Testing for a Shift in Mean Without Having to Estimate Serial-Correlation Parameters

Timothy J. VOGELSANG
Department of Economics, Cornell University, Ithaca, NY 14853-7601 (tjv2@cornell.edu)

Tests for detecting a shift in the mean of a univariate time series that do not require estimation of serial-correlation parameters are proposed. The statistics are valid whether the errors are stationary or have a unit root. The date of the shift may be known or unknown. The statistics are based on a simple transformation of the data and are functions of partial sums of the data. These so-called partial sum statistics are shown to be asymptotically invariant to serial-correlation parameters. The statistics are shown to have good size and power properties asymptotically and in finite samples.

KEY WORDS: HACE; Partial sum; Structural change; Unit root; Wald test.

In this article statistics are proposed that can detect a shift in the mean of a univariate time series. The date of the shift can be known or unknown, and correlated errors are permitted. The statistics are valid whether the errors are stationary or have a unit root. This is an important property because it is often not known whether a series has stationary or unit-root errors. Therefore, the applied researcher can apply the tests while being agnostic about a unit root. The statistics are also asymptotically similar without requiring estimation (parametric or nonparametric) of serial-correlation parameters.

Recently, many statistics have been proposed to test for stability of the mean while permitting serial correlation in the data, including those of Andrews (1993), Andrews and Ploberger (1994), Bai, Lumsdaine, and Stock (in press), Chen and Tiao (1990), Kramer, Ploberger, and Alt (1988), Perron (1991), and Vogelsang (1994a). Bayesian approaches were given by Carlin, Gelfand, and Smith (1992) and McCulloch and Tsay (1993). All of these procedures require either a full specification of the dynamics, the addition of lags of the dependent variable, or consistent estimates of serial-correlation parameters. Poor handling of the dynamics and serial correlation can result in undesirable properties in finite samples.

To construct statistics that do not require estimation of serial-correlation parameters, a simple transformation of the data based on partial sums is used. The transformed data will have at least one unit root by construction. It is the presence of a unit root that yields statistics that are asymptotically invariant to the serial-correlation parameters. As one should expect, when the errors are iid, the introduction of a unit root penalizes the local asymptotic power of the tests compared to using the original data. Surprisingly though, the reduction in local asymptotic power is not that substantial. Even more surprising is that there are cases in which using the transformed data actually *improves* the local asymptotic power compared to test statistics based on the original data. An example is when the date of the mean shift is unknown and a supremum statistic is used. In finite samples, statistics based on the transformed data exhibit better exact size and are often more powerful than statistics based on the untransformed data.

The layout of the article is as follows. In Section 1, the model and statistics are described. Limiting distributions under the null hypothesis are given and tabulated. In Section 2, asymptotic power results are provided and discussed. Section 3 contains the results of finite-sample simulations. Concluding remarks are given in Section 4.

1. THE MODEL AND STATISTICS

1.1 The Data-Generating Process and Assumptions

Consider the following data-generating process (DGP) for a univariate time series $\{y_t\}_1^T$,

$$y_t = \mu + \delta DU_t^c + u_t, \tag{1}$$

where $DU_t^c = 1$ if $t > T_b^c$ and 0 otherwise, T_b^c is the date of the mean shift, and $\{u_t\}$ is a mean-zero error process. Define the partial sums of $\{u_t\}$ as $S_t = \sum_{j=1}^t u_j$. Two sets of assumptions are considered for $\{u_t\}$, depending on whether or not $\{u_t\}$ has a unit root. Let $W(r)$ denote a standard Wiener process defined on [0, 1], let $\Rightarrow$ denote weak convergence in distribution, and let $[x]$ denote the integer part of x.

Assumption 1: u_t is $I(0)$ with $\sigma^2 = \lim_{T\to\infty} E(T^{-1}(\sum_{t=1}^T u_t)^2)$ and $T^{-1/2}S_{[rT]} \Rightarrow \sigma W(r)$.

Assumption 2: $u_t = u_{t-1} + \eta_t, u_0 = 0$, and η_t satisfies Assumption 1 with $\omega^2 = \lim_{T\to\infty} E(T^{-1}(\sum_{t=1}^T \eta_t)^2)$ in place of σ^2.

In addition, define $\sigma_u^2 = \lim_{T\to\infty} E(T^{-1}\sum_{t=1}^T u_t^2)$. To make the asymptotics tractable, it is assumed that $\lambda_c = T_b^c/T$ remains a fixed constant as the sample size grows. In the case of an unknown shift date, regressions will be estimated using a shift date, T_b, that may be different from T_b^c. It is assumed that $\lambda = T_b/T$ remains fixed as the sample size grows.

The statistics are based on the following transformation of the data. Define $z_t = \sum_{j=1}^t y_j$. The time series $\{z_t\}$ is

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the partial sum of $\{y_t\}$, which, applied to (1), gives

$$z_t = \mu t + \delta DT_t^c + S_t, \tag{2}$$

where $DT_t^c = (t - T_b^c)$ if $t > T_b^c$ and 0 otherwise. Note that $\{z_t\}$ is a trending process with unit-root errors with slope μ before T_b^c and slope $\mu + \delta$ after T_b^c.

The null hypothesis is that the mean of $\{y_t\}$ is stable or that the trend of $\{z_t\}$ is stable. In both cases the null hypothesis is $\delta = 0$. The alternative is that $\{y_t\}$ has a shift in mean or equivalently $\{z_t\}$ has a shift in slope. Permitting more than one break in the trend function is not difficult, but because the focus of this article is constructing tests when the form of serial correlation is unknown, the simplicity of the single-break model helps to delineate the results.

When testing the null hypothesis, σ^2 and σ_u^2 are nuisance parameters related to the serial correlation in $\{u_t\}$. In practice σ^2 can be estimated parametrically through the addition of lags of $\{y_t\}$ to the model or nonparametrically using estimators such as those proposed by Andrews (1991) and Andrews and Monahan (1992). These approaches are reviewed next and are later compared to the new statistics.

