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AN ECONOMETRIC ANALYSIS OF BIPROPORTIONAL PROPERTIES IN AN INPUT-OUTPUT SYSTEM*

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ABSTRACT. Various methods, such as biproportional adjustment and econometric estimating have been used to generate time series for input-output tables. In this paper, temporal changes of input-output coefficients are examined in order to analyze their behavior. Within the Chicago Region Econometric Input-Output Model, a set of input-output relationships has been extracted analytically for the period 1980–1997. Using the empirical evidence for Chicago, this paper conducts econometric time series analysis to determine whether or not certain coefficients or sets of coefficients exhibit tendencies toward stability or predictable change or whether others require more extensive econometric estimation.

1. INTRODUCTION

Since its introduction in the *Programme for Growth* series (Cambridge University, Department of Applied Economics, 1963), the RAS or biproportional adjustment technique has become one of the most popular methods for adjusting input-output, social accounting, and demographic matrices. In the input-output literature, the technique has been used for two primary purposes: (1) the initial application, namely, the adjustment of a matrix observed at one time period to

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a new matrix for a subsequent time period, in which only row and column totals for intermediate demand and output are known; and (2) the adjustment of national input-output tables to represent regional tables in the regional input-output literature. The technique has been reviewed extensively; and even though critics have claimed that it is nothing more than a mechanical adjustment process, both Stone (1961, 1962) and Leontief (1941), among others, attempted to provide some theoretical basis to justify its application.

This paper's interest in the technique stems from entirely different premises. A series of input-output tables over time is available from the Chicago Region Econometric Input-Output Model (CREIM), so the issue is not focused on the need to update tables. Rather, the interest lies in whether or not any biproportional properties exist in this time series. Because the input-output tables were generated within a general equilibrium model (CREIM), a further issue centers on the degree to which the additional information attached by the non–input-output components produces tables that are not merely simple extrapolations of the earlier one. Hence, the analysis is not directed to a comparison of two sets of tables where one set is observed and the other estimated—here both sets of tables are derived. In part, the initial stimulus for this approach was a paper of Lecomber (1969) in which he explored RAS projections when two or more matrices were known. In fact, the Chicago input-output tables were adjusted in the spirit that Lecomber proposed for cases where a large number of matrices were known.

In the next section, the RAS technique is presented and reviewed, as well as Lecomber's formulation and some extensions. Section 3 analyzes the estimates from the RAS procedure, focusing on its ability to capture structural changes. Section 4 introduces some experiments using Lecomber's and other formulations conducted on the tables. Some evaluation and concluding comments complete this paper.

2. THE RAS OR BIPROPORTIONAL TECHNIQUE

One of the problems associated with input-output analysis is based on an assumption of constant production relationships (coefficients) over time, especially when this time horizon stretches over a period of more than a decade. At the regional level, the issues are further complicated by the potential for change in trading relationships and problems that may arise if input-output components are nested, linked, or integrated with other models, such as computable general equilibrium models and demo-economic models (Israilevich et al., 1997). The costs of constructing survey-based regional input-output tables over time are prohibitive; as a result, the development of regional input-output tables has relied on two alternatives, nonsurvey and partial survey techniques. In the following subsections, one of the partial survey techniques, the *RAS* or *biproportional adjustment technique*, is presented and reviewed.

Overview of the Technique

The *RAS* or biproportional technique was developed by Stone (1961), Stone and Brown (1962), and Cambridge University, Department of Applied Economics (1963), and was summarized by Bacharach (1970). These studies approached the problem of finding the most efficient way to update the U.K. input-output tables by adopting the following procedure

$$\mathbf{A}_{(t+1)} = \hat{\mathbf{r}}\mathbf{A}_{(t)}\hat{\mathbf{s}} \tag{1}$$

where $\mathbf{A}_{(t)} = \left[a_{ij}^{t}\right]$, and $\mathbf{A}_{(t+1)} = \left[a_{ij}^{t+1}\right]$; $\hat{\mathbf{r}}$ and $\hat{\mathbf{s}}$ can be considered to be multipliers that implicitly transform $\mathbf{A}_{(t)}$ to $\mathbf{A}_{(t+1)}$. Equation (1) can be transformed to the following general expression shown as Equation (2)

$$\mathbf{A}_{(t+1)} = f[\mathbf{A}_{(t)}, \mathbf{u}_{(t+1)}, \mathbf{v}_{(t+1)}, \boldsymbol{x}_{(t+1)}] \tag{2}$$

where $\mathbf{u}_{(t+1)}$ is the vector of total intermediate outputs at time $t + 1$, $\mathbf{v}_{(t+1)}$ is the vector of total intermediate inputs at $t + 1$, and $\mathbf{x}_{t+1}$ is the vector of total outputs at $t + 1$. Given these three sets of data at t+1 and the input coefficient matrix at t, $\mathbf{A}_{(t)}$, $\hat{\mathbf{r}}$, and $\hat{\mathbf{s}}$ can be estimated as follows.

First, an estimated vector of intermediate outputs is obtained using $\mathbf{A}_{(t)}$ and known $\mathbf{x}_{(t+1)}$

$$\mathbf{u}_1 = \mathbf{A}_{(t)}\mathbf{x}_{(t+1)} \tag{3}$$

This estimate $\mathbf{u}_1$ is adjusted to conform to the observed value $\mathbf{u}_{(t+1)}$ through adjustment of the matrix $\mathbf{A}_{(t)}$. A new matrix $\mathbf{A}_1$ will be produced

$$\mathbf{A}_1 = \mathbf{r}_1\mathbf{A}_{(t)} \tag{4}$$

where $\mathbf{r}_1 = \hat{\mathbf{u}}_{(t+1)}\hat{\mathbf{u}}_1^{-1}$, and $\hat{\mathbf{u}}_{(t+1)}$ and $\hat{\mathbf{u}}_1$ are the diagonal matrices of $\mathbf{u}_{(t+1)}$ and $\mathbf{u}_1$, respectively. This new matrix $\mathbf{A}_1$ is now used to obtain the estimate of intermediate input vector $\mathbf{v}_1$ and the matrix $\mathbf{A}_1$ is further adjusted to a new matrix $\mathbf{A}_2$ to ensure equality with observed intermediate inputs $\mathbf{v}_{(t+1)}$

$$\mathbf{v}_1 = \hat{\mathbf{x}}_{(t+1)}\mathbf{A}_1^T\mathbf{i} \tag{5}$$

$$\mathbf{A}_2 = \mathbf{A}_1\mathbf{S}_1 \tag{6}$$

where $\mathbf{s}_1 = \hat{\mathbf{v}}_{(t+1)}\hat{\mathbf{v}}_1^{-1}$, and $\mathbf{A}_1^T$ is the transposed matrix of $\mathbf{A}_1$ and $\mathbf{i}$ is a vector with all elements equal to unity. Then, the process returns to Equation (3), where $\mathbf{u}_2$ is now estimated as follows in Equation (7)

$$\mathbf{u}_2 = \mathbf{A}_2\mathbf{x}_{(t+1)} \tag{7}$$

and so on through Equation (6). Equations (3) through (6) represent one complete iteration. Empirical evidence suggests that the process converges rapidly, usually within ten iterations (Hewings, 1985). After achieving the conversions of $\mathbf{r}_k$ and $\mathbf{s}_k$ at the kth iteration, the estimated $\mathbf{r}$ and $\mathbf{s}$ would be derived as follows

$$\hat{\mathbf{r}} = \hat{\mathbf{r}}_n \ldots \hat{\mathbf{r}}_2 \hat{\mathbf{r}}_1; \hat{\mathbf{s}} = \hat{\mathbf{s}}_1 \hat{\mathbf{s}}_2 \ldots \hat{\mathbf{s}}_n \tag{8}$$

Because the adjustment process shown in Equation (8) operates on the $\mathbf{A}$ matrices, the adjustment process is conservative, making only the minimally necessary adjustments to ensure agreement with the vectors $\mathbf{u}_{(t+1)}$ and $\mathbf{v}_{(t+1)}$.

