Journal of Statistical Planning and Inference 9 (1984) 389–396
North-Holland

NONPARAMETRIC STATISTICAL PROCEDURES FOR THE CHANGEPOINT PROBLEM

Douglas A. WOLFE

Department of Statistics, The Ohio State University, Columbus, OH 43210, USA

Edna SCHECHTMAN

Faculty of Agriculture, Hebrew University of Jerusalem, Rehovot, Israel

Received 22 September 1983

Abstract: Let $X_1, \ldots, X_{r-1}, X_r, X_{r+1}, \ldots, X_n$ be independent, continuous random variables such that X_i, $i=1,\ldots,r$, has distribution function $F(x)$, and X_i, $i=r+1,\ldots,n$, has distribution function $F(x-\Delta)$, with $-\infty<\Delta<\infty$. When the integer r is unknown, this is referred to as a changepoint problem with at most one change. The unknown parameter Δ represents the magnitude of the change and r is called the changepoint. In this paper we present a general review discussion of several nonparametric approaches for making inferences about r and Δ.

AMS Subject Classification: Primary 62G10; Secondary 62G05.

Key words: At most one changepoint; Mann–Whitney statistics; Monte Carlo study.

1. Introduction

Suppose we have a random process that generates independent observations indexed by some non-random factor such as time. As these observations are obtained over varying values of the non-random factor, we suspect that there has been at least one change in our random process during the data collection. It then is of interest to elicit information from the observations concerning the possibility of such a change (or changes) in our process. Such information would include evidence to answer questions like the following:

(1) Has there been a change (or changes) in our process as the non-random factor varied?

(2) If there has been at least one change in our process, at what value (or values) of the non-random factor did it (they) occur?

(3) What type are the changes that have occurred in the process? Of what magnitude or importance are they?

Some specific statistical problems that have previously been considered to adequately fit this changepoint description include: (i) observing industrial output over

0378-3758/84/$3.00 © 1984, Elsevier Science Publishers B.V. (North-Holland)

time, (ii) variation (over time) of share prices on the major stock exchanges, (iii) times between aircraft arrivals at large airports, (iv) literary style in the Lindisfarne Scribes' data (see Pettitt (1979), for example), and (v) magnitude of annual flow in the Nile River.

In almost all of the early literature on this general problem the investigators have concentrated on models that deal with possible process changes that are location or shift in nature. We now formulate such a model. Let $X_1, X_2, \ldots, X_{r-1}, X_r, X_{r+1}, \ldots, X_n$ be independent, continuous random variables such that X_i, $i=1,\ldots,r$, has distribution function $F(x)$, and X_i, $i=r+1,\ldots,n$, has distribution function $F(x-\Delta)$, $-\infty<\Delta<\infty$. When the integer r is known, this is, of course, nothing more than the standard two-sample problem with unknown shift parameter Δ. However, the more complicated model where the integer r is also unknown is referred to as a changepoint problem with at most one change. (Models allowing for more than one change in the process could be similarly constructed. However, we will concentrate in this paper on the AMOC (at most one change) setting.) The parameter Δ represents the unknown magnitude of the postulated change and r is called the changepoint.

2. Literature review – Nonparametric procedures

The basic AMOC problem seems to have been first considered by Page (1954, 1955) for the setting of continuous inspection schemes. Among other things he considered testing the null hypothesis of no change, that is, $H_0: \Delta=0$, against either one- or two-sided alternatives, under the assumption that the initial mean, say θ_0, of the process (i.e., the mean of X_1) is known a priori. Letting $S_0 \equiv 0$ and $S_k=\sum_{j=1}^{k} V_j$, $k=1,\ldots,n$,

$$V_j=\begin{cases} a & \text{if } X_j \geq \theta_0, \\ -b & \text{if } X_j<\theta_0, \end{cases} \tag{2.1}$$

where $a>0$, $b>0$ are constants, possibly dependent on $F(\cdot)$, chosen so that $E_{\theta_0}(V_j)=0$, $j=1,\ldots,n$, Page's decision rule rejects $H_0: \Delta=0$ in favor of the alternative of one change and $\Delta>0$ if

$$T=\underset{0\leq k\leq n}{\text{maximum}}\left\{S_k-\underset{0\leq j<k}{\text{minimum}}(S_j)\right\} \tag{2.2}$$

is too large. Special emphasis was given to the choice of $a=b=1$, in which case the V_j's are simply the signs associated with the $(X_j-\theta_0)$'s, where we identify a positive sign with a zero. The resulting changepoint test is nonparametric distribution-free over the class of continuous variables.

