

CLIMATE CHAOTIC INSTABILITY: STATISTICAL DETERMINATION AND THEORETICAL BACKGROUND

RAYMOND SNEYERS

Institut royal météorologique de Belgique, Avenue Circulaire 3, B-1180 Bruxelles, Belgium

SUMMARY

The paper concerns the determination of statistical climate properties, a problem especially important for climate prediction validation. After a brief review of the times series analyses applied on secular series of observations, an appropriate method is described for characterizing these properties which finally reduces itself to the search for existing change-points. The examples of the Jones North Hemispheric land temperature averages (1856–1995) and of the Prague Klementinum ones (1771–1993) are given and results discussed. Relating the observed chaotic character of the climatological series to the non-linearity of the equations ruling the weather and thus climate evolution, and presenting the example of a solution of the Lorenz non-linear equations showing that non-linearity may be responsible for the instability of the generated process, it seems justified to conclude that there are severe limits to climate predictability at all scales. © 1997 John Wiley & Sons, Ltd.

Environmetrics, **8**, 517–532 (1997)

No. of Figures: 2. No. of Tables: 13. No. of References: 36.

KEY WORDS time series analysis; change-point determination; climate instability theory

1. CLIMATE CHANGE DETECTION

The World Climate Program was created after the first World Climate Conferences held in 1979 in Geneva with the purpose of detecting and predicting man-made climate changes. Moreover, for validation aims, statistical investigation of the available secular climatological series was reduced to the global scale

Whether we know what an unchanged climate is, and, if we do, whether the right tools for detecting its change exist and are correctly used, are questions raised by the implementation of this program. Moreover, the degree to which the global approach is able to give an exhaustive idea of climate evolution at all scales leaves many climatologists doubtful. These questions justify the present statistical investigation on climatological series analysis.

Climate evolution has always been a major concern for climatologists. With empirical, parametric (with normality assumption) or distribution free methodologies, the alternatives generally considered have been either progressive trends or abrupt changes, the former in the case of air temperature or rainfall, and the latter for detecting observational non-homogeneities.

* Correspondence to: R. Sneyers, Institut royal météorologique de Belgique, Avenue Circulaire 3, B-1180 Bruxelles, Belgium.

In particular, testing the existence of non-homogeneities in series of observations has been carried out using the parametric criteria of Abbe (Kendall 1971) based on the ratio of the series variance to the variance of the differences between consecutive elements. Equivalent to the serial correlation test statistic and sensitive to both first-order serial correlation and mean instability, this statistic has apparently not been used for climate instability detection.

The existence of a climate warming was first suggested by empirical running averages analyses (Lewis 1947; Lysgaard 1948; Vandenplas 1948). Though not very convincing, this existence seemed to be confirmed by comparing the mean of observation sequences to the one of earlier reference periods or by testing the significance of linear trends (Sneyers 1954, 1956). Moreover, Vialar (1952) introduced the scanning procedure consisting of testing homogeneity of the two partial series into which a point divides the complete series when running from the beginning to the end of the series. The parametric test statistic was already the one used by Hawkins (1977) and Worsley (1979), but Vialar's aim was limited to the determination of the beginning of a linear trend.

Proceeding in the same way as Vialar, but submitting each partial series to the distribution free trend test of Mann (1945), it was found that for Brussels and Paris, for each season, a point divided the complete series into two stationary partial series, while a significant trend statistic value was found for every complete series. The change in the mean was placed round 1910 for winter and spring and round 1930 for summer and autumn (Sneyers 1958).

Obviously, the trend test method was found to be efficient essentially in the case of one change-point. For the same purpose, using the homogeneity two-sample analysis of variance test, Hawkins (1977), Maronna and Yohai (1978) and Worsley (1979) gave an optimal solution in the parametric case, while with the two-sample homogeneity test of Mann–Whitney (1947), Pettitt (1979) gave the corresponding distribution free solution. Both parametric and distribution free procedures reduce to cumulative summations (cusums) of deviations from the mean of the elements of the series or of the ranks associated with these elements. In this way, the use of the empirical method of cusums of deviations from the mean for testing homogeneity of observation series (Craddock 1979) finds its justification. Using the Pettitt method, Snijders (1983) showed that African rainfall series were perturbed by an abrupt decrease and Sneyers *et al.* (1990) compared the efficiency of the trend and the rank cusums method.

For the case of several change-points, empirical cusums were used by Coops and Schuurmans (1986), for alternating ones, Lombard (1988) estimated their number by applying a Fourier analysis, Sneyers *et al.* (1991) made a first approach of their determination based on the joint use of the trend and the rank cusums method, and Coops (1992) detected their position by applying the Pettitt test on predetermined partial series. In answer to the need for clarification, the theory and determination of change-points have been derived from the definition of simple randomness in Sneyers (1992a, 1992b, 1995).

2. SERIES INTERNAL STRUCTURE ANALYSIS: CHANGE-POINT DETERMINATION

2.1. Testing simple randomness (i.i.d.)

The simplest internal structure of a statistical time series being its simple randomness, the first step is to test both the independence and the identity of distribution defining the simple randomness of its elements. For this purpose, the alternative hypotheses to be considered as most likely are serial

correlation and distribution instability, the latter possibly concerning the mean as well as the dispersion. Moreover, parametric tests being based on the underlying hypotheses of a normal distribution, their use involves here a risk of so-called error of the third kind (correct rejection of the null hypothesis, but for the wrong reason). They have thus to be discarded for this case.

On the contrary, applied on the ranks associated with the series elements which, under the null hypothesis of simple randomness, give unbiased estimates for the value of the distribution function at these elements, whatever this distribution is, distribution free tests avoid the aforementioned drawback. Moreover, they have the advantage of having an efficiency very near to the one of the corresponding parametric test.