Bacharach (1970), responding to the information theoretic approach for updating matrices by Uribe, de Leeuw, and Theil (1965), showed that this RAS method achieves "closeness" and is equivalent to the following minimization problem

$$\text{Min} \sum_{i,j} a_{ij}^{t+1} \log \frac{a_{ij}^{t+1}}{a_{ij}^{t}} \tag{9}$$

subject to

$$\mathbf{A}_{(t+1)}\mathbf{x}_{(t+1)} = \mathbf{u}_{(t+1)}$$

$$\hat{\mathbf{x}}_{(t+1)}\mathbf{A}^{T}_{(t+1)}\mathbf{i} = \mathbf{v}_{(t+1)}$$

The solution is the biproportional estimates, $\mathbf{r}$ and $\mathbf{s}$. Reviewing several methods to achieve closeness between two matrices with row- and column-sum constraints, Hewings and Janson (1980) concluded that, in applications to input-output matrices, the degree to which $\mathbf{A}_{(t+1)}$ can be claimed to be within $\mathbf{A}_{(t)}$'s neighborhood can be only with the empirical observation of $\mathbf{A}_{(t+1)}$.

The economic interpretation of $\hat{\mathbf{r}}$ and $\hat{\mathbf{s}}$ has proven to be contentious: Stone (1962) offered the interpretation that $\mathbf{r}_i$ is a measure of "substitution effects"—the extent to which the input i has substituted for other inputs or has been replaced by them during the time interval and that $\mathbf{s}_j$ is a measure of "fabrication effects" in the production of j—the extent to which the industry j has decreased (increased) its consumption of intermediate inputs per unit of gross output. Leontief (1941) also suggested this biproportional property in input-output tables. His interpretation differs slightly from Stone's: $\mathbf{r}_i$ is defined as a measure of the increased productivity of i in all uses, and $\mathbf{s}_j$ is regarded as a measure of the joint effect of increased productivity in industry j and of a decrease in its rate of investment.

In contrast to these economic interpretations of $\hat{\mathbf{r}}$ and $\hat{\mathbf{s}}$, many researchers discount this 'oversimplified' view of the RAS procedure in which such change is distributed throughout an economy (Miller and Blair, 1985). The critics view the RAS technique as a purely computational procedure that emerges as the solution to a constrained optimization problem subject to the row and column sums, as seen in Equation (9).

Modifications of the RAS Technique

The original RAS procedure uses only one complete matrix $\mathbf{A}_{(t)}$ with some future data $\mathbf{u}_{(t+1)}$, $\mathbf{v}_{(t+1)}$, , and $\mathbf{x}_{(t+1)}$, in order to estimate the future matrix. When more data—especially two or more input coefficient matrices—are available, Lecomber (1969, 1975) and Johansen (1968) proposed modifications of the RAS method to use this additional data in the most efficient way.

Johansen's problem was to estimate $\mathbf{A}_{(t)}$ based on given values for $\mathbf{A}_{(0)} = \left[a_{ij}^0\right]$, and $\mathbf{A}_{(1)} = \left[a_{ij}^1\right]$. He argued that individual coefficients contain more information than the row and column sums and that this information should be taken into account. In other words, the biproportional hypothesis is not only an imperfect representation of the underlying movement of coefficients over time, but also the coefficients for any year would be subject to disturbances and errors in measurement. Thus, the RAS method can be transformed as the error minimization problem revealed in Equation (10)

$$\text{(10)} \qquad \text{Min} \sum_{i,j} \varepsilon_{ij}^2 = \sum \left(a_{ij}^1 - a_{ij}^0 r_i s_j\right)^2$$

Furthermore, the assumption that both $\mathbf{A}_{(0)}$ and $\mathbf{A}_{(1)}$ are subject to disturbances would appear to be more plausible. Consider the model

$$\text{(11)} \qquad a_{ij}^t = \alpha_{ij} r_i^t s_j^t + \varepsilon_{ij}^t$$

where α_{ij} is the true (disturbance-free) coefficient for the (i, j) th pair and ε_{ij}^t is the error term for (i,j) at time t. Moreover, the following initial condition, for t = 0 (base year) is set

$$r_i^0 = s_j^0 = 1$$

From the model of Equation (11) α_{ij}, r_i, and s_j can be estimated by minimizing $\sum_{i,j} \varepsilon_{ij}^2$. Further, this minimization procedure can be simplified by assuming a multiplicative error term; consequently, (11) can be transformed to the log-linear form

$$\text{(12)} \qquad \log a_{ij}^t = \log \alpha_{ij} + \log r_i^t + \log s_j^t + \log \varepsilon_{ij}^t$$

Then log α_{ij}, log r_i^t, and log s_j^t can be estimated by minimizing $\sum_{i,j} \left(\log \varepsilon_{ij}^t\right)^2$.

Given data $\mathbf{A}_{(0)} = \left[a_{ij}^0\right]$ and $\mathbf{A}_{(1)} = \left[a_{ij}^1\right]$, the model of Equation (12) can be further transformed by eliminating α_{ij}

$$\log a_{ij}^1 - \log a_{ij}^0 = \log r_i^1 + \log s_j^1 + \log\left(\frac{\varepsilon_{ij}^1}{\varepsilon_{ij}^0}\right) \tag{13}$$

Similarly to the model (12), $\log r_i^1$, and $\log s_j^1$ in the model (13) can be estimated using ordinary least-squares regression with dummy variables for each r_i and s_j. Unlike the original RAS technique in which only the row and column sums are known, r_i and s_j can be estimated by the minimization of the squared error term for each (i, j). Since this Johansen-Lecomber formulation uses dummy variables for each r_i and s_j, it necessitates dropping one variable from each r_i and s_j set in order to avoid the singularity problem. In other words, one of r_i and one of s_j are set equal to unity—with no change over the period.

In order to improve accuracy, additional modified versions of the RAS technique have been proposed. Allen and Lecomber (1975) introduced the generalized version of RAS in which some of the elements in the forecast matrix $\mathbf{A}_{(t+1)}$ are estimated from exogenous information and the remaining part of the matrix is adjusted by the RAS procedure. Barker (1975) and Snower (1990) have extended the RAS method to incorporate price information. Although each improves the accuracy of the estimation, these modifications substantially increase the data requirement.

Similar to the use of the two-stage least squares approach to the estimation of input coefficients by Gerking (1976a, 1976b, and 1979), Toh (1998) interpreted r_i and s_j as iterative instrumental variable estimates and thus was able to derive the asymptotic standard errors. Toh's main idea is to consider the given row and column sum of the intermediate vectors, $\mathbf{u}_{(t+1)}$ and $\mathbf{v}_{(t+1)}$, as the instrumental variables for r_i and s_j. Toh further proposed RAS as an iterative sectoral optimization model; however, he concluded that the RAS technique is not useful for projection but for the study of structural change, especially when the economy experiences rapid structural change. However, no test of the accuracy of projections using his method has been conducted.

Biproportionality in an input-output system has been explored and extended to investigate different aspects of temporal changes. De Mesnard (1997) analyzed interindustry dynamics (1990) and coefficient variation between original Leontief demand-driven system and Ghosh's (1997) supply-driven system employing biproportional filter in French cases. Although, methodologically, de Mesnard's methods are only an extension of the RAS technique, his results are informative: for any sector, both row and column coefficients are found to change simultaneously.

The RAS Technique and the Field of Influence

The concept of a *field of influence* was developed and described by Sonis and Hewings (1989) to provide a formal and general procedure for the measurement of the analytical impact of changes in the direct coefficients' matrix of an input-output table on the associated Leontief inverse matrix. The procedure involves the calculation of the ratio of two polynomial functions of changes, in contrast to the usual approach using the infinite Taylor-series expansion of the Leontief inverse. Moreover, it is more general in that it can handle a complete range of changes—single element, all elements in a row or column, or all elements in the matrix. In addition, the field of influence is able to define the rank-size hierarchy of inverse important coefficients which are the direct input coefficients whose changes would create the largest volume of change in the input-output system (Hewings, 1984).

