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BIBLIOGRAPHY.

Anderson, K. (1982). *Moment Expansions for Robust Statistics*. Ph.D. dissertation, Stanford University.

Anderson, K., Sobel, M., and Uppuluri, V. R. R. (1982). Quota fulfillment times. *The Canadian Journal of Statistics* **10**, 73-88.

Bickel, P. J. (1967). Some contributions to the theory of order statistics. *Proc. Fifth Berkeley Symp. Math. Statist. Prob.* **1**, 575-591. University of California Press.

David, F. N. and Johnson, N. L. (1954). Statistical treatment of censored data. I. Fundamental formulae. *Biometrika* **41**, 223-240.

David, H. A. (1980). *Order Statistics*. Wiley, New York.

Ghosh, J. K. (1971). A new proof of the Bahadur representation of quantiles and applications. *Annals of Mathematical Statistics* **42**, 1957-1961.

Holst, Lars (1981). On sequential occupancy problems. *Journal of Applied Probability* **18**, 435-442.

Rényi, A. (1970). *Probability Theory*. North-Holland, Amsterdam.

Sen, P. K. (1959). On the moments of the sample quantiles. *Calcutta Statistical Association Bulletin* **9**, 1-19.

Smirnov, N. V. (1952). *Limit Distributions for the Terms of a Variational Series*. American Mathematical Society, Translation No. 67.

Wretman, J. (1978). A simple derivation of the asymptotic distribution of a sample quantile. *Scandinavian Journal of Statistics* **5**, 123-124.

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A CONSERVATIVE NONPARAMETRIC DISTRIBUTION-FREE CONFIDENCE BOUND FOR THE SHIFT IN THE CHANGEPOINT PROBLEM

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Key Words and Phrases: confidence bound; Monte Carlo simulation study; change in location.

ABSTRACT

The problem of testing whether there is a change in location in a sequence of random variables that are taken over time was discussed in several papers. In this paper we develop a conservative nonparametric distribution-free confidence bound for the amount of shift and give some Monte Carlo results to show how conservative the bound is.

1. INTRODUCTION

Sometimes, when observations are taken in an order (i.e. time), one may be interested to know whether they all come from the same distribution or there is a change in location so that the last ones come from a shifted distribution. In the latter case, it might be important to estimate the magnitude of the shift.

Formally, let $X_1, \ldots, X_r, X_{r+1}, \ldots, X_n$ be independent, continuous random variables such that X_i, $i = 1, \ldots, r$, has distribution function $F(x)$, and X_j, $j = r+1, \ldots, n$, has distri-

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bution function $F(x-\theta)$, with $-\infty < \theta < \infty$ unknown. The integer r, which is called the changepoint, is also taken to be unknown. The hypothesis of interest is $H_0 : \theta = 0$ vs. $H_1 : \theta > 0$ $(\theta < 0, \theta \neq 0)$. This problem is considered in several papers (Page (1954, 1955), Chernoff and Zacks (1964), Bhattacharyya and Johnson (1968), Sen and Srivastava (1975a, 1975b, 1976), Hawkins (1977), Pettitt(1979), Schechtman and Wolfe (1981) and Schechtman (1981).)

In this paper we consider the one-sided alternative $H_1: \theta > 0$ and develop a conservative nonparametric distribution - free $100(1-\alpha)\%$ confidence bound for θ.

Let $\underset{\sim}{X} = (X_1, \ldots, X_n)$ and let $U_{i,n-i}(\underset{\sim}{X})$ be the Mann-Whitney form of the Mann-Whitney-Wilcoxon statistic computed for the two samples $(X_1, \ldots, X_i)$ and $(X_{i+1}, \ldots, X_n)$ of sizes i and (n-i), respectively. We base our confidence bound on the test statistic

$$VV = \max_{1 \le i \le n-1} U_i^N(\underset{\sim}{X}) \quad , \text{ where}$$

$$U_i^N(\underset{\sim}{X}) = \left(\frac{U_{i,n-i}(\underset{\sim}{X})}{i(n-i)} - \frac{1}{2} \right) \left(\frac{(n+1)}{12i(n-i)} \right)^{-\frac{1}{2}}$$

This test statistic was considered by Sen and Srivastava (1975a) and by Schechtman and Wolfe (1981) for testing $H_0: \theta = 0$ vs. $H_1: \theta > 0$, where large values of VV are taken to be significant.