It was not until thirteen years later that G.K. Bhattacharyya and Johnson (1968) again approached the changepoint problem from a nonparametric viewpoint. For the case where the initial level θ_0 is unknown they propose rejecting $H_0: \Delta=0$ in favor of $H_1: \Delta>0$ (actually they formulated their model in terms of stochastically increasing variables) for large values of

$$J=\sum_{i=1}^{n} L_i E[-f'(V^{(R_i)})/f(V^{(R_i)})] \tag{2.3}$$

where the $L_i=\sum_{t=1}^{i} l_t$ are cumulative weights with $l_1=0$, $\boldsymbol{R}=(R_1,\ldots,R_n)$ is the vector of ranks of $X_1,\ldots,X_n$, and $V^{(1)}<\cdots<V^{(n)}$ are the order statistics for a random sample of size n from a continuous population with c.d.f. $F(\cdot)$ and density $f(\cdot)$. Thus the G.K. Bhattacharyya–Johnson statistic has the general appearance of an optimal linear rank statistic. (For the less practical case where the initial value θ_0 is known, G.K. Bhattacharyya and Johnson also propose a test based on a statistic having the general form of an optimal linear signed rank statistic.)

We note that the weight l_t can be interpreted (from a Bayesian point of view) as the prior probability that X_t is the initial shifted variable. Thus, for example, with uniform weights $l_t \equiv (n-1)^{-1}$, $t=2,\ldots,n$, corresponding to an uninformative prior, we have

$$L_i=\sum_{t=1}^{i} l_t=(i-1)/(n-1).$$

For these weights, the statistic J is equivalent to

$$J'=\sum_{i=1}^{n}(i-1)E[-f'(V^{(R_i)})/f(V^{(R_i)})]. \tag{2.4}$$

Two particular cases of this general statistic J with uniform weights have received the most attention. They are:

$$J_1=\sum_{i=1}^{n}(i-1)\left[\psi\left\{X_i-\underset{1\leq j\leq n}{\text{median}}(X_j)\right\}\right], \tag{2.5}$$

where $\psi(t)=1, 0$, as $t \geq, <0$, and

$$J_2=\sum_{i=1}^{n}\sum_{j=1}^{n}(i-1)\psi(X_i-X_j). \tag{2.6}$$

Thus the statistics J_1 and J_2 are similar to linear rank statistics with median scores and Wilcoxon scores, respectively. We note (for later reference) that J_1 and J_2 can be written as

$$J_1=\sum_{k=1}^{n-1} M_{k,n-k} \quad \text{and} \quad J_2=\sum_{k=1}^{n-1} U_{k,n-k}, \tag{2.7}$$

where

$$M_{k,n-k}=\sum_{i=k+1}^{n}\psi\left\{X_i-\underset{1\leq j\leq n}{\text{median}}(X_j)\right\} \tag{2.8}$$

and

$$U_{k,n-k}=\sum_{i=k+1}^{n}\sum_{j=1}^{k}\psi(X_i-X_j). \tag{2.9}$$

time, (ii) variation (over time) of share prices on the major stock exchanges, (iii) times between aircraft arrivals at large airports, (iv) literary style in the Lindisfarne Scribes' data (see Pettitt (1979), for example), and (v) magnitude of annual flow in the Nile River.

In almost all of the early literature on this general problem the investigators have concentrated on models that deal with possible process changes that are location or shift in nature. We now formulate such a model. Let $X_1, X_2, \ldots, X_{r-1}, X_r, X_{r+1}, \ldots, X_n$ be independent, continuous random variables such that X_i, $i=1,\ldots,r$, has distribution function $F(x)$, and X_i, $i=r+1,\ldots,n$, has distribution function $F(x-\Delta)$, $-\infty<\Delta<\infty$. When the integer r is known, this is, of course, nothing more than the standard two-sample problem with unknown shift parameter Δ. However, the more complicated model where the integer r is also unknown is referred to as a changepoint problem with at most one change. (Models allowing for more than one change in the process could be similarly constructed. However, we will concentrate in this paper on the AMOC (at most one change) setting.) The parameter Δ represents the unknown magnitude of the postulated change and r is called the changepoint.