Sonis and Hewings (1992) presented the relationship between the RAS procedure and the field of influence concept. The RAS procedure is usually in the following form

$$a_{ij}^{t+1} = r_i a_{ij}^t s_j \tag{14}$$

Let

$$r_i = 1 + \delta i; s_j = 1 + \eta_j$$

With the relative change, Equation (14) can be transformed to

$$a_{ij}^{t+1} = r_i a_{ij}^t s_j = (1+\delta_i) a_{ij}^t (1+\eta_j) = a_{ij}^t (1+\delta_i+\eta_j+\delta_i\eta_j) = a_{ij}^t + \varepsilon_{ij}^t \tag{15}$$

where

$$\varepsilon_{ij}^t = a_{ij}^t (\delta_i + \eta_j + \delta_i \eta_j)$$

Hence, the RAS procedure can be seen as a special case of error analysis, which places it within a broader view of coefficient change. Furthermore, if relative changes in δ_i and η_j are small, the products $\delta_i\eta_j$ can be ignored

$$\varepsilon_{ij}^t \approx a_{ij}^t (\delta_i + \eta_j)$$

In this case, the coefficient change ε_{ij}^t is defined to bear a linear relationship to a previous value. Obviously, the choice of the elements with the largest field of influence will depend in part on the choice of ε_{ij}^t in Equation (15); however, for ranges of ε_{ij}^t observed in survey-based input-output tables, there is a significant degree of stability in the rank-orderings of the fields of influence.

3. BIPROPORTIONALITY OF CHANGES

As indicated earlier, the Chicago input-output tables are extracted from the Chicago Region Econometric Input-Output Model (CREIM), which consists of 36 industrial sectors (see Appendix). This system of 250 equations includes both exogenous and endogenous variables. Endogenous coefficient change serves as the mechanism to clear markets in the quantity-adjustment process (see Israilevich et al., 1997, for more details). The input-output coefficient matrix is not observed directly; however, it is possible to derive analytically a Leontief inverse matrix and, through inversion, the estimated direct coefficient matrix.

Thus, the interest lies in the way these exogenous changes are manifested in the input-output coefficients and the degree to which these input-output coefficients are predictable under the usual conditions associated with the RAS technique. Using the Chicago region input-output tables derived from the CREIM during the period of 1980–1997, the $\hat{\mathbf{r}}$ and $\hat{\mathbf{s}}$ vectors can be estimated by the repeated iteration procedure described in Equations (3) through (6). An important assumption here is that the error terms in derived input-output coefficients from the CREIM are normally distributed, and are independent and identically distributed; thus, the coefficients cannot be "real" observations but are treated as such.

General Observations

Figures 1 and 2 reveal the trends in values of r_i and s_j from Equation (1), ranked by the volume of output in 1980, with largest at the left and smallest at the right. By and large, smaller output sectors (those with lower rank) tend to exhibit greater variance over time whereas the larger sectors tend to have more r_i values that are less than unity in the case with the values of s_j. Overall, a greater volatility in the values r_i than in the entries s_j can be observed. This looks to coincide with the 'hollowing-out' process in the Chicago economy reported by Hewings et al. (1998), in which the level of dependence on local purchases and sales is declining; the tendency of the sectors with larger output to have $r_i < 1$ may be the evidence of substitution, not across sectors, but in the location of purchase; the smaller volatility in the s_j entries indicates that the fabrication effect (technological change) is relatively insignificant.[1] Casual inspection would suggest few pronounced trends in either of these entries. However, these interpretations should wait for the careful investigation of empirical evidence, since they are based on the simulated results.

Figures 3 through 5 provide the sample trends in individual coefficients (a_{ij}, b_{ij}, r_i, s_j), where b_{ij} is the element of Leontief inverse; these trends vary among various (i, j) combinations. For example, the interaction between Sectors 18 (Fabricated Metals) and 19 (Industrial Machinery and Equipment) reveals that the trends in the direct and inverse coefficients are mainly associated with changes in s_j (in this case, Sector 19). The variations in the direct and inverse coefficients can be seen to mirror the changes in s_j. For the interaction between Sectors 10 (Paper and Allied Products) and 11 (Printing and Publishing), on the other hand, the r_i has a greater contribution to the changes in the coefficients, whereas s_j has lesser variations over time. For self-influenced changes (Sector 5, Food and Kindred Products), the row and column effects offset each other, producing little change in the coefficients. Among the inverse-important coefficients, the trends in 17 of the top 25 field of influence coefficients are mainly

[1]A referee pointed out that the RAS technique produces a solution that is parametrically determined (see Bacharach, 1970 and Van der Linden, 1999) and depends on whether the row or columns are adjusted first. In the present analysis, consistent application (columns-first) of the RAS procedure was conducted to ensure consistency across time.

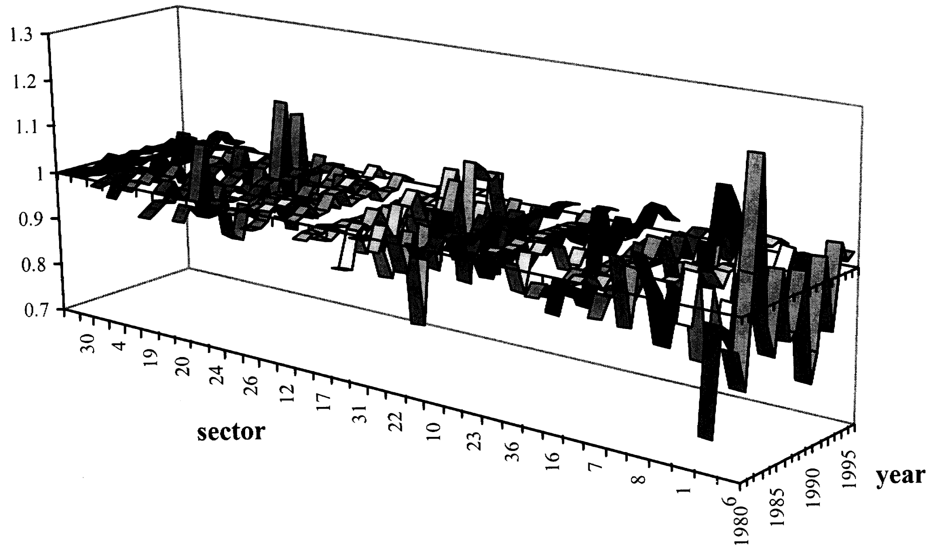

FIGURE 1: Trends of r_i.

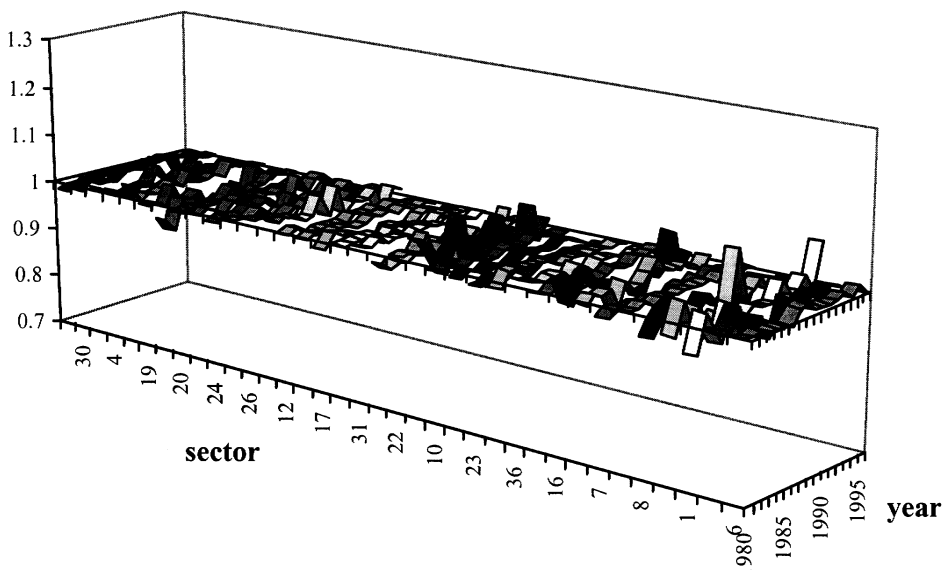

FIGURE 2: Trends of s_j.