Therefore, the chosen tests are the Wald–Wolfowitz test of randomness against serial correlation in its rank version, and the sequentially applied Mann trend test (Sneyers 1975).

In addition, in order to test hypotheses in a homogeneous way, levels for which tests applied on partial series are significant have to be reduced to the size of the complete series, a reduction which is realized through the relation $(1 - \alpha_0) = (1 - \alpha)^{n/n'}$ where α_0 is the reduced level, α is the original one and n and n' are, respectively, the sizes of the complete and of the partial series.

For power consideration, it is obvious that tests should be one-sided or two-sided according to whether the test concerns absence or presence of homogeneity. In particular, for serial correlation, the test will always be one-sided, while for the trend test it will be one-sided for detecting internal trends, but two-sided when testing homogeneity for non-contiguous partial series.

2.2. Change-point determination

A point separating two partial series is a change-point when, being both i.i.d., these series have different distribution. At that point, there is thus a change in the mean or in the dispersion or a change in both the mean and the dispersion.

In the case of one-change-point in the mean, it is obvious that among all the points of the complete series separating it into two partial series, at the change-point the probability for having the extreme value of the mean homogeneity test statistic will be highest. This justifies the use of the Mann–Whitney test statistic for the Pettitt's change-point determination.

However, the result of the test giving no information about the internal stability for the separated partial series, Pettitt's change-point determination does not ensure the detection of a real change-point.

For this reason, a more appropriate determination is obtained by applying a sequential trend analysis consisting of the computation of the standardized Mann trend statistics $u(t_i)$ and $u(t'_j)$ for, respectively, the partial series beginning at x_1 and ending at x_i and the ones beginning at x_i and ending at x_n , for $i, j = 1, 2, \dots, n$. The determination is then given by the k value for which the statistic

$$X_k(t) = u^2(t_k) + u^2(t'_{k+1})$$

is minimum with, at the same time, the absence of any significant value of $u(t)$ for any x_i preceding x_k or of $u(t)$ for any x_j following x_{k+1} . The level to which the shift introduced by the change-point is significant is then given by the value $u(t_n)$ corresponding to the trend statistic for the complete series.

In the case of several existing change-points, the procedure consists in retrieving successively from the complete series the partial series appearing as i.i.d. This may consist of considering the

partial series placed at the beginning or at the end of the series. When the other end of the series has been reached, and the process is returning to the beginning, re-estimation of each change-point is made inside the corresponding joined two consecutive partial series. This procedure is repeated forward and backwards, and when it is stabilized, the final estimates give the solution.

2.3. The test statistics

2.3.1. The independence test statistic

In time series, independence of elements implying one of their ranks, this may be tested through the significance of the serial correlation coefficient between these ranks. Under the hypothesis of randomness, two versions may be used: the circular one, computing the correlation coefficient for the series (x_1, x_2, \dots, x_n) and $(x_2, x_3, \dots, x_n, x_1)$ used by Wald and Wolfowitz, or the non-circular one, relative to the series $(x_1, x_2, \dots, x_{n-1})$ and (x_2, x_3, \dots, x_n) .

Under the hypothesis of simple randomness, these statistics have asymptotic normal distributions with mean and variance

$$E(r) = -(n-1)^{-1} \text{ or } 0 \text{ and } \text{var } r = n^{-1}.$$

For small samples, the normal approximation gives too low values, the bias becoming negligible for sizes approaching 30 and the asymptotic efficiency relative to the corresponding parametric most powerful test reaches 0.91 (Bartels 1982).

Note that if, given its form, the test statistic is the most appropriate against the alternative of an existing serial correlation, in particular, this statistic is sensitive to a mean instability, this instability implying an increased probability for having consecutive deviations to the general mean with the same sign inside the sequences with a deviating mean. This explains the use of the old Abbe criteria as the homogeneity test statistic, due to the fact that it is a mathematical transform of the serial correlation coefficient (Sneyers 1955).

2.3.2. The stability test statistics

For Pettitt's test, under the hypothesis of simple randomness, the statistic $X(P)$ derived from the Mann-Whitney homogeneity test and the level of significance α in a one-sided test for its extreme X_E value are

$$X_k(P) = 2R_k - k(n+1) \text{ and } \alpha = \exp[-6(X_E)^2/(n^3 + n^2)]$$

where $R_k = \sum_{1,k} r(x_i)$, $r(x_i)$ being the rank of the element x_i .

As for the Mann-Whitney test, the asymptotic probabilities α are not valid for small samples. For this case, Snijders (1984) has given the necessary tables. Its asymptotic efficiency relative to the most powerful parametric test of homogeneity depends on the one of the Mann-Whitney test which is 0.955 (Mood 1954).

For the Mann trend test, if for each element x_i , the number of smaller elements preceding x_i is n_i , the test statistic is

$$t = \sum_{1,n} n_i$$

which, under the hypothesis of simple randomness, has mean and variance

$$E(t) = n(n-1)/4 \text{ and } \text{var } t = n(n-1)(2n+5)/72.$$

Moreover, the normal approximation becomes sufficiently valid for $n > 10$ and the asymptotic efficiency relative to the most powerful parametric test against linear trend is 0.98 (Kendall and Stuart 1966).

Note that having $r(x_k) = n_k - n'_k + 1$, the homogeneity test statistic R_k is found to be equivalent to the statistic $(t_k + t'_{k+1} - t_n)$ which does not use separately the information contained in each of its components, as is done with the trend statistic $X_k(t)$. Thus, among those two statistics, the latter one appears to be more appropriate for time series analysis.

2.3.3. Global significance for several change-points

In the case of the detection of several change-points, their global significance level may be based on the Fisher test, the statistic of which may be written as

$$X(F) = -\sum \ln(\alpha_0)$$

the summation operating on the significance levels α_0 of the n_p change-points reduced to the complete series. Under the null hypothesis of randomness, this statistic has a gamma distribution with shape and scale parameters n_p and 1.