2. Literature review – Nonparametric procedures

The basic AMOC problem seems to have been first considered by Page (1954, 1955) for the setting of continuous inspection schemes. Among other things he considered testing the null hypothesis of no change, that is, $H_0: \Delta=0$, against either one- or two-sided alternatives, under the assumption that the initial mean, say θ_0, of the process (i.e., the mean of X_1) is known a priori. Letting $S_0 \equiv 0$ and $S_k=\sum_{j=1}^{k} V_j$, $k=1,\ldots,n$,

$$V_j=\begin{cases} a & \text{if } X_j \geq \theta_0, \\ -b & \text{if } X_j<\theta_0, \end{cases} \tag{2.1}$$

where $a>0$, $b>0$ are constants, possibly dependent on $F(\cdot)$, chosen so that $E_{\theta_0}(V_j)=0$, $j=1,\ldots,n$, Page's decision rule rejects $H_0: \Delta=0$ in favor of the alternative of one change and $\Delta>0$ if

$$T=\underset{0\leq k\leq n}{\text{maximum}}\left\{S_k-\underset{0\leq j<k}{\text{minimum}}(S_j)\right\} \tag{2.2}$$

is too large. Special emphasis was given to the choice of $a=b=1$, in which case the V_j's are simply the signs associated with the $(X_j-\theta_0)$'s, where we identify a positive sign with a zero. The resulting changepoint test is nonparametric distribution-free over the class of continuous variables.

It was not until thirteen years later that G.K. Bhattacharyya and Johnson (1968) again approached the changepoint problem from a nonparametric viewpoint. For the case where the initial level θ_0 is unknown they propose rejecting $H_0: \Delta=0$ in favor of $H_1: \Delta>0$ (actually they formulated their model in terms of stochastically increasing variables) for large values of

$$J=\sum_{i=1}^{n} L_i E[-f'(V^{(R_i)})/f(V^{(R_i)})] \tag{2.3}$$

where the $L_i=\sum_{t=1}^{i} l_t$ are cumulative weights with $l_1=0$, $\boldsymbol{R}=(R_1,\ldots,R_n)$ is the vector of ranks of $X_1,\ldots,X_n$, and $V^{(1)}<\cdots<V^{(n)}$ are the order statistics for a random sample of size n from a continuous population with c.d.f. $F(\cdot)$ and density $f(\cdot)$. Thus the G.K. Bhattacharyya–Johnson statistic has the general appearance of an optimal linear rank statistic. (For the less practical case where the initial value θ_0 is known, G.K. Bhattacharyya and Johnson also propose a test based on a statistic having the general form of an optimal linear signed rank statistic.)

We note that the weight l_t can be interpreted (from a Bayesian point of view) as the prior probability that X_t is the initial shifted variable. Thus, for example, with uniform weights $l_t \equiv (n-1)^{-1}$, $t=2,\ldots,n$, corresponding to an uninformative prior, we have

$$L_i=\sum_{t=1}^{i} l_t=(i-1)/(n-1).$$

For these weights, the statistic J is equivalent to

$$J'=\sum_{i=1}^{n}(i-1)E[-f'(V^{(R_i)})/f(V^{(R_i)})]. \tag{2.4}$$

Two particular cases of this general statistic J with uniform weights have received the most attention. They are:

$$J_1=\sum_{i=1}^{n}(i-1)\left[\psi\left\{X_i-\underset{1\leq j\leq n}{\text{median}}(X_j)\right\}\right], \tag{2.5}$$

where $\psi(t)=1, 0$, as $t \geq, <0$, and

$$J_2=\sum_{i=1}^{n}\sum_{j=1}^{n}(i-1)\psi(X_i-X_j). \tag{2.6}$$

Thus the statistics J_1 and J_2 are similar to linear rank statistics with median scores and Wilcoxon scores, respectively. We note (for later reference) that J_1 and J_2 can be written as

$$J_1=\sum_{k=1}^{n-1} M_{k,n-k} \quad \text{and} \quad J_2=\sum_{k=1}^{n-1} U_{k,n-k}, \tag{2.7}$$