1.2 The Chow Test

For now assume that the shift date is known and Assumption 1 applies. A standard approach in testing the null hypothesis is to estimate (1) by ordinary least squares (OLS) and perform a Chow (1960) test. The Chow statistic is simply the standard Wald statistic for testing $\delta = 0$ in (1) and is denoted by $\text{Chow}_T(T_b^c)$. It is a well-known result that, under the null hypothesis of no shift in mean and Assumption 1, $\text{Chow}_T(T_b^c) \Rightarrow (\sigma^2/\sigma_u^2)\chi_1^2$. The Chow statistic has a limiting distribution that depends on σ^2 and σ_u^2. When $\{u_t\}$ is iid, $\sigma^2 = \sigma_u^2$, and the Chow statistic has a limiting distribution free of nuisance parameters. Otherwise, σ^2 and σ_u^2 need to be estimated. There are two common approaches often used.

The first method assumes that $\{u_t\}$ is an autoregressive moving average (ARMA) process and is based on the regression

$$y_t = \mu + \delta DU_t^c + \sum_{j=1}^{k} \alpha_j y_{t-j} + e_t. \tag{3}$$

Let $\text{Chow}L_T(T_b)$ denote the Chow statistic based on Regression (3). If the errors follow a pure AR(p) process, k is set equal to p. If the errors have an invertible MA component, then enough lags are included to approximate the MA component by a long autoregression. Asymptotically, the Chow statistic will be invariant to σ^2 and σ_u^2 and have a χ_1^2 distribution provided the number of lags is no smaller than p for a pure AR process or provided the number of lags grows at a suitable rate as the sample size grows if an MA component is present (see Said and Dickey 1984).

The second method involves replacing the OLS estimate of the error variance in the Wald statistic with $\hat{\sigma}^2$, a consistent estimate of σ^2. Denote this statistic by $\text{Chow}_T^*(T_b^c)$ defined as $\text{Chow}_T^*(T_b^c) = [\sum_{T_b^c+1}^{T}(y_t - \bar{y})]^2/[\hat{\sigma}^2 T_b^c(T - T_b^c)]$. Under the null hypothesis and Assumption 1, $\text{Chow}_T^*(T_b^c) \Rightarrow \chi_1^2$ using standard results. Typical estimators of σ^2 include those of Andrews (1991) and Andrews and Monahan (1992) such as $\hat{\sigma}^2 = \sum_{j=-T+1}^{T-1} w(j/L)\hat{\gamma}(j)$, where $w(\cdot)$ is a kernel function, $\hat{\gamma}(j) = T^{-1}\sum_{t=1}^{T-j} \hat{u}_t\hat{u}_{t+j}$ are the sample autocovariances of the residuals from Regression (1), and L is a truncation lag parameter.

A difficulty with the preceding approaches is the need to specify in finite samples either k, L, and/or $w(\cdot)$. Although asymptotic theory gives some guidelines as to how k or L must be chosen as the sample size grows, these rules become arbitrary in finite samples, and the performance of the statistics depends crucially on these choices. Recent work has shown that data-dependent methods can often substantially reduce much of the arbitrary nature of these choices. For example, Hall (1994), Ng and Perron (1995), and Perron and Vogelsang (1992) showed that data-dependent choices of k can work well in practice. In those approaches, however, a choice of maximal lag length must still be made. The important work of Andrews (1991) and Andrews and Monahan (1992) has shown that automatic choices of the truncation lag based on the plug-in method can work well in practice. But, their procedures require the choice of an approximating parametric model.

1.3 The Partial Sum (PS) Statistic

Consider estimating regression (2) by OLS and constructing the standard Wald statistic for testing $\delta = 0$. Let $\text{PS}_T(T_b^c)$ denote this Wald statistic divided by the sample size, T. The Wald statistic must be normalized by the sample size to arrive at a nondegenerate asymptotic distribution due to the unit root in $\{S_t\}$. Under the null hypothesis and Assumption 1, it follows from Theorem 1 of Vogelsang (in press) that

$$\text{PS}_T(T_b^c) \Rightarrow H_1(\lambda_c)^2/[H_2K(\lambda_c) - H_1(\lambda_c)^2] \equiv \text{PS}(\lambda_c), \tag{4}$$

where, $H_1(\lambda_c) = \int_{\lambda_c}^1 (r - \lambda_c)W(r)\,dr - \frac{1}{2}(1 - \lambda_c)^2(2 + \lambda_c)\int_0^1 rW(r)\,dr$, $H_2 = \int_0^1 W(r)^2\,dr - 3[\int_0^1 rW(r)\,dr]^2$, and $K(\lambda_c) = (1 - \lambda_c)^3\lambda_c^2(3 + \lambda_c)/12$. This limiting distribution is nonstandard, and asymptotic critical values were simulated using $N(0,1)$ iid random deviates to approximate the Wiener processes. The integrals were approximated by normalized sums of 1,000 steps using 10,000 replications. The random-number generator used was ran1( ) taken from Press, Flannery, Teukolsky, and Vetterling (1992) with initial seed of $-1{,}000$. The tabulated critical values are given in Table 1 for $\lambda_c = .1, .2, \ldots, .9$. Because the distribution does not depend on σ^2 and σ_u^2, there is no need to estimate σ^2 and σ_u^2 to use the PS_T statistic in practice. Section 1.4 shows how to modify the PS_T statistic to account for a unit root in $\{u_t\}$.