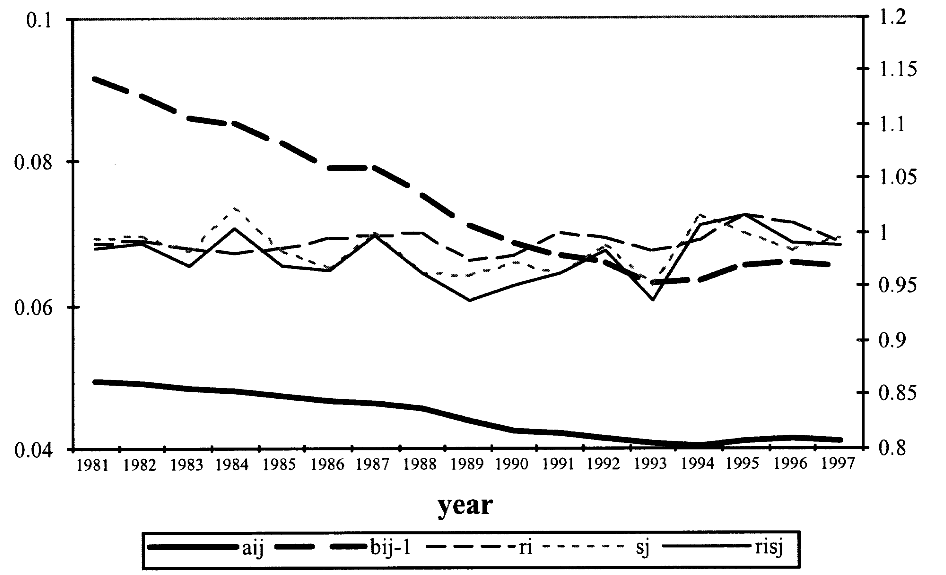

FIGURE 3: Trends in a_{ij}, b_{ij}, r_i, and s_j (Left Axis for a_{ij} and b_{ij}; Right Axis for r_i and s_j) i = 18: Fabricated Metals; j = 19: Industrial Machinery and Equipment.

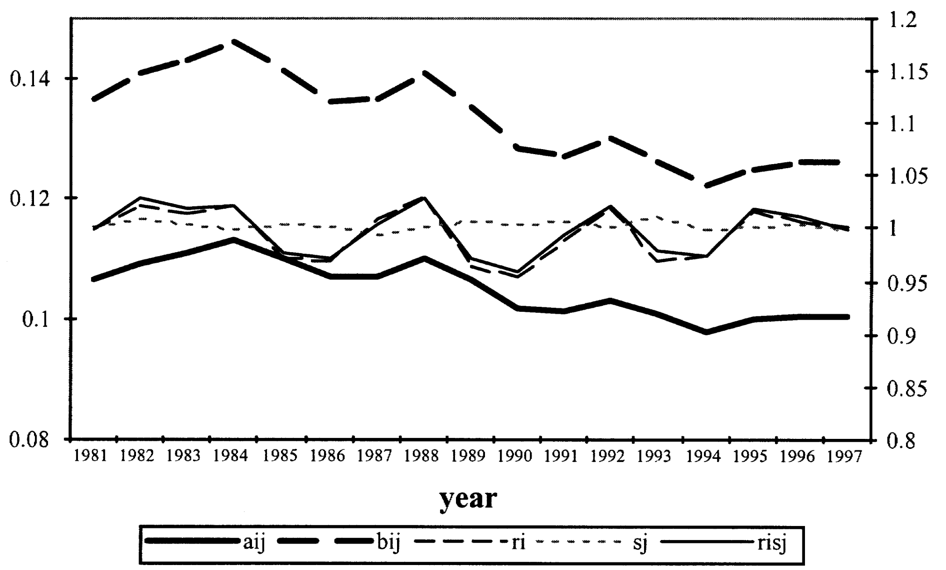

FIGURE 4: Trends in a_{ij}, b_{ij}, r_i, and $s_j s_j$ (Left Axis for a_{ij} and b_{ij}; Right Axis for r_i and s_j) i = 10: Paper and Allied Products; j = 11: Printing and Publishing.

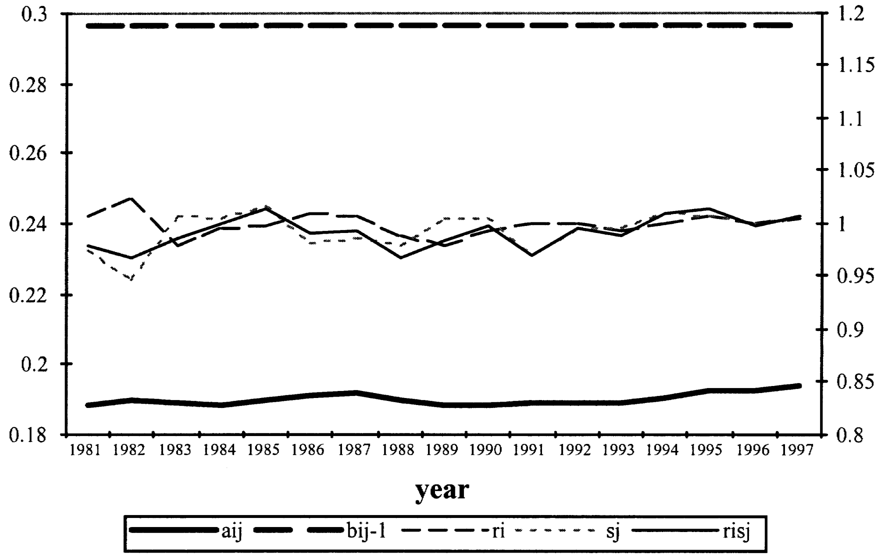

FIGURE 5: Trends in a_{ij}, b_{ij}, r_i, and $s_j s_j$ (Left Axis for a_{ij} and b_{ij}; Right Axis for r_i and s_j) $i = 5; j = 5$: Food and Kindred Products.

associated with the row effects of r_i, whereas 2 and 6 of them are generated by the changes in s_j and the combined effects of r_i and s_j, respectively.[2] This result also confirms the observations in Figure 1 and 2.

Analysis of Estimation Error

As mentioned in the previous section, the RAS procedure does not claim to minimize the sum of squared errors, only to find a matrix that is as close as possible to the prior one, subject to the row and column constraints. Thus, the analyses of estimation error were conducted in order to investigate how well the RAS procedure can capture the changes in the direct input matrices over time. Paelinck and Waelbroeck (1963) identify the errors in the RAS technique as derived from the following three possibilities: (1) aggregation error in the industrial classification that arises in all input-output analysis; (2) error derived from variations in substitution effects over utilizing industries, in violation of the assumed uniformity of these effects; and (3) a false estimate of any one cell, which would force offsetting errors in other elements of its row and column—such errors would spread over a wide area of the matrix. In conjunction with the field of influence concept, the last notion of Paelinck and Waelbroeck (1963) indicates the system-wide impact of changes in each coefficient.

[2]The top 25 field of influence coefficients are used as the inverse-important coefficients, because the value of the field of influence becomes very small and stable after the top 25 coefficients are considered.

The error terms are derived as follows

$$e_{ij}^{t} = a_{ij}^{t} - r_i^t a_{ij}^{t-1} s_j^t \tag{16}$$

Using Jensen's (1980) definition, this ε_{ij}^{t} can be considered as *partitive accuracy*, which measures the cell-by-cell accuracy of the estimation. On the other hand, *holistic accuracy* emphasizes a 'mathematical portrait' interpretation of the estimation, relying on the accuracy of the estimated table as a whole. For this holistic accuracy, at first, the estimation errors of each coefficient in the estimated Leontief inverse are employed

$$\tilde{e}_{ij}^{t} = b_{ij}^{t} - \bar{b}_{ij}^{t}$$

where

$$\left[\bar{b}_{ij}^{t}\right] = \overline{\mathbf{B}}_{(t)} = \left(\mathbf{I} - \hat{\mathbf{r}}_{(t)}\mathbf{A}_{(t-1)}\hat{\mathbf{s}}_{(t)}\right)^{-1}$$

Table 1 shows the estimation performance of the RAS technique on an annual basis, as indicated in Equation (16), using mean absolute deviation (MAD) and mean absolute percentage error (MAPE). For partitive accuracy, some small fluctuations over the years can be observed (except the 85–86 estimation); however, in general the estimation errors are rather small (less than 5 percent). The estimation performances for holistic accuracy are notably improved—less than 1 percent. However, in practice updating an input-output table annually is quite rare, and tables are usually updated over a five or ten year period. The results of the estimation performance over five and ten years reveal that, as the estimation period becomes longer, both the partitive accuracy and holistic accuracy deteriorate.[3] Although the partitive accuracy decreases relatively rapidly, the holistic accuracy remains large (with errors only of two percent over ten years). This tendency of increasing errors for longer estimation periods does not result from the exponential nature of RAS projection; rather, as Toh (1998) noted, the RAS technique is not appropriate for estimation for longer time periods and hence exhibiting large structural changes, because the RAS technique tries to derive the estimated matrix as close as possible to the base-year matrix.