where

$$M_{k,n-k}=\sum_{i=k+1}^{n}\psi\left\{X_i-\underset{1\leq j\leq n}{\text{median}}(X_j)\right\} \tag{2.8}$$

and

$$U_{k,n-k}=\sum_{i=k+1}^{n}\sum_{j=1}^{k}\psi(X_i-X_j). \tag{2.9}$$

We also note that

$$M_{k,n-k} = \text{number of observations among the last } (n-k) \text{ that exceed the median of all } n \text{ observations}$$

is simply a two-sample median statistic applied to the total of n observations viewed as an initial sample of k observations and a second sample of $n-k$ observations. In the same vein, the statistic $U_{k,n-k}$ is just a two-sample Mann–Whitney statistic applied to the same breakdown of the data into two subsamples of sizes k and $n-k$.

A. Sen and Srivastava (1975) also mention (without developing any properties) two additional nonparametric tests as analogues to some parametric likelihood ratio procedures for one-sided alternatives and the case where both the initial level θ_0 and variance σ^2 are unknown. They suggest rejecting $H_0: \Delta = 0$ in favor of $H_1: \Delta > 0$ for large values of

$$D_1 = \underset{1 \le k \le n-1}{\text{maximum}} \{[M_{k,n-k} - E_0(M_{k,n-k})]/[\text{Var}_0(M_{k,n-k})]^{1/2}\}, \tag{2.10}$$

$$\begin{aligned} D_2 &= \underset{1 \le k \le n-1}{\text{maximum}} \{[U_{k,n-k} - E_0(U_{k,n-k})]/[\text{Var}_0(U_{k,n-k})]^{1/2}\} \\ &= \underset{1 \le k \le n-1}{\text{maximum}} \{[U_{k,n-k} - (k(n-k))/2]/[k(n-k)(n+1)/12]^{1/2}\}, \end{aligned} \tag{2.11}$$

where $M_{k,n-k}$ and $U_{k,n-k}$ are as defined in equations (2.8) and (2.9), respectively, and $E_0(M_{k,n-k})$ and $\text{Var}_0(M_{k,n-k})$ are the null mean and variance, respectively, of the statistic $M_{k,n-k}$.

Pettitt (1979) considered both one- and two-sided alternatives to $H_0: \Delta = 0$ using statistics quite similar to D_2 (2.11). For the one-sided alternative $H_1: \Delta > 0$ he proposed rejecting H_0 for large values of

$$K_1 = \underset{1 \le k \le n-1}{\text{minimum}} \left\{ \sum_{i=1}^{k} \sum_{j=k+1}^{n} Q_{ij} \right\}, \tag{2.12}$$

where

$$Q_{ij} = \text{sign}(X_i - X_j) = \begin{cases} 1 & \text{if } X_i > X_j, \\ 0 & \text{if } X_i = X_j, \\ -1 & \text{if } X_i < X_j. \end{cases} \tag{2.13}$$

We note that K_1 can be written as

$$\begin{aligned} K_1 &= \underset{1 \le k \le n-1}{\text{maximum}} \left\{ -\sum_{i=1}^{k} \sum_{j=k+1}^{n} Q_{ij} \right\} \\ &= \underset{1 \le k \le n-1}{\text{maximum}} \{2U_{k,n-k} - k(n-k)\} \\ &= 2 \underset{1 \le k \le n-1}{\text{maximum}} \left\{ U_{k,n-k} - \frac{k(n-k)}{2} \right\}. \end{aligned} \tag{2.14}$$

Thus we see that K_1 and D_2 (2.11) are similar in structure, but they differ in the weightings assigned to the various terms $U_{k,n-k} - [k(n-k)/2]$ leading to the maximums. We see that D_2 weights these differences by

$$[\text{Var}_0(U_{k,n-k})]^{-1/2} = [k(n-k)(n+1)/12]^{-1/2}$$

while K_1 employs equal weightings. Schechtman and Wolfe (1981) studied the relative merits of these two weighting schemes as they developed properties of the one-sided test based on D_2. We return to a discussion of this consideration later in the paper.