1.4 Unit-Root Errors

The asymptotic results in Sections 1.2 and 1.3 are no longer valid when $\{u_t\}$ has a unit root. For example, under the null hypothesis and Assumption 2, Vogelsang (1994a)

Table 1. Asymptotic Distributions: PS_T^m Statistic, Known Shift Date, Stationary Errors

	λ_c								
%	.1	.2	.3	.4	.5	.6	.7	.8	.9
90.0	2.155	2.854	3.257	3.228	2.944	2.381	1.682	1.017	.438
95.0	3.150	4.308	4.809	4.806	4.361	3.446	2.423	1.430	.611
97.5	4.203	5.816	6.502	6.486	5.772	4.715	3.322	1.881	.781
99.0	5.763	8.316	9.027	9.414	8.344	6.610	4.656	2.701	1.017

NOTE: The critical values were calculated via simulation methods using $N(0, 1)$ iid random deviates to approximate the Wiener processes defined in the asymptotic distribution. The integrals were approximated by the normalized sums of 1,000 steps using 10,000 replications.

showed that ChowL_T is no longer chi-square distributed but has a nonstandard distribution, and it is easy to show that the Chow_T^* statistic diverges to ∞. Therefore, a unit root in $\{u_t\}$ complicates inference when using the Chow statistics. Similarly, the limiting distribution of the PS_T statistic is affected by a unit root in the errors. To be precise, under the null hypothesis and Assumption 2, it follows from theorem 2 of Vogelsang (in press) that

$$\text{PS}_T(T_b^c) \Rightarrow M_1(\lambda_c)^2/[M_2K(\lambda_c) - M_1(\lambda_c)^2] \equiv \text{PS}_u(\lambda_c), \tag{5}$$

where $M_1(\lambda_c) = \int_{\lambda_c}^1 (r - \lambda_c)Q(r)\,dr - \frac{1}{2}(1 - \lambda_c)^2(2 + \lambda_c)\int_0^1 rQ(r)\,dr$, $M_2 = \int_0^1 Q(r)^2\,dr - 3[\int_0^1 rQ(r)\,dr]^2$, and $Q(r) = \int_0^r W(s)\,ds$. This distribution is different from (4) but like (4) is free of nuisance parameters. In an unreported simulation, the critical values for (5) were computed. The key feature of the critical values is that they are much larger than the critical values of (4). For example, the 5% critical value with $\lambda_c = .5$ is 38.71, which is much larger than 4.361, the 5% critical value from Table 1. One implication of the large critical values in the unit-root case is that for series with highly persistent errors (a root near or equal unity) using PS_T with stationary critical values will result in an oversized test. If a unit root (or near unit root) cannot be ruled out, one could always take a conservative approach and use the unit-root critical value. This will keep the test from being oversized, but size will be nearly 0 if the errors are in fact stationary. This severe downward bias in size would heavily penalize power, making the PS_T statistic less useful when the errors are stationary.

Fortunately, there is a simple modification of the PS_T statistic that corrects the size distortion caused by a unit root in $\{u_t\}$ that does not severely penalize power when the errors are stationary. The correction is based on the unit-root statistic of Park and Choi (1988) and Park (1990) constructed using the following regression estimated by OLS:

$$y_t = \mu + \delta DU_t^c + \sum_{i=1}^{9} \gamma_i t^i + u_t. \tag{6}$$

The statistic, denoted by $J_T(T_b^c)$, is defined as T^{-1} times the standard Wald statistic for testing the joint hypothesis that $\gamma_1 = \gamma_2 = \cdots = \gamma_9 = 0$. Note that J_T is invariant to δ. The J_T statistic has two useful properties. Regardless of δ, under Assumption 1, $J_T(T_b^c) - 0$ and, under Assumption 2, $J_T(T_b^c) \Rightarrow J(\lambda_c) = \{\int_0^1 W^*(r)^2\,dr/\int_0^1 W^{**}(r)^2\,dr\} - 1$, where $W^*(r)$ and $W^{**}(r)$ are the residuals from the projection of $W(r)$ onto the spaces spanned by $\{1, du^c\}$ and $\{1, du^c, r, r^2, \ldots, r^9\}$, respectively, with $du^c = 1$ if $r > \lambda_c$ and 0 otherwise. Notice that in both cases the limiting distribution of J_T does not depend on serial-correlation parameters. Thus, the PS_T statistic can be modified with J_T while retaining the property that serial-correlation parameters do not have to be estimated.

Let b be a finite constant. Consider a modified version of the PS_T statistic defined as $\text{PS}_T^m(T_b^c) = \text{PS}_T(T_b^c)\exp(-bJ_T(T_b^c))$. When $b = 0$, the modification has no effect on PS_T. Under the null hypothesis, it follows from (4), (5), and the limiting behavior of J_T that

$$\text{under Assumption 1, } \text{PS}_T^m(T_b^c) \Rightarrow \text{PS}(\lambda_c) \tag{7}$$

and

$$\text{under Assumption 2, } \text{PS}_T^m(T_b^c) \Rightarrow \text{PS}_u(\lambda_c)\exp(-bJ(\lambda_c)). \tag{8}$$

When $\{u_t\}$ is stationary, the limiting distribution of PS_T^m is equivalent to that of PS_T, so the modification does not affect the test asymptotically. When $\{u_t\}$ has a unit root, however, the constant b can be chosen so that the critical values of (8) are close to the critical values of (4). In particular, for a given nominal size, b can be chosen so that PS_T^m asymptotically has the same critical value (and hence size) for both stationary and unit-root errors. Thus, PS_T^m can be used without prior knowledge of whether $\{u_t\}$ is stationary or has a unit root. Statistics proposed by Perron (1991) and Vogelsang (1994a) also share this property.

Some justification is required for constructing J_T using 9 as the highest-order polynomial of t in Regression (6) because J_T can be defined using other finite integers for the largest power of t. The value of 9 was chosen based on a local asymptotic analysis of size and power of the PS_T^m statistic. Details of that analysis are not provided for brevity but are available on request. The results are easy to summarize. The size of PS_T^m remained stable regardless of the largest power of t used for J_T, but power steadily increased as the largest power of t in J_T increased. But the increases in power were negligible when the greatest power of t exceeded 9.

To implement the PS_T test, b must be computed. Given a desired significance level, b is chosen so that the critical values are the same for stationary and unit-root errors. Because the distributions given by (7) and (8) are nonstandard, b must be computed using simulation methods. To facilitate computation of b for a range of significance levels, b was computed for percentiles $p = .7, .71, \ldots, .98, .99$

and a $b(p)$ function was fitted through those values. The functional form that provided an excellent fit in all cases is $b(p) = a_0 + a_1p + a_2p^2 + a_3p^3 + \exp(a_4 + 100p)$. Coefficient for the a_i's are given in Table 3. The $b(p)$ function also simplifies computation of p values as follows. First, PS_T is computed using Regression (2). Second, J_T is computed using Regression (6). Third, PS_T^m is computed for a range of p's. The p value of the test is $1-p$ for p such that PS_T^m is equal (or very close) to the corresponding stationary critical value.