Table 2 shows a similar analysis of the estimation errors but only for the coefficients with the top 25 direct fields of influence—the most important coefficients. The results reveal the same tendencies with all the coefficients, in which the estimation error increases as the estimation time becomes longer. However, overall, both the partitive and holistic accuracy are considerably better than the results for all coefficients. These results indicate that the inverse-important coefficients are stable over time.

[3]Detailed tables describing these findings are available from the authors upon request.

TABLE 1: Estimation Accuracy of the RAS Procedure (Annual Estimation)

Years Base	Target	Partitive Accuracy (a_{ij}) MAD	Partitive Accuracy (a_{ij}) MAPE Percent	Holistic Accuracy (b_{ij}) MAD	Holistic Accuracy (b_{ij}) MAPE Percent
80	81	0.00017	4.03	0.00022	0.41
81	82	0.00015	2.57	0.00019	0.36
82	83	0.00012	2.07	0.00016	0.29
83	84	0.00015	2.88	0.00019	0.36
84	85	0.00012	4.41	0.00016	0.39
85	86	0.00016	20.37	0.00019	0.47
86	87	0.00015	3.27	0.00018	0.37
87	88	0.00016	3.27	0.00020	0.45
88	89	0.00017	3.44	0.00020	0.43
89	90	0.00013	2.33	0.00015	0.32
90	91	0.00015	3.60	0.00019	0.46
91	92	0.00011	4.44	0.00013	0.32
92	93	0.00014	2.95	0.00016	0.34
93	94	0.00011	3.83	0.00013	0.29
94	95	0.00012	2.33	0.00015	0.34
95	96	0.00011	2.36	0.00013	0.27
96	97	0.00009	1.46	0.00011	0.21

Note: The 1986 table (**A** matrix) contains a very small coefficient at the intersection of sectors 33 and 36; although the absolute deviation between the actual value in 1986 and the estimated value using the 1985 matrix is small the percentage error is large, contributing to a very large MAPE for that year.

TABLE 2: Estimation Accuracy of the RAS Procedure (Annual Estimation) Top 25 Fields of Influence

Years Base	Target	Partitive Accuracy (a_{ij}) MAD	Partitive Accuracy (a_{ij}) MAPE Percent	Holistic Accuracy (b_{ij}) MAD	Holistic Accuracy (b_{ij}) MAPE Percent
80	81	0.00013	0.61	0.00065	0.19
81	82	0.00012	0.49	0.00046	0.16
82	83	0.00008	0.44	0.00040	0.13
83	84	0.00006	0.32	0.00047	0.16
84	85	0.00005	0.35	0.00047	0.18
85	86	0.00008	0.69	0.00052	0.16
86	87	0.00007	0.39	0.00058	0.18
87	88	0.00008	0.50	0.00050	0.19
88	89	0.00008	0.50	0.00066	0.23
89	90	0.00011	0.71	0.00113	0.33
90	91	0.00007	0.37	0.00027	0.09
91	92	0.00009	0.71	0.00036	0.12
92	93	0.00010	0.57	0.00055	0.18
93	94	0.00007	0.54	0.00070	0.20
94	95	0.00009	0.47	0.00063	0.16
95	96	0.00009	0.42	0.00042	0.11
96	97	0.00005	0.21	0.00029	0.07

Although the findings in this subsection are consistent with the previous studies (Szyrmer, 1989; Miller and Blair, 1985, among others), the results here would raise an interesting question: If the trends of r_i and s_j are relatively easily determined over time using some econometric method, can the RAS technique replace more complex models for the estimation of input-output coefficients? In the following subsection, the trends of r_i and s_j are investigated.

Tests for the Trends of r_i and s_j

As presented in the previous section, the RAS technique can trace the changes in direct input coefficients a_{ij} relatively well, especially for a short period of time and in terms of holistic accuracy. The question arises whether $\hat{\mathbf{r}}$ and $\hat{\mathbf{s}}$ themselves are predictable so that the future input coefficient matrices can be estimated solely by the predicted $\hat{\mathbf{r}}$ and $\hat{\mathbf{s}}$. The following tests investigate the behaviors of r_i and s_j over time.

Table 3 provides a summary of a runs test that was applied to the time series of r_i and s_j in order to test the randomness of the trends. For the majority of sectors, r_i and s_j indicate random trends (no apparent monotonic or cyclical trends). An interesting observation is that the sectors with non-random trends vary between r_i and s_j; this also confirms the different trends in r_i and s_j shown in the Figures 1 and 2. By and large, the trends of r_i and s_j are random. Since the run test is for nonparametric analysis, more extensive tests were adopted to explore the statistical properties of r_i and s_j, assuming that r_i and s_j can be considered random variables.

If r_i and s_j can be considered as a univariate time-series, in which dependence is based only on the past (autoregressive) trends, the models applied to these data need to be examined in terms of their integration (nonstationary) process in order to specify the models. Investigation focused on models of $I(1)$, integrated of order one, referred to as a unit root process. The importance of the tests is that if the process is a random walk,

$$y_t = y_{t-1} + \varepsilon_t$$

the current observation is the simple sum of random disturbance terms, which possibly can be explained by exogenous variables. Furthermore, if a variable is truly $I(1)$, then shocks to it will have permanent effects. Augmented Dicky-Fuller (ADF) tests, with lagged differences for auto-regressive specification were employed to test the processes of r_i and s_j with the following three model specifications, because no a priori specification is known

$$y_{\mathrm{t}} = y_{t-1} + \varepsilon_t \text{ (random walk model)} \tag{17}$$

$$y_{\mathrm{t}} = \mu + y_{t-1} + \varepsilon_t \text{ (random walk with drift)}$$

$$y_{\mathrm{t}} = \mu + \beta_t + y_{t-1} + \varepsilon_t \text{ (random walk with drift and trend)} \tag{18}$$

where y_{t} is either r_i and s_j at time t, and t = 1981,...,1997.

TABLE 3: Summary of Run Test for r_i and s_j

Sector	r_i	s_j
1	12	10
2	6	9
3	8	**13**
4	8	8
5	9	6
6	10	7
7	6	8
8	6	6
9	8	7
10	8	8
11	6	11
12	9	**5**
13	9	11
14	11	9
15	**4**	10
16	**4**	9
17	10	7
18	9	8
19	10	11
20	10	7
21	10	6
22	8	7
23	9	10
24	6	8
25	8	7
26	6	10
27	9	8
28	6	10
29	**4**	7
30	7	7
31	6	10
32	7	7
33	8	12
34	8	10
35	6	9
36	8	8

Note: reject null hypothesis of randomness if either r or s is either ≤5 or ≥13 ($\alpha = 0.063$)

Allowing maximum lags up to *t-5*, in a pure random walk specification (17) for all r_i and s_j all except Sector 3 (Mining), the null hypothesis of a unit root is not rejected at 5 percent level. In a random walk with drift specification, the results were more varied: Sectors 1, 3, and 31 for r_i and Sectors 4, 5, 13, 19, 20, and 22 for s_j reject the null hypothesis. Model (18), the random walk with drift and trend model, resulted in most sectors not rejecting the null hypothesis; the exceptions were Sectors 1, 25, 28, 31, and 33 for r_i and Sectors 1, 3, 4, 5, 16, 17, and 34 for s_j. In either model, the majority of sectors did not reject the null hypothesis of unit root.

The entire sample consists of only 17 observations (1980 to 1997), and ADF tests with lagged differences consume observations, so that with five lagged differences the sample size was decreased to only 11 observations. In order to address this problem and to analyze the sensitivity of the lagged differences, ADF tests with lesser lags (maximum of three) were conducted, and the results are shown in Table 4. In the majority of cases, no changes occurred in the behavior of a sector in terms of whether the null hypothesis was accepted or rejected. However, there were some variations: with a maximum lag of three, a smaller number of sectors rejected the null hypothesis across the models. Overall, one can claim that the majority of the sectors do not reject the null, indicating unit root behavior, and hence nonstationary process. The results suggest the option of constructing *autoregressive integrated moving average* [ARIMA(p,d,q)] models for each r_i and s_j; however, the ARIMA model requires larger sample sizes (Harvey, 1989, 1990). Furthermore, because r_i and s_j can capture the sudden changes of economic structure and technological advancement, the model for these using constant coefficients might perform poorly, either for forecasting or for analyzing the effect of policy change (Maddala and Kim, 1998). Likewise, if there is a break (sudden trend shift) in the deterministic trend, then unit root tests will lead to a misleading conclusion (Perron, 1989). Again, given the size of samples, the results here need to be considered carefully.