On the other hand when, among the partial series separated by change-points, non-contiguous change-points have near averages, the null hypothesis of homogeneity may be verified by testing the simple randomness of the joined series being considered.

3. EXAMPLES

3.1. The Jones North Hemisphere land temperature averages (1856–1995)

The data give in °C the temperature deviations from the 1961–1990 normals (Jones 1994).

3.1.1. Testing independence and stability in the annual mean

From the monthly averages prepared by Jones (1994), annual and seasonal averages have been computed. For the annual averages, Figure 1(a), giving the rank expectation of the common distribution function for each element of the series, under the i.i.d. null hypothesis, shows a grouping of low values at the beginning of the series and of high values at its end. Figure 1(b), representing the variation of the $X(P)$ statistic, suggests 1920 as a unique change-point. However, in Figure 1(c), the value of $u(t')$ for 1920 found through the sequential trend analysis excludes this hypothesis.

The internal structure analysis applied to the complete series gives the following results:

	n	m	s	$u(r)$	$u(t)$	$u_x(t)$
1856–1995	140	−0.180	0.364	4.72	7.58	7.58/7.86

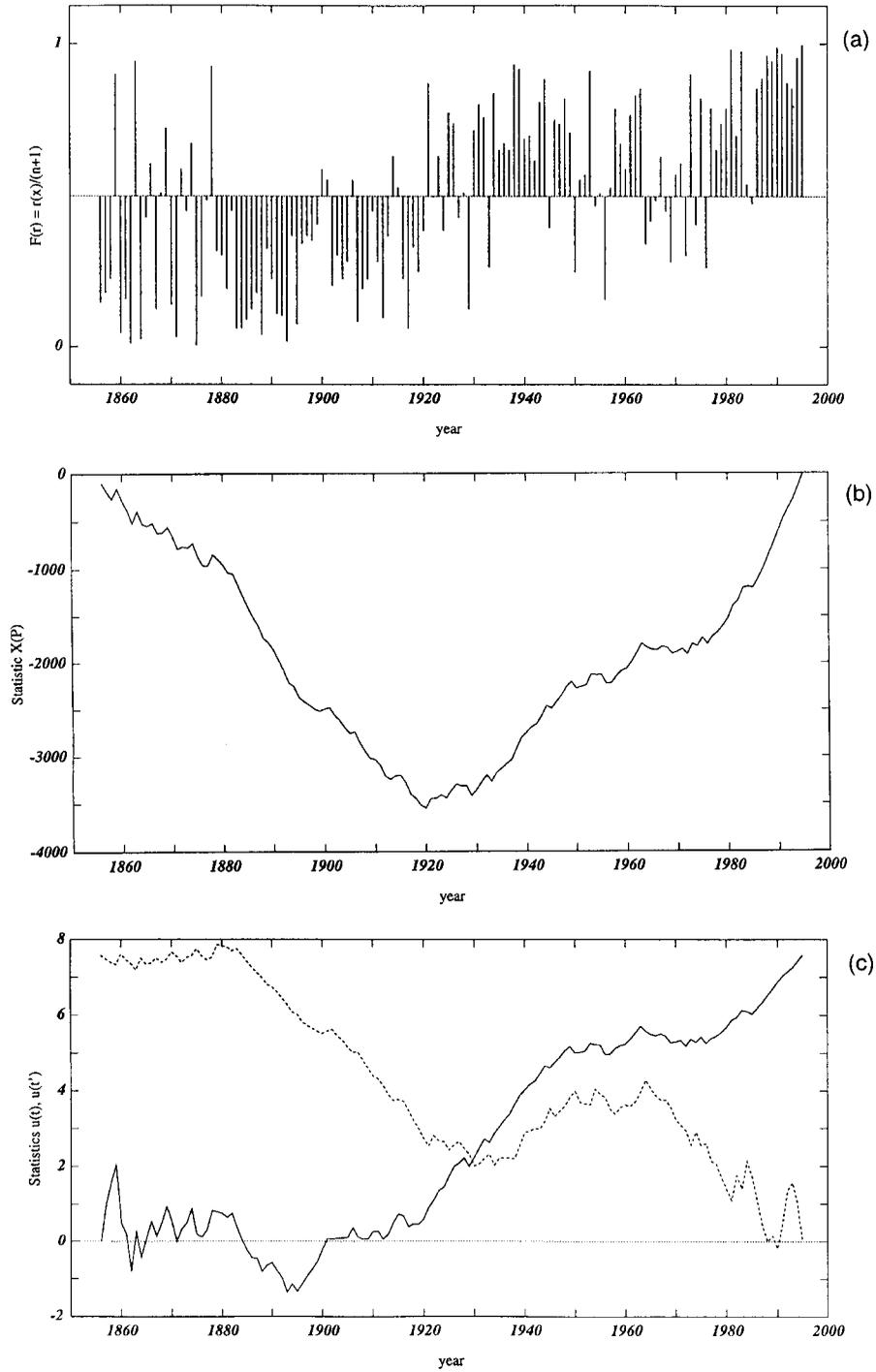


Figure 1. North Hemisphere annual land air temperature (1856–1995)

Table I. Change-point analysis of the Jones data. P are the years ending each i.i.d. partial series, n is the size of the series, m and s its average and standard deviation. The standardized serial and trend statistic are denoted by $u(r)$ and $u(t)$, $u_x(t)$ are the extreme values of the standardized trend statistic found forwards and backwards through the sequential analysis. N is the value of the D'Agostino (1971) or Shapiro–Wilk (Shapiro and Wilk, 1965) normality test statistic, $u(t_0)$ is the value of the trend test statistic for the sequence formed by the two contiguous partial series joined by the change-point, and α_0 is the significance level for this trend statistic reduced to the complete series

P	n	m	s	$u(r)$	$u(t)$	$u_x(t)$	N	$u(t_0)$	α_0
1918	63	-0.416	0.303	-0.74	0.47	- / 2.40	-1.360	1.99	0.044
1929	11	-0.166	0.235	1.01	0.00	- / -	0.988	2.14	0.071
1949	20	0.033	0.182	-0.47	0.29	- / -	0.944	-2.26	0.035
1976	27	-0.122	0.227	-0.03	-0.17	2.24 / -	0.964	1.94	0.093
1987	11	0.134	0.255	-0.57	0.16	- / -	0.918	2.13	0.116
1995	8	0.496	0.211	1.12	0.00	- / -	0.933		

where n is the size of the complete series, m and s , are its average and standard deviation, $u(r)$ and $u(t)$ are the standardized serial and trend statistics, and $u_x(t)$ are the extreme values of the standardized trend statistic found forwards and backwards through the sequential analysis.