Finally, Pettitt (1979) proposes rejecting $H_0: \Delta = 0$ in favor of the two-sided alternative $H_1: \Delta \neq 0$ for large values of

$$\begin{aligned} K_2 &= \underset{1 \le k \le n-1}{\text{maximum}} \left| \sum_{i=1}^{k} \sum_{j=k+1}^{n} Q_{ij} \right| \\ &= 2 \underset{1 \le k \le n-1}{\text{maximum}} \left\{ \left| U_{k,n-k} - \frac{k(n-k)}{2} \right| \right\}. \end{aligned} \tag{2.15}$$

Schechtman and Wolfe (1981) propose and study the two-sided analogue of K_2 based on the unequal weightings as utilized in the one-sided statistic D_2 (2.11). They suggest rejecting $H_0: \Delta = 0$ in favor of $H_1: \Delta \neq 0$ for large values of

$$D_3 = \underset{1 \le k \le n-1}{\text{maximum}} \left\{ \left| U_{k,n-k} - \frac{k(n-k)}{2} \right| \Big/ [k(n-k)(n+1)/12]^{1/2} \right\}. \tag{2.16}$$

Some of the asymptotic properties of the changepoint procedures based on K_1 (2.12) and K_2 (2.15) are obtained in Pettitt (1979) and P.K. Sen (1978). The large sample properties of the tests associated with D_2 (2.11) and D_3 (2.16) are discussed in P.K. Sen (1978) and Schechtman and Wolfe (1981).

Other nonparametric approaches to testing hypotheses about a changepoint include an asymptotically distribution-free procedure proposed by P.K. Sen (1977) and based on aligned rank statistics. Sen (1980) has also extended these ideas to develop tests based on aligned rank order statistics for the problem of a possible change in the regression slope occurring at an unknown time point. In addition, P.K. Bhattacharya and Frierson (1981) recently used Parent's (1965) idea of sequential ranks to construct a nonparametric control chart that is useful for detecting a changepoint when the data are collected sequentially. (This is a somewhat natural method for obtaining the observations in certain changepoint settings.) Other related asymptotic results for sequential rankings have been obtained by Lombard (1981).

3. Monte Carlo comparisons of nonparametric tests for a changepoint

Certain of the necessary small sample-size null distribution tables for D_2 (2.11) and D_3 (2.16) are provided in Schechtman (1982). In addition Schechtman (1980) presents the results of a substantial Monte Carlo study of the relative power properties of some of the nonparametric test procedures presented in Section 2, as well as

some parametric competitors. We discuss here a few of the findings from the nonparametric portion of that investigation.

We considered the single sample size $n=20$ and alternatives to $H_0: \Delta=0$ of the form (r, Δ), where r is the changepoint and Δ is the size of the shift. Five different underlying distributions, namely, uniform, normal, exponential, double exponential, and Cauchy, were studied. For each of these distributions we looked at $r=1$, 5 and 10 in conjunction with each of four values of Δ corresponding to solving the equation $P(X_{20}>X_1)=0.6$, 0.7, 0.8 and 0.9, where X_1 and X_{20} have c.d.f.'s $F(x)$ and $F(x-\Delta)$, respectively. (Since the power functions of all the tests considered in the Monte Carlo study are, for a fixed $F(\cdot)$ and fixed value of Δ, symmetric in the changepoint r about $r=n/2=10$, the results of the Monte Carlo simulations for $r=1$ and 5 apply equally well to $r=19$ and 15, respectively.)

For each power comparison, 5000 samples of size 20 each were generated, so as to guarantee that the resulting power estimates would have errors no greater than 0.018 with approximately 99% confidence. For the one-sided alternative $\Delta>0$, we considered the tests based on D_1 (2.10), D_2 (2.11), J_1 (2.5), J_2 (2.6), and K_1 (2.12). For the two-sided alternative $\Delta\neq 0$, we included the statistics D_3 (2.16) and K_2 (2.15).