In Section 2 we develop the conservative $100(1-\alpha)\%$ confidence bound which is nonparametric distribution-free over the class of all continuous distributions, and in Section 3 we report some results from a simulation study which show how conservative the bound is for different underlying distributions and for several values of θ and r.

2. THE CONFIDENCE BOUND

In this section we shall develop a lower bound $\hat{\theta}$ which will be reasonably sure (with probability at least $1-\alpha$ independent of the underlying distribution) not to fall above θ.

Let b satisfy

$$P_0(VV \le b) = 1 - \alpha$$

(see Schechtman (1981) for tables of the approximate null distribution of VV).

Then

$$1 - \alpha = P_0\left(\max_{1 \le i \le n-1} U_i^N(\underset{\sim}{x}) \le b \right) =$$

$$= P_0\left(U_{i,n-i}(\underset{\sim}{x}) \le b \sqrt{\frac{(n+1)i(n-i)}{12}} + \frac{i(n-i)}{2} \text{ for all } i\right) =$$

$$= P_0(U_{i,n-i}(\underset{\sim}{x}) \le a_i \quad \text{for all } i)$$

where

$$a_i = b \sqrt{\frac{(n+1)i(n-i)}{12}} + \frac{i(n-i)}{2}, \quad i = 1, \ldots, n-1 .$$

<u>Lemma 1</u> Let $a_1, a_2, \ldots, a_{n-1}$ be (n-1) arbitrary numbers, then for any r

$$P_{\theta,r}(U_{i,n-i}(\underset{\sim}{x}^i, \theta) \le a_i \quad \text{for all } i) \ge$$

$$P_0(U_{i,n-i}(\underset{\sim}{x}) \le a_i \quad \text{for all } i)$$

where $(\underset{\sim}{x}^i, \theta) = (x_1, \ldots, x_i, x_{i+1} - \theta, \ldots, x_n - \theta)$ and $P_{\theta,r}$ is the probability computed under H_1, with θ being the location parameter and r being the point of change.

<u>Proof</u>: $P_{\theta,r}(U_{i,n-i}(\underset{\sim}{x}^i, \theta) \le a_i$ for all i) =

$$= P_{\theta,r}(U_{i,n-i}(x_1, \ldots, x_i, x_{i+1}-\theta, \ldots, x_r-\theta, x_{r+1}-\theta, \ldots, x_n-\theta) \le$$

$$\le a_i \quad \text{for } 1 \le i < r ,$$

$$U_{r,n-r}(x_1, \ldots, x_r, x_{r+1}-\theta, \ldots, x_n-\theta) \le a_r ,$$

$$U_{i,n-i}(x_1, \ldots, x_r, x_{r+1}, \ldots, x_i, x_{i+1}-\theta, \ldots, x_n-\theta) \le$$

$$\le a_i \quad \text{for } r < i \le n-1) .$$

Let $V(\underset{\sim}{x})$ be a vector valued function defined as follows:

$$V(\underset{\sim}{x}) = (V_1(\underset{\sim}{x}), \ldots, V_{n-1}(\underset{\sim}{x})), \quad \text{where}$$

$$V_i(\underset{\sim}{x}) = (x_1, \ldots, x_i, x_{i+1}-\theta, \ldots, x_r-\theta, x_{r+1}, \ldots, x_n)$$

$$\text{for } 1 \le i < r ,$$

$$V_r(\underset{\sim}{x}) = \underset{\sim}{x}$$

$$V_i(\underset{\sim}{x}) = (x_1, \ldots, x_r, x_{r+1}+\theta, \ldots, x_i+\theta, x_{i+1}, \ldots, x_n)$$

$$\text{for } r < i \le n-1 .$$

Since $(\underset{\sim}{X}^r,\theta)|_{\theta,r} \overset{d}{=} \underset{\sim}{X}|_{\theta=0}$ we can apply the function V to both sides of the $\overset{d}{=}$ sign and get:

$$V(\underset{\sim}{X}^r,\theta)|_{\theta,r} \overset{d}{=} V(\underset{\sim}{X})|_{\theta=0} \quad . \tag{2.1}$$