1.5 Chow and PS Statistics When the Shift Date is Unknown

When the shift date is unknown, testing for a shift in mean falls into the class of tests in which a nuisance parameter (λ_c) is present only under the alternative ($\delta \neq 0$). Andrews and Ploberger (1994) derived the class of optimal tests in this framework in models with stationary errors and nontrending regressors. In the context of testing for structural change, this amounts to computing the Chow and PS_T^m statistics for all possible break dates over some range $\Lambda = \{T_b^*, T_b^* + 1, \ldots, T - T_b^*\}$ and then forming a composite statistic. Let $\lambda^* = T_b^*/T$, and assume that λ^* remains fixed as the sample size grows. λ^* is typically called the amount of trimming. Let $h_T(T_b)$ generically denote either of the Chow statistics, Chow_T^* or $\mathrm{Chow}L_T$. Two statistics proposed by Andrews and Ploberger (1994) are the mean and mean-exponential statistics defined as $\mathrm{mean}h_T = T^{-1}\Sigma_{T_b\in\Lambda}h_T(T_b), \exp h_T = \log\{T^{-1}\Sigma_{T_b\in\Lambda}\exp(\frac{1}{2}h_T(T_b))\}$. The PS statistics are defined as $\mathrm{meanPS}_T = \{T^{-1}\Sigma_{T_b\in\Lambda}\mathrm{PS}_T(T_b)\}\exp(-bJ_T^*)$ and $\mathrm{expPS}_T = \log\{T^{-1}\Sigma_{T_b\in\Lambda}\exp(\frac{1}{2}\mathrm{PS}_T(T_b))\}\exp(-bJ_T^*)$, where $J_T^* = \inf_{T_b\in\Lambda} J_T(T_b)$. J_T^* is a statistic that could be used to test for a unit root while allowing a shift in mean at an unknown date. A third statistic that is not in the class of optimal statistics is the supremum statistic of Andrews (1993) defined as $\sup h_T = \sup_{T_b\in\Lambda} h_T(T_b)$ and $\mathrm{supPS}_T = \{\sup_{T_b\in\Lambda} \mathrm{PS}_T(T_b)\}\exp(-bJ_T^*)$. One nice feature of the supremum statistic is that it yields an estimate of the shift date. See Bai (1993) for a discussion of the properties of such estimates. Because the results of Andrews and Ploberger (1994) do not permit unit-root errors, none of the PS statistics are in the class of optimal tests.

When $\{u_t\}$ is stationary, limiting distributions of the Chow_T^* and $\mathrm{Chow}L_T$ statistics are given by Andrews (1993) and Andrews and Ploberger (1994). When $\{u_t\}$ has a unit root, limiting distributions of the $\mathrm{Chow}L_T$ statistics are given by Vogelsang (1994a), and the Chow_T^* statistics diverge to ∞. The limiting distributions of the PS statistics follow from the continuous-mapping theorem. Under Assumption 1, $\mathrm{meanPS}_T \Rightarrow \int_{\lambda^*}^{1-\lambda^*} \mathrm{PS}(\lambda)\,d\lambda$, $\mathrm{expPS}_T \Rightarrow \log\{\int_{\lambda^*}^{1-\lambda^*} \exp(\frac{1}{2}\mathrm{PS}(\lambda))\,d\lambda\}$, and $\mathrm{supPS}_T \Rightarrow \sup_{\lambda\in(\lambda^*,1-\lambda^*)}\mathrm{PS}(\lambda)$. Under Assumption 2, $\mathrm{meanPS}_T \Rightarrow \{\int_{\lambda^*}^{1-\lambda^*} \mathrm{PS}_u(\lambda)\,d\lambda\}\exp(-bJ^*)$, $\mathrm{expPS}_T \Rightarrow \log\{\int_{\lambda^*}^{1-\lambda^*}\exp(\frac{1}{2}\mathrm{PS}_u(\lambda))\,d\lambda\}\exp(-bJ^*)$, and $\mathrm{supPS}_T \Rightarrow \{\sup_{\lambda\in(\lambda^*,1-\lambda^*)} \mathrm{PS}_u(\lambda)\}\exp(-bJ^*)$, where $J^* = \inf_{\lambda\in(\lambda^*,1-\lambda^*)} J(\lambda)$. The distributions of the PS statistics are nonstandard, and critical values were simulated using methods similar to those used for Table 1 and are tabulated in Table 2 for $\lambda^* = .01$ and .10. Coefficients of the $b(p)$ functions are given in Table 3.

Table 2. Asymptotic Distributions: MeanPS$_T$, ExpPS$_T$ and SupPS$_T$ Statistics, Unknown Shift Date, Stationary Errors

	$\lambda^* = .01$			$\lambda^* = .10$		
%	MeanPS	ExpPS	SupPS	MeanPS	ExpPS	SupPS
90.0	2.000	1.403	5.426	1.832	1.303	5.424
95.0	2.608	2.089	7.680	2.411	2.029	7.680
97.5	3.278	3.075	10.416	3.095	3.044	10.416
99.0	4.089	4.723	14.453	3.856	4.626	14.453

NOTE: The critical values were calculated via simulation methods using $N(0, 1)$ iid random deviates to approximate the Wiener processes defined in the asymptotic distributions. The integrals were approximated by the normalized sums of 1,000 steps using 10,000 replications.