Table 1
Monte Carlo power comparisons, one-sided alternative

	(i) $r=5$					
Distribution	$P(X_{20}>X_1)$	D_1	D_2	J_1	J_2	K_1
Double exponential	0.7	0.564	0.638	0.476	0.545	0.595
	0.8	0.825	0.905	0.617	0.783	0.881
Normal	0.7	0.205	0.267	0.205	0.269	0.258
	0.8	0.406	0.517	0.344	0.463	0.490
Cauchy	0.7	0.292	0.274	0.265	0.254	0.271
	0.8	0.532	0.517	0.410	0.439	0.494
Exponential	0.7	0.155	0.295	0.158	0.253	0.264
	0.8	0.294	0.523	0.261	0.441	0.487
	(ii) $r=10$					
Distribution	$P(X_{20}>X_1)$	D_1	D_2	J_1	J_2	K_1
Double exponential	0.7	0.734	0.780	0.784	0.797	0.825
	0.8	0.962	0.977	0.962	0.973	0.987
Normal	0.7	0.266	0.354	0.313	0.400	0.415
	0.8	0.517	0.656	0.564	0.698	0.733
Cauchy	0.7	0.389	0.346	0.431	0.367	0.403
	0.8	0.739	0.640	0.754	0.659	0.708
Exponential	0.7	0.233	0.352	0.271	0.369	0.407
	0.8	0.506	0.652	0.549	0.671	0.721

Table 2
Monte Carlo power comparisons, Two-sided alternative

		$r=5$		$r=10$	
Distribution	$P(X_{20}>X_1)$	D_3	K_2	D_3	K_2
Double exponential	0.7	0.490	0.454	0.649	0.736
	0.8	0.829	0.793	0.944	0.968
Normal	0.7	0.163	0.160	0.240	0.297
	0.8	0.362	0.340	0.532	0.613
Cauchy	0.7	0.179	0.180	0.243	0.300
	0.8	0.379	0.360	0.512	0.605
Exponential	0.7	0.203	0.183	0.232	0.297
	0.8	0.409	0.367	0.509	0.607

Some of the Monte Carlo power estimates for the one- and two-sided tests with nominal level $\alpha=0.05$ are given in Tables 1 and 2, respectively. (The *relative* values of the estimated powers for samples from uniform distributions were similar to those shown for underlying normal distributions. For all distributions studied, the results for nominal levels $\alpha=0.01$ and 0.10 were much like those presented here for $\alpha=0.05$.)

Two general conclusions can be drawn from this Monte Carlo study. First, for any amount of shift Δ and any of the five distributions the estimated powers for all of the test procedures were largest at $r=n/2=10$ and smallest at $r=5$. Of course, this is not too surprising since a shift occurring near the middle of a sequence of observations should be much easier to detect than one occurring at the beginning or the end of the sequence.

Second, for $r=5$ the test procedures based on D_2 and D_3 are most often superior among the studied nonparametric procedures for the one- and two-sided alternatives, respectively. This advantage appears to be generally an increasing function of $P(X_{20}>X_1)$. On the other hand, for $r=10$ the test procedures based on K_1 and K_2, as proposed by Pettitt (1979), are superior among all competing nonparametric procedures. In general, tests associated with the linear rank type statistics J_1 and J_2 did not fare as well for one-sided alternatives as did their analogues based on maximums.

5. Other problems for the changepoint setting

While it is clear that there has been considerable activity in the area of hypothesis tests for a single changepoint, very little work has appeared on nonparametric point or interval estimation for the unknown parameters r and Δ. Pettitt (1980) discusses the point estimation of a changepoint r and Schechtman (1983) develops a conserva-

tive nonparametric distribution-free confidence bound for the magnitude of the shift Δ. Much remains yet to be done, however, including addressing some of the following topics:

(a) confidence intervals and bounds for the changepoint r,

(b) relative properties of naturally competing nonparametric point estimators for r and Δ,

(c) nonparametric inference procedures for data involving possibly more than one changepoint,

(d) nonparametric comparisons of potential changepoints in several independent sequences of variables,

(e) an investigation into the optimal way to assign weights to the differences $[U_{k,n-k}-k(n-k)/2]$ as used in both D_2 (2.11) and K_1 (2.14).

References

Bhattacharya, P.K. and D. Frierson, Jr. (1981). A nonparametric control chart for detecting small disorders. *Ann. Statist.* 9, 544–554.

Bhattacharyya, G.K. and R. Johnson (1968). Nonparametric tests for shifts at an unknown time point. *Ann. Math. Statist.* 39, 1731–1743.