Now we apply the vector valued function $U(\underset{\sim}{x}) = (U_{1,n-1}(\underset{\sim}{x}),\ldots,$ $U_{n-1,1}(\underset{\sim}{x}))$ to both sides of equation (2.1) and get:

$$U(V(\underset{\sim}{X}^r,\theta))|_{\theta,r} \overset{d}{=} U(V(\underset{\sim}{X}))|_{\theta=0} \quad .$$

Notice that

[illegible]

$$= U_{i,n-i}(x_1,\ldots,x_r,x_{r+1}+\theta,\ldots,x_i+\theta,x_{i+1},\ldots,x_n)$$
$$\text{for } r<i\leq n-1.$$

Therefore,

$$P_{\theta,r}(U_{i,n-i}(\underset{\sim}{x}^i,\theta) \leq a_i \quad \text{for all } i) = \tag{2.2}$$
$$= P_0(U_{i,n-i}(x_1,\ldots,x_i,x_{i+1}-\theta,\ldots,x_r-\theta,x_{r+1},\ldots,x_n) \leq a_i$$
$$\text{for } 1 \leq i < r \quad ,$$
$$U_{r,n-r}(\underset{\sim}{x}) \leq a_r \ ,$$
$$U_{i,n-i}(x_1,\ldots,x_r,x_{r+1}+\theta,\ldots,x_i+\theta,x_{i+1},\ldots,x_n) \leq a_i$$
$$\text{for } r < i \leq n-1) \ .$$

For any numbers $y_1, \ldots, y_n$,

(a) $U_{i,n-i}(y_1,\ldots,y_i,y_{i+1}-\theta,\ldots,y_r-\theta,y_{r+1},\ldots,y_n) \leq U_{i,n-i}(\underset{\sim}{y})$

$$\text{for } 1 \leq i < r \ ,$$

and

(b) $U_{i,n-i}(y_1,\ldots,y_r,y_{r+1}+\theta,\ldots,y_i+\theta,y_{i+1},\ldots,y_n) \leq U_{i,n-i}(\underset{\sim}{y})$

$$\text{for } r < i \leq n-1$$

Therefore the event

$$A = \{U_{i,n-i}(x_1,\ldots,x_i,x_{i+1}-\theta,\ldots,x_r-\theta,x_{r+1},\ldots,x_n) \leq a_i \quad \text{for } 1\leq i<r,$$
$$U_{r,n-r}(\underset{\sim}{x}) \leq a_r \ ,$$
$$U_{i,n-i}(x_1,\ldots,x_r,x_{r+1}+\theta,\ldots,x_i+\theta,x_{i+1},\ldots,x_n) \leq a_i \quad \text{for } r<i\leq n-1\}$$

Contains the event

[illegible]

then from Lemma 1,

$$P_{\theta,r}(U_i^N(\underset{\sim}{x}^i,\theta) \leq b \quad \text{for all } i) =$$
$$= P_{\theta,r}(U_{i,n-i}(\underset{\sim}{x}^i,\theta) \leq a_i \quad \text{for all } i) \geq$$
$$\geq P_0(U_{i,n-i}(\underset{\sim}{x}) \leq a_i \quad \text{for all } i) =$$
$$= P_0(U_i^N(\underset{\sim}{x}) \leq b \quad \text{for all } i),$$

or, equivalently,

$$P_{\theta,r}(\max_{1\leq i\leq n-1} U_i^N(\underset{\sim}{x}^i,\theta) \leq b) \geq P_0(\max_{1\leq i\leq n-1} U_i^N(\underset{\sim}{x}) \leq b) = \tag{2.3}$$
$$= P_0(VV \leq b) = 1 - \alpha \ .$$

We note that (2.3) is true for all r, i.e. r is a nuisance parameter and will be omitted in what follows.

Let $W_{j,k}^{(i)} = X_j - X_k$, $\quad i = 1,\ldots,n-1$; $k = 1,\ldots, i$
$j = i+1,\ldots, n.$

and for each i let $W^{(i)}_{(1)} \le W^{(i)}_{(2)} \le \dots \le W^{(i)}_{(i(n-i))}$ be the sequences of $\{W^{(i)}_{j,k}\}_{j,k}$ in non-decreasing order.