Table 3. Coefficients of $b(p) = a_0 + a_1p + a_2p^2 + a_3p^3 + \exp(a_4 + 100p)$, $p \in [.7, .99]$

	a_0	a_1	a_2	a_3	a_4
	Known break date				
$\lambda_c = .1$	−3.287	13.019	−16.709	7.222	−101.771
$\lambda_c = .2$	−17.349	67.094	−85.795	36.813	−100.735
$\lambda_c = .3$	−15.413	60.710	−78.576	34.258	−100.106
$\lambda_c = .4$	−15.707	61.057	−77.770	33.458	−100.195
$\lambda_c = .5$	−23.800	92.880	−119.223	51.354	−100.535
$\lambda_c = .6$	−17.147	67.696	−87.783	38.295	−100.645
$\lambda_c = .7$	−11.781	46.619	−60.600	26.572	−100.523
$\lambda_c = .8$	−13.197	51.372	−65.926	28.316	−101.326
$\lambda_c = .9$	−.885	3.981	−5.421	2.504	−101.434
	Unknown break date: $\lambda^* = .01$				
MeanPS	−4.172	19.661	−26.995	12.745	−100.291
ExpPS	−12.752	54.809	−74.422	34.978	−99.400
SupPS	−12.717	53.196	−70.687	32.333	−99.387
	Unknown break date: $\lambda^* = .10$				
MeanPS	−5.183	23.397	−31.575	14.629	−100.252
ExpPS	−8.986	42.543	−60.427	29.432	−99.324
SupPS	−13.491	55.858	−73.769	33.502	−99.372

2. CONSISTENCY AND LOCAL ASYMPTOTIC POWER

Consistency and local asymptotic power properties of the PS statistics are explored in this section and comparisons made with the Chow statistics. Suppose that $\{u_t\}$ is stationary. When the break date is known, it directly follows from theorem 3 of Vogelsang (in press) that the PS_T^m statistic is consistent. When the break date is unknown, consistency of supPS_T trivially holds provided $\lambda_c \in (\lambda^*, 1 - \lambda^*)$. Consistency of the mean and exponential statistics follows from straightforward arguments. A formal proof is omitted but can be found in the working paper by Vogelsang (1994b). When $\{u_t\}$ has a unit root, none of the tests are consistent. This is not surprising because it is not possible to construct a consistent test for a shift in mean of a unit-root process.

Now consider local asymptotic power of the tests. Comparisons are drawn between the Chow and PS tests. The insight to be gained by comparing local asymptotic power stems from the fact that the $\mathrm{Chow}_T^*(T_b^c)$, $\mathrm{meanChow}_T^*$, and $\mathrm{expChow}_T^*$ are optimal test statistics when the errors are iid and Gaussian. The local asymptotic analysis was carried out using local alternatives of the form $\delta_T = cT^{-1/2}$ with the DGP $y_t = \mu + \delta_T DU_t^c + u_t$. The transformed model under the local alternative becomes $z_t = \mu t + \delta_T DT_t^c + S_t$. Under these local alternatives limiting distributions of the statistics can be obtained using straightforward arguments along with the continuous-mapping theorem. When $\{u_t\}$ is stationary, the local asymptotic distributions are nondegenerate, and power depends on c/σ. When $\{u_t\}$ has a unit root, the local asymptotic distributions are identical to the limiting null distributions. This is no surprise because the tests are inconsistent when $\{u_t\}$ has a unit root. Representations of the limiting distributions when $\{u_t\}$ is stationary are not informative and are not reported. The local asymptotic distributions, however, were used to compute asymptotic power curves.

Asymptotic power curves with stationary errors are presented in Figures 1 and 2. In the case of a known shift date, the asymptotic power function of the Chow statistic was computed analytically because the local asymptotic distribution is equivalent to a noncentral chi-squared distribution with 1 df. In the other cases the local asymptotic distributions are nonstandard, and the asymptotic power functions were simulated. All of the power curves were computed using rejections based on 5% asymptotic critical values. In all cases, $\lambda_c = .5$, and for an unknown break date $\lambda^* = .01$.

Figure 1 displays the the asymptotic power curves of $\mathrm{Chow}_T(T_b^c)$ and $\mathrm{PS}_T^m(T_b^c)$ when the break date is known for c/σ ranging from 0 to 12. As predicted by the theory, $\mathrm{Chow}_T(T_b^c)$ yields a more powerful test than $\mathrm{PS}_T^m(T_b^c)$. Surprisingly though, the power of the PS test is not significantly lower than the power of the Chow test. When the shift date is unknown, the power differences are smaller. The power of all the composite statistics is shown in Figure 2. Notice that the power of the three PS tests is similar but below the power of the mean and exponential Chow tests as the theory predicts. For the supremum statistics, things are much different. SupPS_T is more powerful than $\mathrm{supChow}_T$ for small to moderate values of c/σ. This is a very striking result, and indicates that supPS_T can lead to a more powerful test than $\mathrm{supChow}_T$.

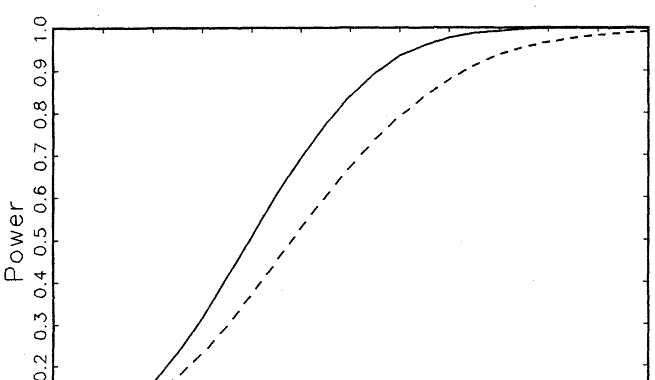

Figure 1. Local Asymptotic Power With Known Break Date: λ_c = .5, 5% Nominal Size: ———, Chow; - - - , PS^m.