4. EXPERIMENTS

In order to investigate the biproportional properties of input-output systems further, the following experiments were implemented and analyzed. First, using the Johansen-Lecomber formulation, r_i and s_j are estimated by regression models for each year. Second, systems of equations approaches are examined, and the *vector autoregression* (VAR) model is employed to analyze the estimates of the RAS technique.

Lecomber Revisited

As presented above, the Johansen-Lecomber formulation of the RAS model is based on the regression model, Equation (13). While it preserves the biproportional structure of the estimation procedure, this formulation has the advantage of minimizing the error terms. The comparison of the estimation error between the RAS technique and the Johansen-Lecomber formulation can elucidate the properties of the estimation procedures. In order to estimate the Lecomber model, r_i and s_j for Sector 6 (Tobacco Products) are set to unity (zero in the model) in order to avoid the singularity problem in regression estimation as noted in Section 2. Sector 6 was chosen because the tobacco industry in Chicago hardly exists and its outputs are negligible throughout the period of 1980–1997; therefore, no changes in r_i and s_j can be assumed.

Table 5 displays the partitive and holistic accuracy of annual estimation using the Johansen-Lecomber estimation, corresponding to Table 1 for the RAS procedure. Generally, the MAPEs for partitive accuracy in the Lecomber model

TABLE 4: Summary of Augmented Dickey-Fuller Test on r_i and s_j (Maximum Lag = 3)

	r_i			s_j		
Sector	Random Walk	RW with Drift	RW with Drift and Trend	Random Walk	RW with Drift	RW with Drift and Trend
1	–0.39	**–4.15**	**–3.99**	1.53	0.21	–2.74
2	0.19	–2.25	–2.14	–0.18	–1.91	–1.88
3	–0.16	–2.01	–2.13	–0.14	**–4.46**	**–7.10**
4	0.48	–2.34	**–4.17**	–0.39	–1.78	–1.67
5	0.12	–1.52	–1.35	0.45	–1.81	–2.46
6	0.56	–1.61	–1.22	0.30	–1.87	–2.35
7	1.07	–1.50	–2.54	0.38	–0.98	–1.79
8	0.12	–2.39	–2.68	–0.29	–1.80	–1.25
9	1.05	–1.24	–1.81	–0.13	–1.49	–1.89
10	–0.51	–2.65	–1.19	0.18	–1.63	–1.18
11	0.10	–0.64	–1.41	–0.13	–1.60	–1.60
12	0.07	–2.75	–2.82	–0.03	–1.71	–1.94
13	–0.29	–2.82	–3.00	0.37	**–3.31**	–3.37
14	0.04	–2.39	–2.34	–0.34	–1.98	–1.18
15	0.46	–2.75	–2.40	–0.14	–1.73	–1.80
16	0.79	–1.37	–1.58	–0.26	–1.33	–0.93
17	0.08	–2.89	–2.79	–0.37	–2.53	–2.34
18	0.94	–0.62	–1.25	–0.38	–2.08	–2.00
19	0.61	–1.98	–2.33	–0.34	–1.18	–0.95
20	2.96	–0.69	–1.71	–0.75	–2.71	–2.55
21	0.27	–2.04	–2.86	–0.45	–2.14	–2.05
22	0.41	–1.57	–2.01	–0.50	–1.63	–2.37
23	0.03	–2.41	–2.31	–0.03	–2.58	–2.55
24	0.39	–1.89	–2.20	0.31	–1.79	–1.70
25	–0.30	–2.73	–3.21	0.39	–2.07	–2.10
26	–0.29	–2.03	–2.52	0.23	–2.23	–2.10
27	0.42	–2.24	–3.54	0.32	–2.19	–1.31
28	–0.01	–2.76	–2.79	–0.40	–1.89	–2.17
29	–0.34	–2.96	–2.43	0.09	–2.98	–2.83
30	0.34	–2.29	–1.17	–0.05	–2.08	–2.19
31	0.76	–1.72	–1.84	–0.55	–1.56	–3.37
32	0.79	–1.53	–1.87	–1.12	–1.31	–1.59
33	0.50	–1.05	**–3.76**	–0.24	–1.90	–1.79
34	–0.03	–1.39	–1.43	0.24	**–4.10**	**–4.92**
35	–0.21	–1.80	–2.01	–0.27	–0.95	–1.04
36	–0.59	–2.02	–2.61	–0.77	–2.29	–3.21

Critical Values ($\alpha = 0.05$): Random Walk = –1.95; Random Walk with Drift = –3.00; Random Walk with Drift and Trend = –3.60.

are comparable to the ones in the RAS procedure; however, for holistic accuracy, the Lecomber model exhibits larger errors throughout the estimates. Furthermore, these larger MAPEs fluctuate across the estimates in a wider range than for the RAS estimates. The Johansen-Lecomber formulation is basically a regression model, so the estimation procedure provides statistical outputs, such

TABLE 5: Estimation Accuracy of Lecomber Model (Annual Estimation)

Years		Partitive Accuracy (a_{ij})		Holistic Accuracy (b_{ij})	
Base	Target	MAD	MAPE Percent	MAD	MAPE Percent
80	81	0.00043	4.63	0.00140	3.30
81	82	0.00030	3.36	0.00102	2.45
82	83	0.00022	2.65	0.00085	1.91
83	84	0.00033	4.13	0.00115	2.96
84	85	0.00044	5.80	0.00129	3.34
85	86	0.00068	16.84	0.00206	5.38
86	87	0.00047	4.75	0.00145	3.44
87	88	0.00037	3.71	0.00112	2.87
88	89	0.00030	3.71	0.00089	2.13
89	90	0.00024	2.80	0.00066	1.58
90	91	0.00038	4.20	0.00112	2.85
91	92	0.00031	5.06	0.00094	3.53
92	93	0.00032	3.89	0.00123	3.59
93	94	0.00026	4.47	0.00101	2.88
94	95	0.00037	3.79	0.00129	3.12
95	96	0.00020	2.75	0.00068	1.47
96	97	0.00019	2.06	0.00060	1.55

See note for Table 1.

as significance of the coefficients. The results from the Lecomber model without insignificant coefficients—set to unity (Lecomber model is in log-linear form: $\beta = 1$ when $\log \beta = 0$) for corresponding r_i and s_j—indicate that both partitive and holistic accuracy are less than in Table 5; omitting insignificant coefficients has a relatively small (one to two percent increase in MAPE), effect on the estimation. As seen in the results from the standard RAS procedure, the estimation accuracy deteriorates as the estimation span becomes longer, for both partitive and holistic accuracy. Although the partitive accuracy for the Lecomber model is comparable to the RAS results (better in several cases) again, the holistic accuracy is less accurate than the RAS counterparts.

Estimation errors were further investigated for the inverse-important coefficients, that is, the top 25 coefficients with the largest direct fields of influence. Table 6 presents the results. As in the RAS results the estimation accuracy—for both partitive and holistic—improves for the annual estimation. However, the MAPEs for partitive and holistic accuracy are much larger than in the RAS results, and partitive and holistic accuracy have similar values, unlike in the RAS procedure where the MAPEs for holistic accuracy are smaller than the ones for partitive accuracy. The increase of the MAPEs with longer periods is smaller than that in the RAS results, but the estimation accuracy for partitive and especially for holistic accuracy is lower than when using the RAS technique.