Let
$$\hat{\theta} = \max_{1\le i\le n-1} W^{(i)}_{(-[b\sqrt{\frac{(n+1)i(n-i)}{12}} - \frac{i(n-i)}{2}])} \tag{2.4}$$
where $[\cdot]$ is the greatest integer function.

Theorem 2.1 A conservative nonparametric distribution - free $100(1-\alpha)\%$ confidence bound for θ is given by $\theta \ge \hat{\theta}$, where θ is defined in (2.4).

Remark: for i close to 1 or n, $-[b\sqrt{\frac{(n+1)i(n-i)}{12}} - \frac{i(n-i)}{2}]$ may be negative. For notational convenience we define $W^{(i)}_{(t)} = -\infty$ for non-positive t.

Proof:

$$1-\alpha \le P_\theta\left(\max_{1\le i\le n-1} U^N_i(\underset{\sim}{x}^i, \theta) \le b\right) =$$

$$= P_\theta\left(U_{i,n-i}(\underset{\sim}{x}^i,\theta) \le a_i \text{ for all } i\right) =$$

$$= P_\theta\left(\begin{matrix}(\#\text{ of times } x_j-\theta > x_k \\ k=1,\dots,i;\ j=i+1,\dots,n)\end{matrix} \le a_i \text{ for all } i\right) =$$

$$= P_\theta\left(\begin{matrix}(\#\text{ of times } x_j-x_k > \theta \\ k=1,\dots,i;\ j=i+1,\dots,n)\end{matrix} \le a_i \text{ for all } i\right) =$$

$$= P_\theta\left(\theta \ge W^{(i)}_{([i(n-i)-a_i+1])} \text{ for all } i \text{ such that}\right.$$

a_i is not an integer, and

$$\left.\theta \ge W^{(i)}_{(i(n-i)-a_i)} \text{ for all } i \text{ such that } a_i \text{ is an integer}\right) =$$

$$= P_\theta\left(\theta \ge W^{(i)}_{(-[b\sqrt{\frac{(n+1)i(n-i)}{12}} - \frac{i(n-i)}{2}])} \text{ for all } i\right) =$$

$$= P_\theta\left(\theta \ge \max_{1\le i\le n-1} W^{(i)}_{(-[b\sqrt{\frac{(n+1)i(n-i)}{12}} - \frac{i(n-i)}{2}])}\right).$$

Q.E.D.

3. A SIMULATION STUDY

Since our confidence bound is conservative, we need to check that it is not trivial, i.e. that $P(\theta \ge \hat{\theta}) < 1$. In order to do it we ran a simulation study as follows: we used 5 underlying distri-

TABLE I : CONFIDENCE COEFFICIENTS, MEANS OF CONFIDENCE BOUNDS AND STANDARD DEVIATIONS

$1-\alpha = 0.9$

	r = 5			r = 10		
	$\hat{P}$	Mean($\hat{\theta}$)	s.d($\hat{\theta}$)	$\hat{P}$	Mean($\hat{\theta}$)	s.d($\hat{\theta}$)
Normal						
$P(X_n > X_1) = 0.7$, $\theta = 0.742$	.957	-0.140	.512	.947	-0.026	.481
$P(X_n > X_1) = 0.8$, $\theta = 1.190$	.970	0.177	.512	.972	0.331	.472
Cauchy						
$P(X_n > X_1) = 0.7$, $\theta = 1.460$	.955	-0.613	1.891	.951	-0.355	1.749
$P(X_n > X_1) = 0.8$, $\theta = 2.755$	.972	0.275	1.912	.965	0.670	1.921
Double Exponential						
$P(X_n > X_1) = 0.7$, $\theta = 0.873$	.975	0.225	.361	.972	0.315	.311
$P(X_n > X_1) = 0.8$, $\theta = 1.467$	.982	0.727	.398	.979	0.842	.328
Exponential						
$P(X_n > X_1) = 0.7$, $\theta = 0.511$	.960	-0.120	.444	.946	-0.065	.423
$P(X_n > X_1) = 0.8$, $\theta = 0.916$	.972	.161	.486	.968	0.254	.431
Uniform						
$P(X_n > X_1) = 0.7$, $\theta = 0.225$	.955	-.037	.151	.943	0	.143
$P(X_n > X_1) = 0.8$, $\theta = 0.367$	.973	.063	.154	.967	0.109	.140

butions: Normal, Cauchy, exponential, double exponential and uniform. For each of these distributions we generated 5000 samples of size 20 for each of the 4 cases:

1) $r = 5$ and θ is such that $P(X_n > X_1) = 0.7$
2) $r = 5$ and θ is such that $P(X_n > X_1) = 0.8$
3) $r = 10$ and θ is such that $P(X_n > X_1) = 0.7$
4) $r = 10$ and θ is such that $P(X_n > X_1) = 0.8$

TABLE II : CONFIDENCE COEFFICIENTS, MEANS OF CONFIDENCE BOUNDS AND STANDARD DEVIATIONS

	$1-\alpha = 0.95$, $r = 5$			$r = 10$		
	$\hat{P}$	Mean($\hat{\theta}$)	s.d($\hat{\theta}$)	$\hat{P}$	Mean($\hat{\theta}$)	s.d($\hat{\theta}$)
Normal						
$P(X_n > X_1) = 0.7$, $\theta = 0.742$	.982	-0.310	.518	.972	-0.175	.488
$P(X_n > X_1) = 0.8$, $\theta = 1.190$	.986	0.004	.521	.982	0.195	.481
Cauchy						
$P(X_n > X_1) = 0.7$, $\theta = 1.460$	.976	-1.158	2.151	.976	-0.842	1.967
$P(X_n > X_1) = 0.8$, $\theta = 2.755$	.986	-0.263	2.211	.983	0.206	2.130
Double Exponential						
$P(X_n > X_1) = 0.7$, $\theta = 0.873$	.986	0.114	.378	.985	0.229	.331
$P(X_n > X_1) = 0.8$, $\theta = 1.467$	.991	0.616	.423	.986	0.764	.324
Exponential						
$P(X_n > X_1) = 0.7$, $\theta = 0.511$	.981	-0.277	.493	.969	-0.204	.470
$P(X_n > X_1) = 0.8$, $\theta = 0.916$	.986	0.011	.537	.982	0.127	.471
Uniform						
$P(X_n > X_1) = 0.7$, $\theta = 0.225$	.977	-0.091	.154	.969	-0.047	.146
$P(X_n > X_1) = 0.8$, $\theta = 0.367$	.985	0.011	.156	.983	0.068	.142

where X_1 has c.d.f. $F(x)$ and X_n had c.d.f. $F(x-\theta)$.

The values of θ for the different distributions are calculated in detail in Schechtman (1981).

For each generated sample we calculated the confidence bound and then we estimated $P = P(\theta \geq \hat{\theta})$ using

$$\hat{P} = \frac{(\#\text{ of simulated samples for which } \theta \geq \hat{\theta})}{5000}$$

The estimated probabilities are given in Tables I and II for nominal $1 - \alpha = 0.9$ and 0.95, respectively. In addition, for each of these estimated confidence coefficients we report the average of the confidence bounds $\hat{\theta}_i$ $(i=1,\ldots,5000)$ and the sample standard deviation $\left(\sum_{i=1}^{5000} \frac{(\hat{\theta}_i - \bar{\hat{\theta}})^2}{4999}\right)^{\frac{1}{2}}$ as well as the true value of θ.

In most cases, as r gets closer to $n/2$, the lower bound is less conservative (i.e. the estimated probabilities get closer to the nominal $1-\alpha$). It can also be seen that as $P(X_n > X_1)$ gets larger, the lower bound is more conservative. (The case, where $P(X_n > X_1) = 0.5$ is the case for which X_1, X_n are i.i.d. (i.e. $\theta = 0$) and $P(\theta > \hat{\theta}) = 1 - \alpha$) .

BIBLIOGRAPHY

Bhattacharyya, G.K., & Johnson, P.A. (1968). Nonparametric Tests for Shift at an Unknown Time Point. Annals of Mathematical Statistics 39, 1731 - 1743.

Chernoff, H. & Zacks, S. (1964). Estimating the Current Mean of a Normal Distribution which is Subjected to Changes in Time. Annals of Mathematical Statistics 35, 999-1013.