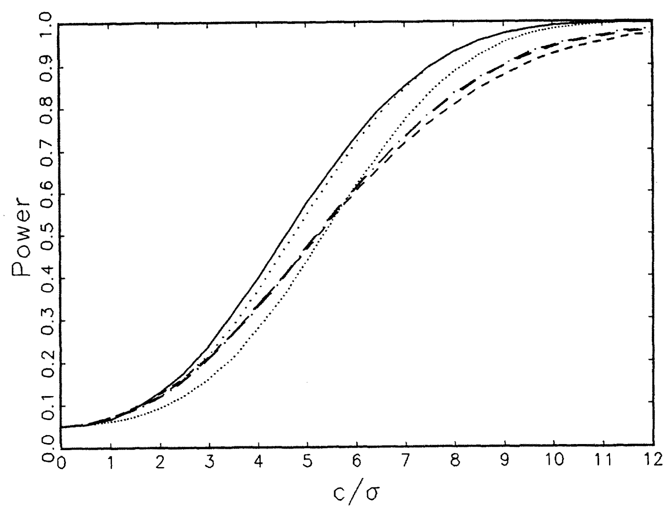

Figure 2. Local Asymptotic Power With Unknown Break Date: λ^* = .01, 5% Nominal Size: ———, Mean Chow; · · ·, ExpChow; · · · · , SupChow; – –, MeanPS; – ·, ExpPS; · – ·, SupPS.

3. FINITE–SAMPLE SIZE AND POWER

This section presents results from simulation experiments meant to assess the finite-sample size and power of the PS tests compared to the Chow tests. Size results are first discussed for AR(1) and MA(1) errors. Size and power are then examined for more general ARMA(p, q) errors using parameterizations based on point estimates from some typical economic time series. To mimic how the tests are used in practice, the statistics are applied as if the true dynamic structure of the errors is unknown.

3.1 Finite-Sample Size With AR(1)/MA(1) Errors

In this subsection the errors are modeled as either AR(1) or MA(1) processes. The following DGP was used for these simulations: $y_t = u_t, u_t = \alpha u_{t-1} + v_t + \theta v_{t-1}, u_0 = v_0 = 0$, where $v_t \sim$ iid $N(0,1)$. The parameter μ was set equal to 0 because all the statistics are exactly invariant to μ. A sample size of $T = 100$ was used for all simulations. The same set of random numbers was used in all simulations to minimize the influence of sampling error when comparing experiments. The initial seed for the random-number generator was set to -100 in all experiments. The number of replications was 2,000. Results from this section are only reported for a known break date, $T_b^c = 50$. Qualitatively similar results were obtained for other break dates and unknown break date. For pure AR(1) errors $\alpha = -.9, -.8, \ldots, .8, .9, \theta = 0$, and for pure MA(1) errors $\theta = -.9, -.8, \ldots, .8, .9, \alpha = 0$. Asymptotic 5% critical values were used in all cases.

The Chow_T^* statistic was implemented by estimating σ^2 using the automatic bandwidth procedure of Andrews (1991) with the quadratic spectral kernel. The $\mathrm{Chow}L_T$ statistic was implemented using Regression (3) with k chosen using the following data-dependent method following Perron and Vogelsang (1992). A maximal lag length, $k$max, is first chosen. Regression (3) is estimated using $k$max lags. The significance of the coefficient on the last lag is tested using a 5% two-tailed t test. If the coefficient is insignif-

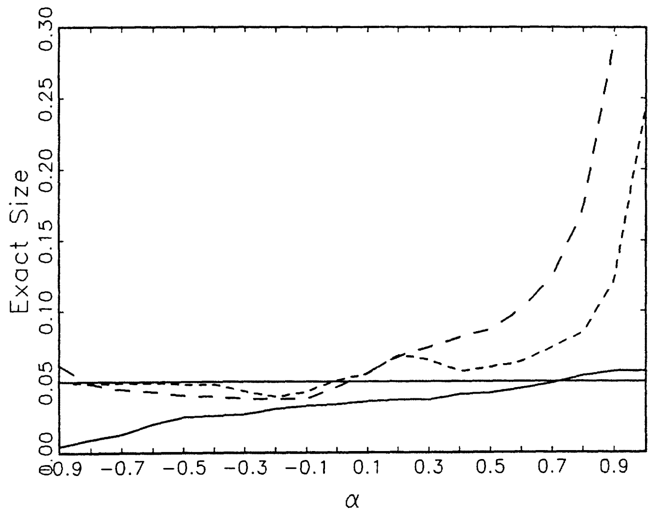

Figure 3. Finite Sample Size AR(1) Errors, Known Break Date: λ_c = .5, 5% Nominal Size, T = 100: ———, PS_T^m, - - -, $ChowL_T$; – –, $Chow_T^*$.

icant, the number of lags is reduced by 1. Again the coefficient on the last included lag is tested. This procedure continues until a significant lag is found. If the coefficient on the $k$max lag is significant, $k$max is increased and the procedure is restarted. Ng and Perron (1995) gave a theoretical justification for this approach. For these simulations, $k\text{max} = 5$ was used.

The exact size results are summarized in Figures 3 and 4. In Figure 3, exact size is plotted for AR(1) errors. Notice that size is excellent for the PS_T^m statistic and is never much greater than .05. This illustrates how effectively the J_T adjustment controls size as the errors go from stationary to a unit root. On the other hand, the exact size of the Chow tests is often inflated well above .05, and the distortions become more severe as α approaches 1. This illustrates how uncertainty regarding the stationarity of the errors can be problematic for the exact size of the Chow tests. When the errors are MA(1), exact size is much more stable and is close to .05 for all the statistics provided that $\theta \geq 0$, as can be seen in Figure 4. For $\theta < 0$, exact size is always below .05. If prewhitening is used as suggested by Andrews and Monahan (1992), some of the size distortion of Chow_T^* for AR(1) errors can be reduced; however, exact size will still be much greater than .05 for α near or equal to 1. This size distortion cannot be completely reduced because Chow_T^* diverges to ∞ as T increases when $\alpha = 1$. Exact size of $\mathrm{Chow}L_T$ can be controlled by taking a conservative approach and using the unit-root critical value as suggested by Vogelsang (1994a). This will be an effective approach if the errors are AR(1) and α is close to 1 but will reduce power when α is not close to 1 or the errors are MA(1). Because the PS_T^m statistic is not a conservative test, it does not penalize power when the errors are stationary.