Both $u(r)$ and $u(t)$ are highly significant for serial correlation and trend. Moreover, having $u(t_k) = 0.60$ and $u(t'_{k+1}) = 2.53$ for the 1920 change-point derived through the $X_E(P)$ Pettitt's statistic minimum, this excludes 1920 as a single change-point.

The results of the progressive method of change-point determination are given in Table I. From these results it may be concluded that the complete series is divided into six i.i.d. partial series, by five change-points jointly significant at the $2.3(10^{-3})$ level, derived from $X(F) = 13.65$. Moreover, the corresponding probabilities of the N values being sufficiently higher than 0.05, the normality assumption may be accepted for the six series.

Note that no account has been taken from the relative high $u_x(t)$ value observed for the 1918 partial series, the value resulting from existing change-points in 1884 and 1892, only jointly significant at the reduced level of 0.25. For the 1976 partial series, the same reason concerns the $u_x(t)$ value 2.24.

Table II gives the results of homogeneity testing based on i.i.d. partial series; for the sequence of consecutive sub-series 1929, 1949, 1976, 1987, $X(F)$ gives the joint significance level $\alpha(P) = 0.021$. It follows that between the two main increasing shifts of 1918 and 1987, the means of the partial series oscillate between two common high and low values.

3.1.2. Testing independence and stability in the residual dispersion

The analysis of the series of absolute deviations from the mean computed for each of the i.i.d. partial series gives the following results for the complete series:

	n	m_d	s_d	$u(r)$	$u(t)$	$u_x(t)$	s_r
1856–1995	140	0.197	0.165	1.03	-2.42	-3.67 / -2.72	0.257

where n is the size of the series, m_d and s_d are the mean and standard deviation, s_r is the residual standard deviation computed as the square root of $(m_d^2 + s_d^2)$. In particular, squaring 0.257 and

Table II. Testing homogeneity of partial series. Symbols have the same meaning as in Table I

P	m	$u(r)$	$u(t)$	$u_x(t)$
(1929, 1976)	-0.135	0.35	0.40	- / -
(1949, 1987)	0.069	-0.16	0.75	- / -
(1918, 1929, 1976)			4.22	
(1949, 1987, 1995)			3.04	

Table III. Sequential trend analysis. P are the years ending the partial series, m_d and s_d are the mean and standard deviation of each series. The standardized serial and trend statistic are denoted by $u(r)$ and $u(t)$, $u_x(t)$ are the extreme values of the standardized trend statistic found forwards and backwards through the sequential analysis. Symbol s_r denotes the residual standard deviation

P	n	m_d	s_d	$u(r)$	$u(t)$	$u_x(t)$	s_r
1879	24	0.336	0.236	0.05	0.35	- / -	0.411
1995	116	0.168	0.130	-1.36	-0.17	- / -	0.212

comparing it to the square of the standard deviation for the complete series 0.364 shows that the shifts at the change-points explain 50 per cent of the total variance in the mean.

The complete series being obviously not i.i.d., the applied necessary sequential trend analysis leads to the determination of one change-point placed in 1879 separating the two partial series with test results given in Table III.

Owing to the acceptable assumption of normality, and with the variance ratio given by the squared ratio of the two s_r values being equal to 3.76 for 24 and 116 degrees of freedom, it may be concluded that the heterogeneity is significant at a level less than $5(10^{-4})$.

3.1.3. Analysis of seasonal temperature averages

The trend analysis of the complete series for winter, spring, summer and autumn gives the results shown in Table IV. We observe a strong seasonal variation with lowest and highest serial correlation statistic, respectively, for winter and summer and with lowest and highest trend statistic, respectively, for summer and for both spring and autumn.

The final results for the seasonal means compared with the year mean are given in Table V. In this way, it appears that the seasonal contribution to the annual increase is in decreasing order for winter, spring and autumn; while for summer, if the warmest periods, spread evenly over the complete period, have equal averages, the contribution to the annual increase results from the fact that the coldest period is placed near the beginning of the complete series. Finally, the recent annual shift of 1987 seems to be mainly due to the spring and winter mean increase, while the alternating means found in the year are explained by the ones of summer and autumn.

Table IV. Analysis of seasonal temperature averages. The meaning of the symbols are the same as in Table I

Season	Series	n	m	s	$u(r)$	$u(t)$	$u_x(t)$
Winter	1857-1995	139	-0.272	0.741	1.80	5.35	5.35/5.51
Spring	1856-1995	140	-0.270	0.474	4.25	7.09	7.09/7.18
Summer	1856-1995	140	-0.007	0.278	6.18	2.05	-4.49/6.10
Autumn	1856-1995	140	-0.175	0.445	3.98	7.00	7.00/7.00

Table V. Seasonal means compared to year mean, δ gives the shift between the first and the last partial series temperature average