Although the Lecomber estimates vary by setting r_i and s_j to unity in a different sector, the findings illustrate that the estimation by the RAS technique

TABLE 6: Estimation Accuracy of Lecomber Model (Annual Estimation) Top 25 Fields of Influence

		Partitive Accuracy (a_{ij})		Holistic Accuracy (b_{ij})	
Years Base	Target	MAD	MAPE Percent	MAD	MAPE Percent
80	81	0.00042	1.57	0.00528	1.85
81	82	0.00014	1.63	0.00368	1.32
82	83	0.00015	1.38	0.00297	1.04
83	84	0.00013	1.26	0.00253	0.86
84	85	0.00036	3.10	0.00446	1.65
85	86	0.00026	3.00	0.01277	4.25
86	87	0.00019	2.46	0.01256	3.93
87	88	0.00031	1.41	0.00292	1.11
88	89	0.00014	0.61	0.00310	1.15
89	90	0.00020	0.95	0.00253	0.83
90	91	0.00044	2.27	0.00447	1.54
91	92	0.00022	3.29	0.00304	1.02
92	93	0.00015	1.96	0.00313	1.10
93	94	0.00012	1.20	0.00433	1.38
94	95	0.00039	1.39	0.00628	2.00
95	96	0.00022	1.04	0.00437	1.35
96	97	0.00012	0.96	0.00288	0.85

is quite accurate compared to the Lecomber regression model, especially in terms of holistic accuracy. This implies that, although both models possess a similar biproportional structure (taking into account row and column effects), adjusting coefficients by row and column sums will provide better estimation than the estimation by each coefficient—in this case less information means better results. This can be considered contradictory to received theory; however, after all, regression allows error terms (while minimizing them), whereas the RAS procedure adjusts a matrix with a pair of constraints (row and column sums) to derive the closest matrix to the base matrix. These row and column constraints provide better results, especially for the inverted matrix, indicating the existence of row and column structures in the matrices. One aspect of the Lecomber model may contribute to this difference in estimation capability. Because the regression model of Equation (13) minimizes $\sum_{i,j}\left[\log\left(\varepsilon_{ij}^{1}/\varepsilon_{ij}^{0}\right)\right]$, the estimated r_i and s_j do not minimize $\sum_{i,j}\left(\varepsilon_{ij}^{0}\right)^2$, nor $\sum_{i,j}\left(\varepsilon_{ij}^{1}\right)^2$. In other words, this formulation does not guarantee that the estimated result has the least-square-error property; thus, the derived r_i and s_j have poor estimation capability.

Systems of Equations Approaches

The Johansen-Lecomber formulation can be considered as a regression model of biproportional estimates for a pair of time periods, as in the RAS procedure. However, if a time-series model can be formed over the observation period it can use more information on the structural changes of input-output tables than the Lecomber model. There are at least two candidates to formulate such a model: the seemingly unrelated regression (SUR) model and the vector autoregression (VAR) model.

An input-output table can be considered as a system with biproportional influence row- and column-wise, therefore, the univariate time-series model for each coefficient may be improved upon by generating the estimates jointly. The SUR model can be employed to form such a system of equations. However, because there are no exogenous variables on the right-hand side and each equation can only be a univariate time-series model, the SUR model in this case is equivalent to a vector autoregression model.

A VAR model can be used to estimate the future vector (or, vectorized matrix) using the past trend of the vector. Typically, the VAR model can be formulated as follows

$$\mathbf{y}_t = \mu + \Gamma_1 \mathbf{y}_{t-1} + \ldots + \Gamma_p \mathbf{y}_{t-p} + \varepsilon_t \tag{19}$$

where $\mathbf{y}_t$ and ε_t are $n \times 1$ vectors, μ is the $n \times 1$ mean vector, and $\Gamma_1, \ldots, \Gamma_p$ are $n \times n$ parameter matrices. The elements of $\Gamma_1, \ldots, \Gamma_p$ can be estimated by multivariate least-squares, which is exactly the same as applying ordinary least-squares to each equation. If input-output tables reflect this kind of autoregressive process over time, then this type of VAR model can be applied to forecast the future vectors (and hence matrices), and can be employed to analyze the autoregressive properties of input-output tables.

Consider the following VAR model

$$\begin{pmatrix} \log a_{ij}^{t} \\ \log r_i^{t+1} \\ \log s_j^{t+1} \end{pmatrix} = \Gamma_1 \begin{pmatrix} \log a_{ij}^{t-1} \\ \log r_i^{t} \\ \log s_j^{t} \end{pmatrix} + \ldots + \Gamma_p \begin{pmatrix} \log a_{ij}^{t-p} \\ \log r_i^{t-p+1} \\ \log s_j^{t-p+1} \end{pmatrix} + \varepsilon_t \tag{20}$$

This formulation is equivalent to

$$\begin{aligned} \log a_{ij}^{t} &= \varphi_{11}^{1} \log a_{ij}^{t-1} + \varphi_{12}^{1} \log r_i^{t} + \varphi_{13}^{1} \log s_j^{t} + \ldots \\ \log r_i^{t+1} &= \varphi_{21}^{1} \log a_{ij}^{t-1} + \varphi_{22}^{1} \log r_i^{t} + \varphi_{23}^{1} \log s_j^{t} + \ldots \\ \log s_j^{t+1} &= \varphi_{31}^{1} \log a_{ij}^{t-1} + \varphi_{32}^{1} \log r_i^{t} + \varphi_{33}^{1} \log s_j^{t} + \ldots \\ &+ \varphi_{11}^{p} \log a_{ij}^{t-p} + \varphi_{12}^{p} \log r_i^{t-p+1} + \varphi_{13}^{p} \log s_j^{t-p+1} + \varepsilon_1^{t} \\ &+ \varphi_{21}^{p} \log a_{ij}^{t-p} + \varphi_{22}^{p} \log r_i^{t-p+1} + \varphi_{23}^{p} \log s_j^{t-p+1} + \varepsilon_2^{t} \\ &+ \varphi_{31}^{p} \log a_{ij}^{t-p} + \varphi_{32}^{p} \log r_i^{t-p+1} + \varphi_{33}^{p} \log s_j^{t-p+1} + \varepsilon_3^{t} \end{aligned} \tag{21}$$

The first equation in (21) is analogous to the autoregressive form of the Lecomber model. The model for each element (i, j pair) is estimated using the value of a_{ij}, and the estimated values of r_i and s_j from the RAS procedure. The *Schwarz*

(Bayesian) information criterion (SIC or BIC) can be employed to determine the number of lags (maximum lag set at three) for each model of Equation (20).

Table 7 presents the results for the selected elements with the top ten largest fields of influence; the table shows only the results for the first equation in (21).[4] Overall, all top ten elements have a high coefficient of determination R^2. These results imply that r_i and s_j have statistically significant explanatory power on a_{ij}. For the autoregressive structure, only the sixth (34, 4) and the ninth (28, 30) pairs exhibit longer lags (in this case 3); however, the coefficients for the second and third lags are insignificant. Otherwise, the lag structures for other elements are only a single lag. In the third, fourth, eighth, and tenth elements, the coefficient for s_j is insignificant, indicating a smaller influence on a_{ij}. In general, the results reinforce the Lecomber's formulation for the most important coefficients.

Two things must be noted: first, because the estimated values of r_i and s_j by the RAS procedure are used as data, the coefficients for r_i and s_j indicate the accuracy of the estimates; if r_i and s_j can perfectly estimate a_{ij} based on its lagged value, the coefficients for them should be unity. Second, most of the coefficients for a_{ij} are very close to unity, implying possible unit root behavior of a_{ij}. The results from the augmented Dicky-Fuller tests for each a_{ij} suggest that only 22 percent, 10 percent, and 15 percent of a_{ij} are stationary processes (random walk, random walk and drift, and random walk with drift and trend models, respectively). Given the results of the unit root tests for r_i and s_j in Section 3, the results from the VAR models above must be viewed with caution. It may be possible that some models with nonstationary variables can generate spurious regression results. However, given the simplicity of the RAS procedure, further investigation of the VAR specifications are infeasible and beyond the scope of this study.

Additionally, serious drawbacks may prevent the further investigation of the VAR formulation. First, as formulated in Equation (19), this type of VAR model requires a large number of observations for the adequate degree of freedom and the lagged variables. Second, the essential condition for the VAR estimators is that the series in $\mathbf{y}_t$ should be jointly stationary. This is rarely true for time-series data, and attempting to tackle the problem by taking differences is rarely satisfactory (Harvey, 1990).

5. EVALUATION AND SUMMARY

In this section, the major findings in this paper are evaluated as an adjustment procedure and as a tool for structural analysis. Further extensions and concluding comments are also provided.

[4]Due to the computational limitation, only the elements with the ten largest fields of influence were used for the experiment.

