 replications. We considered two cases for the DGP: a random walk without breaks: Δy_t i.i.d. N(0,1); and a white noise also without breaks: y_t i.i.d. N(0,1). We considered both the change in mean model and the change in trend model. The maximum possible number of breaks, R_U, was set at 1, 2, 3 and 4. For samples of 50 and 100 observations, the results for the change in mean and change in trend models are very similar. When the DGP is a random walk, the SBC selects the maximum permitted number of breaks on the overwhelming majority of occasions. On the other hand, we see that this criterion correctly selects zero breaks a very large percentage of the time when the DGP is a white noise.

3. Conclusion

To the extent that the SBC accurately reflects what would be inferred from visual inspection, it appears that such inspection will provide a very unreliable guide to determining the number of breaks when the DGP is integrated of order one. Indeed, it appears that in that case spuriously many breaks will be inferred. Our analysis reinforces the conclusion that separate determination of the number of breaks in, and the order of integration of, a time series is likely to prove very difficult in practice.

References

Hendry, D.F. and A.J. Neale, 1991, A Monte Carlo study of the effects of structural breaks on tests for unit roots, in: P. Hackl and A.H. Westlund, eds., Economic structural change: Analysis and forecasting (Springer, New York) 95–119.

Nelson, C.R. and C.I. Plosser, 1982, Trends and random walks in macroeconomic time series, Journal of Monetary Economics 10, 139–162.

Nunes, L.C., C.-M. Kuan and P. Newbold, 1994, Spurious break, Econometric Theory, in press.

Perron, P., 1989, The great crash, the oil price shock and the unit root hypothesis, Econometrica 57, 1361–1401.

Reichlin, L., 1989, Structural change and unit root econometrics, Economics Letters 31, 231–233.

Yao, Y.-C., 1988, Estimating the number of change-points via Schwarz' criterion, Statistics and Probability Letters 6, 181–189.

Zivot, E. and D.W.K. Andrews, 1992, Further evidence on the great crash, the oil-price shock, and the unit-root hypothesis, Journal of Business and Economic Statistics 10, 251–270.