<i>P</i>	Year	Winter	Spring	Summer	Autumn
1880				0.200	
1895					-0.566
1916				-0.280	
1918	-0.416				
1919			-0.531		
1920		-0.580			
1922					-0.287
1929	-0.135			-0.034	
1941				0.200	
1949	0.069				
1950					0.173
1972			-0.191		
1976	-0.135				
1978				-0.034	-0.070
1980		-0.163			
1987	0.069		0.129		
1995	0.496	0.608	0.549	0.200	0.173
δ	0.980	1.190	1.080	0.000	0.739

Table VI. Trend analysis of the residual dispersion series for the seasons. $EV(P)$ gives, in percentage, the explained variance by the change-point shifts; for other symbol explanations see Table III

Season	Series	<i>n</i>	<i>m_d</i>	<i>s_d</i>	<i>u(r)</i>	<i>u(t)</i>	<i>u_x(t)</i>	<i>s_r</i>	$EV(P)$
Winter	1857-1995	139	0.496	0.409	0.61	-1.63	-3.24/-	0.643	25%
Spring	1856-1995	140	0.289	0.225	0.28	-2.86	-2.86/- 2.86	0.366	40%
Summer	1856-1995	140	0.146	0.130	2.63	-0.65	-3.03/2.91	0.195	51%
Autumn	1856-1995	140	0.254	0.212	1.95	-0.41	-2.77/2.86	0.330	45%

Table VII. Residual standard deviations for the seasons

<i>P</i>	Year	Winter	Spring	Summer	Autumn
1879	0.336				
1881		0.902			
1894			0.490		
1922				0.210	
1929					0.460
1949				0.096	
1982					0.274
1995	0.227	0.573	0.307	0.210	0.460

Table VI describes the trend analysis of the residual dispersion series of the seasons. It appears that $EV(P)$ is strongly inversely correlated with the residual variance.

Just as for the mean, the final results for the residual standard deviation are given in Table VII, where the annual shift seems to be mainly due to the winter shift. If spring, summer and autumn shifts have apparently a general circulation explanation, for year and winter, the shifts might be

related to the generalized introduction around 1880 of Stephenson screens for measuring the air temperature.

3.2. The Prague Klementinum air temperature averages (1771–1993)

In comparison with the NH global temperature series, that of Prague has the complementary interests of being more localized and extending further into the past.

3.2.1. Testing independence and stability in the mean

For the mean, the graphs of Figure 2 show this time high values placed at the beginning and at the end of the series separated by a sequence of low values. The variation of the Pettit statistic $X(P)$ suggests the existence of a decreasing change-point before 1840 compensated by an increasing one around 1940, but the sequential trend analysis validates apparently only the first one.

Moreover, the analyses of the annual and seasonal data, expressed in °C, are given in Table VIII, where, for comparison, $u'(t)$ has been computed for the series beginning in 1856.

It appears that, as for the NH series, the serial correlation statistic has highest value for summer and lowest values for winter and autumn. On the other hand, if the trend statistics $u'(t)$ computed for the series beginning in 1856 give, as for the NH series, the largest significant values for spring and autumn, for the complete series, with exception of the winter series, no trend is significant at the 5 per cent level. It follows that if the NH temperature trend is present in Prague, the lower significance levels result from larger existing standard deviations, while this trend seems to be compensated by the temperatures of the years preceding 1856.

The seasonal means are given in Table IX. The δ amplitude shows here also that winter and spring contributions to the year average are the most important. Moreover, if recent winters appear to be the warmest ones of the series, for the remaining seasons and for the year, the beginning and the end of the series are equally warm.

3.2.2. Testing independence and stability in the dispersion

The analysis of the residual dispersion series are given in Table X, which show, just like for the NH series, that the explained variance by the change-points is highest for spring and autumn.

The final results of the residual deviations are given in Table XI. All the changes (with the exception of winter) lead to variance ratios at least significant at the 5 per cent level.

Table VIII. Analysis of the annual and seasonal temperature means in Prague Klementinum. For the meaning of symbols see Table I

Season	Period	n	m	s	$u(r)$	$u(t)$	$u'(t)$	$u_x(t)$
Year	1771–1993	223	9.50	0.91	3.74	1.43	5.89	−4.51/7.04
Winter	1772–1993	222	0.12	2.12	1.88	2.05	3.31	−2.38/4.08
Spring	1771–1993	223	9.30	1.29	2.32	1.43	4.68	−3.55/5.79
Summer	1771–1993	223	18.84	1.05	3.05	−0.57	3.14	−4.74/4.40
Autumn	1771–1993	223	9.71	1.08	1.37	0.22	4.38	−4.75/4.96