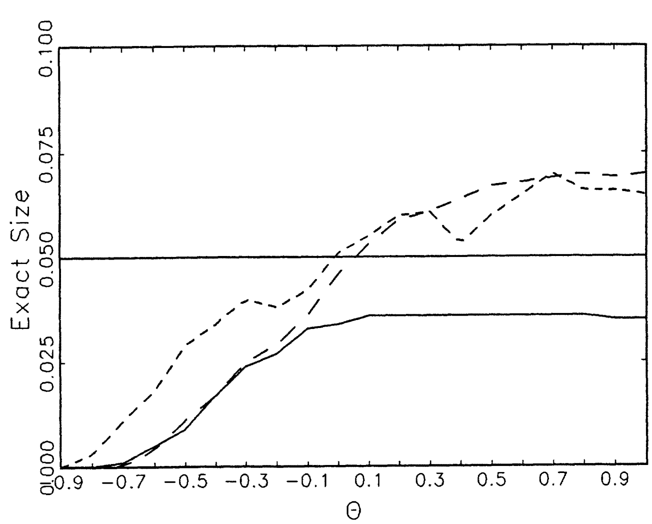

Figure 4. Finite Sample Size MA(1) Errors, Known Break Date: λ_c = .5, 5% Nominal Size, T = 100: ———, PS_T^m, - - -, $ChowL_T$; – –, $Chow_T^*$.

3.2 Finite-Sample Size/Power With ARM(p, q) Errors

In this subsection, finite-sample size and power simulations are reported in the case of ARMA(p, q) errors. This situation is the one most likely encountered in practice. Results are reported for both a known shift date and an unknown shift date. Given the myriad of models that fall into the class of ARMA(p, q) models, results are reported using DGP's based on point estimates from fitting ARMA models to several economic time series. The series are monthly unemployment rates from 1948–1992 for three groups—all civilians, males age 20 and over and females age 20 and over—and the yearly real exchange rate between the United States and Finland from 1900–1987 using gross domestic product deflators as the price indexes. The unemployment-rate series were taken from U.S. Department of Labor, Bureau of Labor Statistics (BLS) (1982), and recent issues of *Employment and Earnings* (BLS 1983–1993). The data sources for the real-exchange-rate series were given by Perron and Vogelsang (1992). ARMA models were fitted to these series and point estimates obtained. Details of the estimation were given by Vogelsang (1994b).

Using the point estimates of the fitted ARMA models, size and power were simulated using 2,000 replications of the DGP: $y_t = \delta DU_t^c + u_t, u_t = \alpha_1 u_{t-1} + \alpha_2 u_{t-2} + \alpha_3 u_{t-3} + v_t + \theta_1 v_{t-1} + \theta_2 v_{t-2}$, where $v_t \sim$ iid $N(0, \sigma_v^2)$. The parameter values are reported in Table 4 for the civilian-unemployment series and Table 5 for the real-exchange-rate series. Notice for the civilian-unemployment-rate series that the sum of the AR parameters is close to 1 in all cases and θ_1 is large and negative in the ARMA(3, 1) parameterization. For the real-exchange-rate series, the sum of the AR parameters is near .5 and θ_1 is positive. So, these point estimates represent ARMA models that are either clearly stationary with a positive MA component or are close to a unit root with a negative MA component. Sample size $T = 540$ was used for the civilian-unemployment-rate simulations, and sample size $T = 100$ was used for the U.S./Finland real-exchange-rate simulations. The shift date was assumed known for the civilian-unemployment simulations but assumed unknown for the real-exchange-rate simulations. For the civilian-unemployment-rate simulations, $\delta = .0, .5, \ldots, 2.5$ and for the U.S./Finland real-exchange-rate simulations, $\delta = .0, .025, \ldots, .1$. The magnitudes of δ were chosen relative to the point estimates of

Table 4. Finite-Sample Size and Size-Adjusted Power, ARMA(p,q) Errors, Known Shift Date

								Probability of rejection		
Series	α_1	α_2	α_3	θ_1	θ_2	σ_v	δ	PS_T^m	$ChowL_T$	$Chow_T^*$
Civilian	1.57	−.36	−.23	−.58	.00	.23	.0	.037	.057	.230
unemployment rate							.5	.131	.169	.149
1948–1992, monthly							1.0	.368	.502	.406
ARMA(3, 1)							1.5	.646	.838	.700
							2.0	.835	.974	.904
							2.5	.942	.997	.981
Civilian	1.80	−.81	.00	−.82	.22	.23	.0	.044	.065	.247
unemployment rate							.5	.116	.154	.139
1948–1992, monthly							1.0	.331	.459	.369
ARMA(2, 2)							1.5	.585	.784	.656
							2.0	.774	.952	.868
							2.5	.898	.992	.964

NOTE: DGP: $y_t = \delta DU_t^c + u_t$, $u_t = \alpha_1 u_{t-1} + \alpha_2 u_{t-2} + \alpha_3 u_{t-3} + v_t + \theta_1 v_{t-1} + \theta_2 v_{t-2}$, $v_t \sim$ iid $N(0, \sigma_v^2)$, 2,000 replications, $T = 540$, $\lambda_c = .58$.

σ_v. For the ChowL_T statistic, $k$max $= 15$ for the civilian-unemployment-rate simulations and $k$max $= 5$ for the real-exchange-rate simulations. As before, the Chow$_T^*$ statistic was implemented using the automatic bandwidth of Andrews (1991) with the quadratic spectral kernel. For the size simulations, 5% asymptotic critical values were used, and power was size adjusted by using finite-sample critical values. Power was size adjusted to make power comparisons easier.

The results are given in Tables 4 and 5 and can be summarized as follows. Exact size of the PS statistics is close to or below .05 in all cases. On the other hand, size of ChowL_T is often above .05, and Chow$_T^*$ is oversized in all cases. Prewhitening for ChowL_T^* was also used, but it resulted in size of nearly 0. Overall, the PS statistics have much better exact size than the Chow statistics.