Lombard, F. (1981). An invariance principle for sequential nonparametric test statistics under contiguous alternatives. *South African Statist. J.* 15, 129–152.

Page, E.S. (1954). Continuous inspection schemes. *Biometrika* 41, 100–115.

Page, E.S. (1955). A test for a change in a parameter occurring at an unknown point. *Biometrika* 42, 523–526.

Parent, E.A., Jr. (1965). Sequential ranking procedures. Doctoral Dissertation, Stanford University.

Pettitt, A.N. (1979). A non-parametric approach to the changepoint problem. *Appl. Statist.* 28, 126–135.

Pettitt, A.N. (1980). Estimating a changepoint using nonparametric type statistics. *J. Statist. Comput. Simul.* 11, 261–274.

Sen, A. and M.S. Srivastava (1975). On tests for detecting changes in mean. *Ann. Statist.* 3, 98–108.

Sen, P.K. (1977). Tied-down Wiener process approximations for aligned rank order processes and some applications. *Ann. Statist.* 5, 1107–1123.

Sen, P.K. (1978). Invariance principles for linear rank statistics revisited. *Sankhyā Ser. A.* 40, 215–236.

Sen, P.K. (1980). Asymptotic theory of some tests for a possible change in the regression slope occurring at an unknown time point. *Zeit. Wahrsch. Verw. Geb.* 52, 203–218.

Schechtman, E. (1980). A nonparametric test for the changepoint problem. Doctoral Dissertation, The Ohio State University.

Schechtman, E. (1982). A nonparametric test for detecting changes in location. *Comm. Statist. – Theor. Meth. A* 11(13), 1475–1482.

Schechtman, E. (1983). A conservative nonparametric distribution-free confidence bound for the shift in the changepoint problem. *Comm. Statist.-Theor. Meth. A* 12(21), 2455–2464.

Schechtman, E. and D.A. Wolfe (1981). Distribution-free tests for the changepoint problem. Technical Report, The Ohio State University.

Journal of Statistical Planning and Inference 9 (1984) 397–399
North-Holland

AUTHOR INDEX*

Volume 9 (1984)

Ahmad, I.A. and P.-E. Lin, Fitting a multiple regression function (2) 163–176

Anderson, D., *see* J. Srivastava (2) 229–252

Arnold, B.C., A. Becker, U. Gather and H. Zahedi, On the Markov property of order statistics (2) 147–154

Athreya, K.B., M. Ghosh, L.Y. Low and P.K. Sen, Laws of large numbers for bootstrapped U-statistics (2) 185–194

Aubuchon, J.C. and P. Hettmansperger, A note on the estimation of the integral of $f^2(x)$ (3) 321–331

Becker, A., *see* B.C. Arnold (2) 147–154

Bell, C.B., Inference for goodness-of-fit problems with nuisance parameters (Applications to signal detection) (3) 273–284

Bhapkar, V.P., On nonparametric profile analysis of several multivariate samples (3) 285–296

Castillo, A., *see* F.J. O'Reilly (2) 195–198

Ceranka, B. and M. Kozłowska, Some methods for constructing C-designs (2) 253–258

Chandrasekar, B. and B.K. Kale, Unbiased statistical estimation functions for parameters in presence of nuisance parameters (1) 45– 54

Chen, T.T., Y. Hochberg and A. Tenenbein, Analysis of multivariate categorical data with misclassification errors by triple sampling schemes (2) 177–184

Cohen, A. and H.B. Sackrowitz, Testing hypotheses about the common mean of normal distributions (2) 207–227

Doksum, K.A. and S. Nabeya, Estimation in proportional hazard and log-linear models (3) 297–303

Fligner, M.A., *see* S.W. Rust (3) 345–353

Gather, U., *see* B.C. Arnold (2) 147–154

Genizi, A., E. Lahav and J. Putter, Estimation of the variance between units of given size from experimental data (1) 83–101

Ghosh, M., *see* K.B. Athreya (2) 185–194

Govindarajulu, Z., Stopping times of one-sample rank order sequential probability ratio tests (3) 305–320

Gupta, R.C., Estimating the probability of winning (losing) in a gambler's ruin problem with applications (1) 55– 62

* The issue number is given in front of page numbers.

0378-3758/84/$3.00 © 1984, Elsevier Science Publishers B.V. (North-Holland)