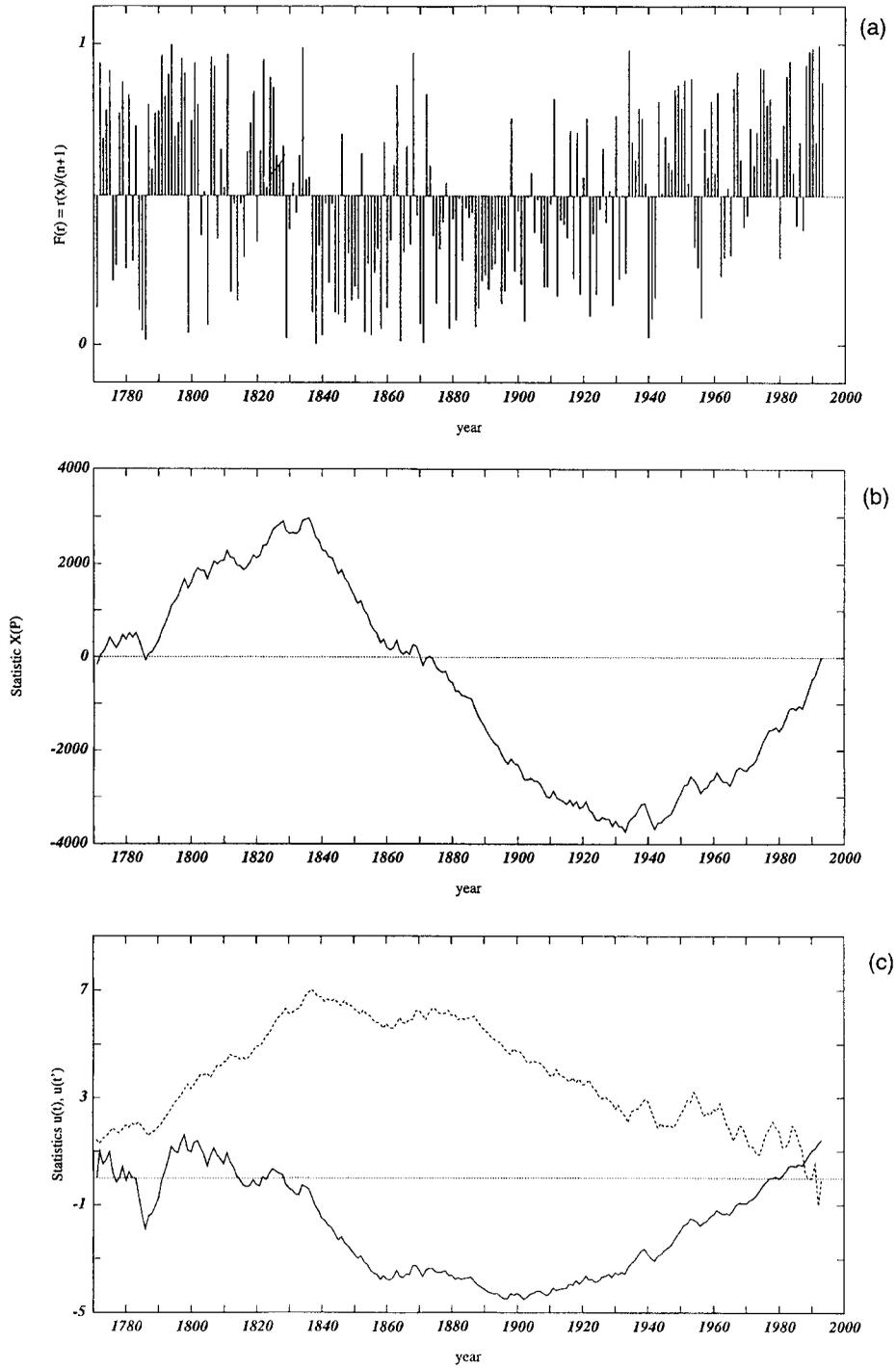


Figure 2. Prague Klementinum annual air temperature (1771–1993)

Table IX. Seasonal means of the Prague Klementinum. Symbol δ gives the temperature amplitude separating the warmest and the coldest sequence

<i>P</i>	Year	Winter	Spring	Summer	Autumn
1824		0.22			
1834	9.87				
1838			9.70	19.17	
1841					10.06
1888			8.54		
1893		-0.56			
1902	8.94				
1926					9.15
1934				18.41	
1941			9.11		
1942	9.35				
1965		0.22			
1993	9.87	1.33	9.70	19.17	10.06
δ	0.93	1.89	1.16	0.76	0.91

Table X. Residual dispersions of the Prague Klementinum series. For symbol explanation see Table III

Season	Period	<i>m</i>	<i>s</i>	<i>u(r)</i>	<i>u(t)</i>	<i>u_x(t)</i>	<i>s_r</i>	EV(<i>P</i>)
Year	1771-1993	0.620	0.490	-0.22	0.42	-2.53/2.19	0.790	25%
Winter	1772-1993	1.600	1.230	0.52	-1.45	3.52/ - 2.54	2.020	9.4%
Spring	1771-1993	0.929	0.714	0.65	-1.32	-2.80/ - 1.73	1.170	18%
Summer	1771-1993	0.763	0.606	-0.94	-2.31	-2.68/ - 2.32	0.974	13%
Autumn	1771-1993	0.792	0.588	-1.55	-1.59	-1.72/ - 2.30	0.986	17%

Table XI. The residual standard deviations of the Prague Klementinum series

<i>P</i>	Year	Winter	Spring	Summer	Autumn
1834			1.32		
1844				1.19	
1872	0.954				
1892		2.12			
1942					1.04
1993	0.620	1.88	1.11	0.85	0.77

4. CLIMATE MODELLING

The main purpose of a statistical analysis of a time series is the determination of those properties allowing a full statistical characterization of the generating process and the possibility of a powerful statistical prediction of its evolution.

In addition to the preceding search in time, the climate problem needs an investigation on the connections existing in space. For the time scale, the two examples considered show that if during certain periods a regular alternation exists, for other periods, abrupt changes occur in a quite arbitrary way which makes its integration in any regular statistical model impossible.

Table XII. Analysis of the global, regional and local temperature series

Series (latitude)	$u(t)$	$u(t_\delta)$	EV(NH)
NH		7.58	
Oulu (N. Finland) (65°)	2.11	0.11	18%
Central England (55°)	3.14	-0.67	20%
Brussels (50°)	7.43	3.71	17%
Prague (50°)	5.89	3.36	36%
Krakow (50°)	6.30	4.29	30%
Austria (48°)	5.22	2.55	31%
Genova (Italy) (44°)	1.05	-3.05	17%
USA (35°)	1.08	-1.10	33%

For the space scale, the correlation coefficient has been computed for the common part of the two complete series as well as for each of the i.i.d. partial NH series with the corresponding ones of Prague. The first correlation shows that the total Prague variability explains 36 per cent of the complete variance of the NH series, while for the NH i.i.d. partial series, if the explained variance is, as expected, reduced in the mean to 18 per cent, it varies strongly from one to the other partial series. In particular, the contribution to each NH partial series of the corresponding Prague partial ones, in percentage of variance, is found to be: 1918 (28); 1929 (52), 1949 (10); 1976 (5); 1987 (20); 1995 (1). It follows that, if the explained variance by the Prague series proves the existence of a more or less high correlation extended at the regional scale, its variability in time found at the scale of the i.i.d. partial series, although assignable to randomness, shows the large independence existing between global and local climate evolution.