Now turn to the size-adjusted power results. For the civilian-unemployment-rate simulations, the Chow statistics are generally more powerful than the PS$_T^m$ statistic. The difference in power is minor for small and large shifts but larger for medium-sized shifts. Things are much different for the real-exchange-rate simulations in which the break date is unknown. The PS$_T^m$ statistics are more powerful than the ChowL_T statistics in all cases. For small and medium shifts the PS$_T^m$ statistics are more powerful than the Chow$_T^*$ statistics. For large shifts, however, the Chow$_T^*$ statistics are more powerful than the PS statistics. If prewhitening is used for the Chow$_T^*$ statistic, size-adjusted power was similar to not using prewhitening. But, because prewhitening drives exact size to near 0, unadjusted power of the Chow$_T^*$ statistics was very poor.

Overall, in terms of the size/power trade-off, the PS statistics perform better in finite samples compared to the Chow statistics. Given that the PS statistics do not require a priori information about the dynamic structure of the errors, they should prove very useful in practice. As an illustration, the PS statistics were applied to the unemployment-rate and real-exchange-rate series. The results are reported in Table 6. The PS$_T^m$(1973:12) statistic was used for the unemployment-rate series, and the meanPS$_T$, expPS$_T$, and supPS$_T$ statistics were applied to all the series. As can be seen in the table, the null hypothesis of a stable mean can be rejected in many cases.

Table 5. Finite-Sample Size and Size-Adjusted Power, ARMA(p,q) Errors, Unknown Shift Date

				Probability of rejection								
Series			δ	*MnPS*	*MnCL*	*MnC**	*ExpPS*	*ExpCL*	*ExpC**	*SupPS*	*SupCL*	*SupC**
U.S./Finland real	$T = 100$	$\lambda_c = .42$	.00	.035	.060	.108	.030	.075	.169	.028	.070	.138
exchange rate	$\alpha_1 = .87$	$\alpha_2 = -.33$	.025	.200	.196	.193	.199	.163	.179	.191	.137	.166
1900–1987, yearly	$\alpha_3 = .0$		.05	.554	.538	.544	.550	.487	.522	.546	.424	.487
ARMA(2, 0)	$\theta_1 = .0$	$\theta_2 = .0$	.075	.825	.812	.846	.829	.805	.852	.827	.765	.828
	$\sigma_v = .04$		.10	.944	.930	.976	.947	.952	.982	.947	.935	.977
U.S./Finland real	$T = 100$	$\lambda_c = .42$	.00	.043	.084	.130	.039	.115	.193	.037	.088	.144
exchange rate	$\alpha_1 = .45$	$\alpha_2 = .0$	.025	.152	.142	.161	.145	.122	.132	.146	.114	.129
1900–1987, yearly	$\alpha_3 = .0$		.05	.409	.374	.425	.409	.330	.378	.410	.307	.362
ARMA(1, 1)	$\theta_1 = .43$	$\theta_2 = .0$	.075	.674	.603	.728	.671	.600	.705	.672	.585	.695
	$\sigma_v = .04$		.10	.848	.741	.907	.848	.803	.915	.847	.797	.908
U.S./Finland real	$T = 100$	$\lambda_c = .42$	.00	.037	.065	.117	.033	.084	.178	.032	.068	.141
exchange rate	$\alpha_1 = -.5$	$\alpha_2 = .48$	.025	.176	.173	.179	.170	.149	.155	.172	.122	.149
1900–1987, yearly	$\alpha_3 = -.27$		.05	.487	.463	.487	.486	.423	.448	.486	.359	.426
ARMA(3, 1)	$\theta_1 = .94$	$\theta_2 = .0$	.075	.771	.729	.796	.774	.730	.782	.773	.681	.766
	$\sigma_v = .04$		.10	.913	.882	.948	.919	.916	.960	.917	.891	.954

NOTE: DGP: $y_t = \delta DU_t^c + u_t$, $u_t = \alpha_1 u_{t-1} + \alpha_2 u_{t-2} + \alpha_3 u_{t-3} + v_t + \theta_1 v_{t-1} + \theta_2 v_{t-2}$, $v_t \sim$ iid $N(0, \sigma_v^2)$, 2,000 replications.

Table 6. Empirical Results Using the PS Statistics: P Values

Series	Period	T_b^c	$PS_T^m(T_b^c)$	$MeanPS_T$	$ExpPS_T$	$SupPS_T$	T_b^{sup}
Civilian unemployment rate	1948–1992 (monthly)	1973:12	.01	.07	.07	.07	1973:6
Males age 20+ unemployment rate	1948–1992 (monthly)	1973:12	.03	.12	.12	.15	1976:12
Females age 20+ unemployment rate	1948–1992 (monthly)	1973:12	.02	.12	.12	.11	1972:2
U.S./Finland exchange rate	1900–1987 (yearly)			.04	.04	.04	1946

NOTE: T_b^{sup} is the estimated break date obtained using $SupPS_T$.

4. CONCLUSIONS

By using a simple transformation of a univariate time series, a test statistic for detecting a shift in mean based on partial sums of the data was constructed that is asymptotically invariant to serial correlation in the data. Estimates of serial-correlation nuisance parameters are not required to use the test in practice. Perhaps more importantly, the tests are valid whether the errors are stationary or have a unit root. These PS statistics have good asymptotic power compared to Chow tests, which are optimal when the shift date is known and the errors are iid and Gaussian. In the case of an unknown break, the statistics of Andrews and Ploberger (1994) were used, and again the PS statistics have surprisingly good asymptotic power. If the supremum statistic of Andrews (1993) is used, the PS statistic has greater asymptotic power than the Chow statistic for small to medium breaks. In finite samples, the PS statistics have excellent properties. Exact size is never oversized, and size-adjusted power in many cases is greater than the Chow tests. On the other hand, the Chow tests become substantially oversized as the persistence in the errors increases. In practice, the PS statistics should prove useful because they are easy to compute and do not require a choice of lag length, kernel, or truncation lag and offer a simple way to test for a stable mean.

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