Gardner, L.A., Jr. (1969). On Detecting Changes in the Menas of Normal Variates. Annals of Mathematical Statistics 40, 116-126.

Hawkins, D.M. (1977). Testing a Sequence of Observations for a Shift in Location. J. Amer. Statist. Assoc. 72, 180-186.

Hinkley, David V. (1970). Inference about the changepoint in a sequence of random variables. Biometrika 57, 1-17.

Kander, Z. and Zacks, S. (1966). Test procedures for possible changes in parameters of statistical distribution occuring at unknown time points. Annals of Mathematical Statistics 37, 1196-1210.

Page, E.S. (1954). Continuous Inspection Schemes. Biometrika 41, 100-115.

Page, E.S. (1955). A Test for a Change in a Parameter Occurring at an Unknown Point. Biometrika 42, 523-527.

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BIBLIOGRAPHY

Bhattacharyya, G.K., & Johnson, P.A. (1968). Nonparametric Tests for Shift at an Unknown Time Point. Annals of Mathematical Statistics 39, 1731 - 1743.

Chernoff, H. & Zacks, S. (1964). Estimating the Current Mean of a Normal Distribution which is Subjected to Changes in Time. Annals of Mathematical Statistics 35, 999-1013.

Gardner, L.A., Jr. (1969). On Detecting Changes in the Menas of Normal Variates. Annals of Mathematical Statistics 40, 116-126.

Hawkins, D.M. (1977). Testing a Sequence of Observations for a Shift in Location. J. Amer. Statist. Assoc. 72, 180-186.

Hinkley, David V. (1970). Inference about the changepoint in a sequence of random variables. Biometrika 57, 1-17.

Kander, Z. and Zacks, S. (1966). Test procedures for possible changes in parameters of statistical distribution occuring at unknown time points. Annals of Mathematical Statistics 37, 1196-1210.

Page, E.S. (1954). Continuous Inspection Schemes. Biometrika 41, 100-115.

Page, E.S. (1955). A Test for a Change in a Parameter Occurring at an Unknown Point. Biometrika 42, 523-527.

Pettitt, A.N. (1979). A Nonparametric Approach to the Changepoint Problem. Applied Statistics 28, 126-135.

Schechtman, E. (1980). A Nonparametric Test for the Changepoint Problem. Ph.D. Thesis, Department of Statistics, The Ohio State University.

Schechtman, E. & Wolfe, D.A. (1981). Distribution-Free Tests for the Changepoint Problem. Submitted for publication.

Schechtman, E. (1981). A Nonparametric Test for Detecting Changes in Location. To appear in Communications in Statistics.

Sen, A. & Srivastava, M.S. (1975a). On Tests for Detecting Change in Mean. Annals of Statistics 3, 98-108.

Sen, A. & Srivastava, M.S. (1975b). Some One-Sided Tests for Change in Levels. Technometrics 17, 61-64.

Sen, A. & Srivastava, M.S. (1976). On Tests for Detecting Change in Mean when Variance is Unknown. Annals of the Institute of Statistical Mathematics 27, 479-486.

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MODIFIED CRAMER-VON MISES AND ANDERSON-DARLING TESTS FOR WEIBULL DISTRIBUTIONS WITH UNKNOWN LOCATION AND SCALE PARAMETERS

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ABSTRACT

The standard Cramer-von Mises and Anderson-Darling goodness-of-fit tests require continuous underlying distributions with known parameters. In this paper, tables of critical values are generated for both tests for Weibull distributions with unknown location and scale parameters and known shape parameters. The powers of the Cramer-von Mises, Anderson-Darling, Kolmogorov-Smirnov, and Chi-Square tests for this situation are investigated. The Cramer-von Mises test has most power when the shape is 1.0 and the Anderson-Darling test has most power when the shape is 3.5. Finally, a relation between critical value and inverse shape parameter is presented.

1. INTRODUCTION

One of the more important families of distributions in reliability studies is the Weibull family. The family is represented by the density function

$$f(x) = \frac{K(x-c)^{K-1}}{\theta^K} \exp\left\{-\left(\frac{x-c}{\theta}\right)^K\right\}, \qquad (1)$$

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