The same conclusion appears to be true at the space scale when comparing global, regional and local series as suggested by the computation of the standardized test statistics $u(t)$ and $u(t_\delta)$, respectively, for the considered series and for their difference with the NH series, EV(NH) giving the explained NH variance by the complete series (see Table XII).

From these results, it appears, in particular, that the highest trend statistics for European stations are grouped in the 50° latitude region, where the weather evolution depends essentially on the alternating influence of polar or tropical air masses. This suggests that the recent warming observed mainly in this region is the consequence of changes in the frequency of the invasions of these different air masses. Moreover, if we accept this hypothesis, it seems reasonable to consider that the compensating cooling which, in the Prague temperature series, occurred at the beginning of the eighteenth century, results from a reverse effect in the frequencies of the air masses' influence and that an equivalent compensating effect occurred for the global averages. For all the climatological interest of these results, particularly with the apparent limitation to the region of the 50° latitude for the recent warming, which weakens the hypothesis of the major influence on the climate evolution of the CO₂ air concentration increases, their statistical value seems to be too poor for the elaboration of an efficient statistical climate model. The same answer is valid for the adjustment of eventual stochastic ones.

5. UNDERLYING THEORETICAL BACKGROUND

Noting that climate is the integration of the daily weather evolution, the right alternative to be considered is use of the equations ruling the physics of this daily weather. Here it should be

recalled that these equations are those of (i) the general atmospheric circulation, (ii) the general oceanic circulation, and (iii) the interaction between these two circulations as well as with the earth surface heterogeneities.

Considering the differential equations defining the points (i) and (ii), the immediate remark is that all of them are non-linear which suggests immediately that this might be the reason for the observed chaotic character of the climate evolution.

A simple indication of the relevance of this suggestion may be found in the example given by the equations considered by Lorenz (1963):

$$dx/dt = \sigma(-x + y), \quad dy/dt = rx - y - zx \quad \text{and} \quad dz/dt = xy - bz$$

defining a special case of turbulent convection, where x is the vertical speed, y, z the temperature inside the system, t the time, and σ, r and b appropriate physical constants.

Computing the x values for every t value distant from 0.25 by integration of these equations, Vannitsem and Nicolis (1991) obtained a series of 399 values to which the application of the sequential analysis gives the final following change-points P and means x for the i.i.d. partial series described in Table XIII. Moreover, the residual variance has been found to be constant for the complete series.

Table XIII. Change-point analysis of Vannitsem and Nicolis series. P is the order number of the element of the series ending the i.i.d. series, x is the mean

P	x	P	x	P	x	P	x
40	3.07	131	-5.53	257	6.41	335	-0.57
52	-5.53	178	6.41	273	-0.57	325	6.41
75	3.07	210	-5.53	287	6.41	385	-0.57
86	-5.53	218	6.41	309	-0.57	399	6.41
93	3.07	242	-5.53	329	6.41		

The similarity with the above considered climatological examples is obvious with, on one side, alternating partial series means inside given sequences, but with arbitrary abrupt changes for the means after $P = 131$ for the high ones and after $P = 257$ for the low ones.

The existence of a chaotic component in the evolution of x is thus obvious here. Of course, it should not be discounted that in longer developments this chaotic component would appear to be linked to a longer range regularity. However, the results found for the considered climatological series, suggesting that the perturbation of any regularity by chaotic components is an intrinsic property for each particular level of time or space scales, mean accepting the fact that the evolution of such series remains practically out of control, making validation of any model statistical or stochastic, practically impossible.

6. CONCLUSION

A rigorous sequential statistical method has been presented, especially appropriate for the search for the statistical properties of series when their evolution is essentially characterized by change-points separating i.i.d. partial series.

Applied to the cases of the NH global land temperature and of the local Prague one, if there is evidence of correlation properties resulting from the scale of the weather systems ruling weather evolution, the main conclusion reached by statistical analysis is the chaotic character of this evolution. Moreover, the fact that it seems reasonable to think that this character results from the existence of several modes of stable circulations, for the atmospheric system and for the oceanic one, increases the complexity of the situation. On the other hand, the balance between the global input and output of energy might be considered as constant; it is possible that inside the system, variations of forcing factors may influence the climate at some time and (or) some space scale. However, this raises the question of the level at which these forcing factors may have an action and how far our knowledge of the meteorological phenomena is exhaustive (Quinet and Vanderborgh 1996).

In any case, the final question remains what differences exist between a natural abrupt change in climatic conditions and a man-made one and the degree of confidence with which climate predictions have to be accepted.

ACKNOWLEDGEMENTS

We are indebted to Chris Folland, Head of the Climate Variability Research Group (Hadley Centre, Meteorology Office, UK) for kindly providing the most recent Jones global temperature averages, and also to Vladimir Bruzek (Czech Hydrometeorological Institute) and to Dr S. Vannitsem, for providing the Prague Klementinum data and the Lorenz solution, respectively. We are indebted to Ms L. Frappez for the computer preparation of the graphs.