TABLE 7: Summary of VAR Estimation (for a_{ij} Equation: Top 10 Fields of Influence)

F of I Rank	Sector i	Sector j	Lag	R^2	SIC[§]	Granger Text F-statistic	Coefficients (Standard Error) a_{ij}	r_i	s_j
1	28	27		0.9993	–32.76	390.2*			
			1				0.999*	0.887*	0.904*
							(0.0001)	(0.033)	(0.094)
2	28	4		0.9964	–30.31	77.87*			
			1				0.999*	0.840*	1.306*
							(0.0004)	(0.090)	(0.202)
3	2	27		0.9969	–29.35	67.27*			
			1				0.999*	0.939*	0.695
							(0.0003)	(0.083)	(0.327)
4	2	4		0.9957	–28.69	45.11*			
			1				0.999*	0.920*	0.678
							(0.0004)	(0.114)	(0.354)
5	34	27		0.9952	–32.26	238.56*			
			1				0.999*	0.942*	1.641*
							(0.0002)	(0.043)	(0.148)
6	34	4		0.9925	–30.37	38.37*			
			1				1.193*	0.909*	2.046*
							(0.471)	(0.090)	(0.335)
			2				–0.605	–0.114	–0.458
							(0.643)	(0.399)	(1.064)
			3				0.412	0.510	0.586
							(0.381)	(0.396)	(0.703)
7	36	27		0.9995	–31.60	1396.20*			
			1				0.999*	1.012*	0.476*
							(0.0001)	(0.021)	(0.129)
8	36	4		0.9950	–28.35	135.81*			
			1				1.000*	0.912*	0.369
							(0.0002)	(0.056)	(0.269)
9	28	30		0.9997	–29.21	68.28*			
			1				0.907*	1.058*	0.902*
							(0.408)	(0.076)	(0.065)
			2				0.104	–0.053	0.149
							(0.508)	(0.454)	(0.377)
			3				–0.011	0.032	–0.033
							(0.342)	(0.454)	(0.327)
10	35	27		0.9996	–29.74	3845.27*			
			1				0.999*	1.006*	0.311
							(0.0001)	(0.012)	(0.156)

[§]SIC: Schwarz Information Criterion

* indicates rejecting the null hypothesis (no Granger causality; or insignificant coefficient) at 5 percent level.

Evaluation

Overall, the results indicate that the evidence of different types of contribution by r_i and s_j to the changes in direct and inverse coefficients exist; however, the trends of r_i and s_j can be mostly considered as random movements. This might be caused by either the structure of the CREIM, especially the quantity-adjustment mechanism, or the fluctuation of the exogenous variables in the CREIM. In this regard, further analysis of empirical evidence and the model structure is necessary to develop firm conclusions. The use of disturbance term suggested by Lecomber (1969) and Johansen (1968) has some advantage to introduce a stochastic process in the trends of coefficients. Although this formulation does not provide the estimation accuracy of the RAS procedure and requires the sacrifice of one sector for the estimation, further investigation using more sophisticated techniques, for example the ARIMA model, could make this procedure more accurate in estimation but requires longer time-series data.

Barker (1985) made a strong case for the use of other macro variables in any adjustment process; in fact, Van der Linden and Dietzenbacher (1995) suggested that—although the productivity and intensity change alone are not sufficient to explain all the changes, this does not imply the economic interpretations of r_i and s_j is unjustified. In this regard, Lecomber's (1969) suggestion that even if other variables are introduced there are still advantages in assuming that such effects act uniformly across rows and down columns seems unnecessarily restrictive. However, from a more practical perspective the RAS procedure provides minimum data requirement for updating an input-output table over a relatively short time interval.

As Lecomber (1975) and Allen and Lecomber (1975) suggested, the generalized RAS approach—in which key elements are estimated by exogenous variables, say the top 25 fields of influence, and the remaining elements by the RAS procedure—will improve the estimation accuracy without creating the need for a large estimation model.[5] Nevertheless, the RAS procedure can only provide evidence of structural changes, not the mechanisms and interpretation of these changes.

6. CONCLUSIONS

There are several issues that need to be explored further. First, can a taxonomy of tables be developed in a way that enables the analyst to classify the tables themselves or the changes in terms of certain tendencies? If this were the case, analysis could move in the direction suggested by Jensen et al. (1988); certain coefficients or sets of coefficients may exhibit tendencies toward stability or predictable change whereas others may require more extensive econometric estimation. Second, new approaches can be employed for evaluating sets of

[5]Further discussion on the improvement of estimation accuracy with extra information can be found in Oosterhaven, Piek, and Stelder (1986), responding to the critique by Miller and Blair (1985).

tables. The work of de Mesnard (1990, 1997, 2000) promotes the perspective of a broader set of biproportional and bicausative matrices; in all these approaches, greater creativity needs to be employed in endowing the methods with richer economic meanings and interpretations. Another alternative would be to adopt a nonlinear redistributive dynamic approach. The following models can be used to trace the movement of coefficients over time

$$y_t = ay_{t-1}(1 - y_{t-1}) + \varepsilon_t$$

$$y_t = ay_{t-1}(1 - y_{t-2}) + \varepsilon_t$$

These logistic forms, with different lag functions, provide an alternative way to handle annual changes with a greater focus on changes over a longer time period. Some initial explorations within this paradigm have been reported in Sonis, Dridi, and Hewings (2001) with promising results, although extensions beyond simple 3 × 3 matrices will require greater sophistication in estimation techniques. A third alternative would be to exploit the input-output time series drawing on Markov properties

$$\mathbf{A}_{t+1} = \mathbf{R}_L\mathbf{A}_t \tag{22}$$

$$\mathbf{A}_{t+1} = \mathbf{A}_t\mathbf{S}_R \tag{23}$$

The statistical evaluation of the matrix multipliers $\mathbf{R}_L$ and $\mathbf{S}_R$ requires the estimation of a linear system of equations; this formulation differs from the VAR model with vectorized matrices, because the unknown variables are 36 × 36 = 1,296. Equations (22) and (23) provide for two alternatives, based on row and column properties; however, the respective $\mathbf{R}$, $\mathbf{S}$ matrices are derived from the time series of interactions between matrices over each of the two time periods. However, unlike the more traditional biproportional technique, the adoption of Markov adjustments would exploit a full matrix adjustment process as opposed to the diagonalized matrix adjustment procedures portrayed in Equation (1). This type of analysis using Markov matrices is similar to the causative matrices of Jackson et al. (1990). All of these approaches share similar perspectives with the notion of temporal changes in input-output systems introduced in Sonis and Hewings (1998) but the formal linkages between these methodologies remain to be developed.

Since the publication of Lecomber's (1969) paper, nonlinear regression and the application of maximum-likelihood estimators offer new opportunities to explore the nature of change in the time series of input-output matrices. However, we should recall the findings of Feldman, McClain, and Palmer (1987) and Sonis, Hewings, and Guo (1996) that in an evaluation of sources of structural change, changes in final demand rather than in input-output coefficients were frequently more important components of change in output in a time-series evaluation of U.S. input-output tables.

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APPENDIX

Sectoring Scheme in the CREIM Model

Sector	Title	SIC
1	Livestock, Livestock Products, and Agricultural Products	01, 02
2	Agriculture, Forestry, and Fisheries	07, 08, 09
3	Mining	10, 12, 13, 14
4	Construction	15, 16, 17
5	Food and Kindred Products	20
6	Tobacco	21
7	Apparel and Textile Products	22, 23
8	Lumber and Wood Products	24
9	Furniture and Fixtures	25
10	Paper and Allied Products	26
11	Printing and Publishing	27
12	Chemicals and Allied Products	28
13	Petroleum and Coal Products	29
14	Rubber and Misc. Plastics Products	30
15	Leather and Leather Products	31
16	Stone, Clay, and Glass Products	32
17	Primary Metals Industries	33
18	Fabricated Metal Products	34
19	Industrial Machinery and Equipment	35
20	Electronic and Electric Equipment	36
21	Transportation Equipment	37
22	Instruments and Related Products	38
23	Miscellaneous Manufacturing Industries	39
24	Railroad Transportation and Transportation Services	40–47
25	Communications	48
26	Electric, Gas, and Sanitary Services	49
27	Wholesale and Retail Trade	50–57, 59
28	Finance and Insurance	60–64, 66, 67
29	Real Estate	65
30	Lodging, Business, Engineering, Management, and Legal Services	70, 73, 81, 87, 89
31	Eating and Drinking Places	58
32	Auto Repair, Services, and Parking	75
33	Motion Pictures, and Amusement and Recreation Services	78, 79
34	Other Services (Health, Education, Social, etc.)	
35	Federal Government Enterprises	
36	State and Local Government Enterprises	