REFERENCES

- Bartels, R. (1982). 'The rank version of the von Neumann's ratio test for randomness', *Journal of the American Statistical Association*, **77**, 40–46.
- Coops, A. J. (1992). 'Analysis of temperature series in Europe in relation to the detection of the enhanced greenhouse effect', *Theoretical and Applied Climatology*, **46**, 89–98.
- Coops, A. J. and Schuurmans, C. J. E. (1986). 'Detection of the CO₂ effect upon the climate of western Europe', *Theoretical and Applied Climatology*, **37**, 111–125.
- Craddock, J. M. (1979). 'Methods of comparing annual rainfall records for climatic purposes', *Weather*, **34**, 139–147.
- D'Agostino, R. B. (1971). 'An omnibus test for normality for moderate and large size samples', *Biometrika*, **58**, 341–348.
- Hawkins, D. M. (1977). 'Testing a sequence of observations for a shift in location', *Journal of the American Statistical Association*, **72**, 180–182.
- Jones, P. D. (1994). 'Hemispheric surface air temperature variations: A reanalysis and an update to 1993', *Journal of Climate*, **7**, 1794–1802.
- Kendall, M. G. (1971). 'Studies in the history of probability and statistics, XXVI: The work of Ernst Abbe', *Biometrika*, **58**, 369.
- Kendall, M. G. and Stuart, A. (1966). *The Advanced Theory of Statistics, Vol. 3.*, Griffin, London.
- Lewis, L. F. (1947). 'Variations of temperature in London 1764–1939', *Meteorological Magazine*, 135–138.
- Lombard, F. (1988). 'Detecting change-points by Fourier analysis', *Technometrics*, **30**, 3, 305–310.
- Lorenz, E. N. (1963). 'Deterministic nonperiodic flow', *Journal of Atmospheric Science*, **20**, 130–141.
- Lysgaard, L. (1948). 'Recent climatic fluctuations', *Nature*, **161**, 442–443.
- Mann, H. B. (1945). 'Non parametric test against trend', *Econometrika*, **13**, 245–259.
- Mann, H. B. and Whitney, D. R. (1947). 'On a test whether one of two random variables is stochastically larger than the other', *Annals of Mathematical Statistics*, **18**, 52–60.

- Maronna, R. and Yohai, V. J. (1978). 'A bivariate test for the detection of a systematic change in the mean', *Journal of the American Statistical Association*, **73**, 363, 640–645.
- Mood, A. M. (1954). 'On the asymptotic efficiency of certain non-parametric two-sample tests', *Annals of Mathematical Statistics*, **25**, 514–522.
- Pettitt, A. N. (1979). 'A non-parametric approach to the change-point problem', *Applied Statistics*, **28**, 126–135.
- Quinet, A. and Vanderborght, J. (1996). 'Clear-sky nocturnal temperatures forecast and the greenhouse effect', *Journal of Applied Meteorology*, **35**, 3, 401–415.
- Shapiro, S. S. and Wilk, M. B. (1965). 'An analysis of variance test for normality (complete samples)', *Biometrika*, **52**, 591–611.
- Sneyers, R. (1954). *Les Caractères Climatiques de la Période 1939–1953 et le Temps en 1953*, Inst. R. Météor. de Belgique, Publ. B, no. 11.
- Sneyers, R. (1955). 'Sur la détermination de l'homogénéité des séries climatologiques', *Journal Scientifique de la Météorologie, Paris*, **28**, 359–372.
- Sneyers, R. (1956). *Sur Quelques Propriétés Statistiques de la Température de l'air en Belgique*, Inst. R. Météor. de Belgique, Publ. A, no. 4.
- Sneyers, R. (1958). *Connexions Thermiques entre Saisons Consécutives à Bruxelles-Uccle*, Inst. R. Météor. de Belgique, Publ. B, no. 23.
- Sneyers, R. (1990). *On the Statistical Analysis of Series of Observations*, WMO, Technical Note no. 143, Geneva, Switzerland. Original French version, WMO, 1975.
- Sneyers, R. (1992a). 'On the use of statistical analysis for the objective determination of climate change', *Meteorologische Zeitschrift*, **1**, 247–256.
- Sneyers, R. (1992b). 'Use and misuse of statistical methods for the detection of climate change', in Proceedings 12th Conference on probability and statistic in the atmospheric sciences, American Meteorological Society and 5th International Meeting on statistical climatology, Atmosph. Envir. Canada, joint session, June 22–26, Toronto, pp., J76–J81.
- Sneyers, R. (1995). 'Climate instability determination. Discussion of method and examples', in Proceedings, 6th International Meeting on statistical climatology, June 19–23, Galway, Ireland, 547–550.
- Sneyers, R., Vandiepenbeeck, M., Vanlierde, R. and Demarél, G. (1990). 'Climatic changes in Belgium as appearing from homogenized series of observations made in Brussels-Uccle (1833–1988)' in Brázdil, R. (ed), *Proceedings of the International Conference Climate Change in the Historical and the Instrumental Periods*, Masaryk University Brno, pp. 170–174.
- Sneyers, R., Böhm, R. and Vannitsem, S. (1991). 'Climatic changes in the Austrian Alps during the period 1775–1989', in *6th Italian Glaciological Meeting*, Gressonney, 26–28 September, in *Geografia Fisica e Dinamica Quaternaria*, vol. 15, pp., 195–199.
- Snijders, T. A. B. (1983). 'A study of the variability in space and time of Upper Volta rainfall', in II. International Meeting on Statistical Climatology, 26–30 September, Lisboa, pp. 3.5.1.–3.5.7.
- Snijders, T. A. B. (1984). 'Antithetic variates for Monte Carlo estimation of probabilities', *Statistica Neerlandica*, **38**, 55–73.
- Vandenplas, A. (1948). *Variation séculaire de la température de l'air à Bruxelles-Uccle*, Institut royal météorologique de Belgique, Misc. 35.
- Vannitsem, S. and Nicolis, C. (1991). 'Detecting climatic transitions: statistical and dynamical aspects', *Beitraege zur Physik der Atmosphaere*, **64**, 3, 245–254.
- Vialar, J. (1952). 'Étude statistique des températures de Paris-Saint-Maur pour la période 1874–1945', *J. Sc. Météor.*, **4**, 13, 20–37.
- Worsley, K. J. (1979). 'On the likelihood-ratio test for a shift in location of normal populations', *Journal of the American Statistical Association*, **74**, 365–367.